



OPTIMAL PLACEMENT OF PHASOR MEASUREMENT UNIT FOR STATE ESTIMATION
IMPROVEMENT

M Sc. THESIS

BY

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HAWASSA UNIVERSITY,

HAWASSA, ETHIOPIA

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OPTIMAL PLACEMENT OF PHASOR MEASUREMENT UNIT FOR STATE ESTIMATION
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ABSTRACT— Installation of modern sensing devices is an integral requirement in the transition of a conventional power grid to smart grid. Phasor measurements synchronized through GPS presents one such alternative to enhance the monitoring and control feature in the power grid. Due to the high financial implications, these units should be deployed optimally in the grid. The methods reported in the literature for optimal placement usually provide multiple solutions. This thesis presents criteria of optimal solution for state estimation enhancement. The criterion is based on sensitivity analysis and analyzing way of increasing the rank of the gain matrix to install the phasor measurement unit on improving the state estimation results. A reduced form integer linear Programming algorithm is to be formulated according the criterion given for optimal placement problem for improved state estimation. The reason behind this is that all the downstream applications of contingency analysis; steady-state, voltage, and dynamic as well as the optimal power flow are dependent on the accuracy of the state estimation results. Because of its key role in energy management system, state estimation is set to be a vital function in the development of smart grid. For the Process Power education tool (PET) and Matlab are used and validated on testing on IEEE 14-bus system as a base case and a southern region 21-bus system as a study case.

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LIST OF SYMBOLS

Acronyms

CRO	Chemical Reaction Optimization
DE	Differential Evolution
DFT	Discrete Fourier Transfer
FOA	Fruit fly Optimization Algorithm
FRACSEC	Fraction of Second
GA	Genetic Algorithm
GPS	Global Positioning Satellite
HMI	Human Machine Interface
ILP	Integer Linear Programing
ILS	Iterated Local search
IP	Internet Protocol
IQP	Integer Quadratic Programing
MILP	Mixed Integer Linear Programing
OPP	Optimal Placement Problem
OST	Observable Spanning Tree
PDC	Phasor Data Concentrator
PET	Power Education Tools
PMU	Phasor Measurement Unit
PPS	Pulse Per second
PSAT	Power System Analysis Toolbox
PSO	Particle Swarm Optimization
RTU	Remote Terminal Unit
SA	Simulated Annealing
SCADA	Supervisory Control And Data accusation

SE	State Estimation
SOC	Second Of Century
SORI	System Observability Redundancy Index
SR	Southern Region
TOA	Traditional Observability Analysis
TS	Tabu Search
TVE	Total Vector Error
UTC	universal Time Coordinate
WAMS	Wide Area Monitoring System
WLS	Weighted Least Square

1. INTRODUCTION

1.1. BACKGROUND

Phase angles of voltage phasors of power network buses have always been of special interest to power system engineers. It is well known that active (real) power flow in a power line is very nearly proportional to the sine of the angle difference between voltages at the two terminals of the line. As many of the planning and operational considerations in a power network are directly concerned with the flow of real power, measuring angle differences across transmission has been of concern for many years. The earliest modern application involves direct measurement of phase angle differences. The next available positive-going zero-crossing of a phase voltage was used to estimate the local phase angle with respect to the time reference. Using the difference of measured angles on a common reference at two locations, the phase angle difference between voltages at two buses was established. Measurement accuracies achieved in these systems were of the order of 40 μ s. Single-phase voltage angles were measured and, of course no attempt was made to measure the prevailing voltage phasor magnitude. Neither was any account taken of the harmonics contained in the voltage waveform. These methods of measuring phase angle differences are not suitable for generalization for wide-area phasor measurement systems, and remain one-of-a-kind systems which are no longer in use. The modern era of phasor measurement technology has its genesis in research conducted on computer relaying of transmission lines. Early work on transmission line relaying with microprocessor-based relays showed that the available computer power in those days (1970s) was barely sufficient to manage the calculations needed to perform all the transmission line relaying functions. A significant portion of the computations was dedicated to solving six fault loop equations at each sample time in order to determine if any one of the ten types of faults possible on a three-phase transmission line are present. The search for methods which would eliminate the need to solve the six equations finally yielded a new relaying technique which was based on symmetrical component analysis of line voltages and currents. Using symmetrical components, and certain quantities derived from them, it was possible to perform all fault calculations with a single equation. This new symmetrical component-based algorithm for protecting a transmission line was described. As a part of this theory, efficient algorithms for computing symmetrical components of three-phase voltages and currents were described, and the calculation of positive-sequence voltages

and currents gave an impetus for the development of modern phasor measurement systems. It was soon recognized that the positive sequence measurement (a part of the symmetrical component calculation) is of great value in its own right. Positive sequence voltages of a network constitute the state vector of a power system, and it is of fundamental importance in all of power system analysis. This invention can be viewed as the starting point of modern synchronized phasor measurement technology. The first prototypes of the modern “phasor measurement units” (PMUs) using GPS were built at Virginia Tech in early 1980s, and two of these prototypes are shown in Fig. 1.1.

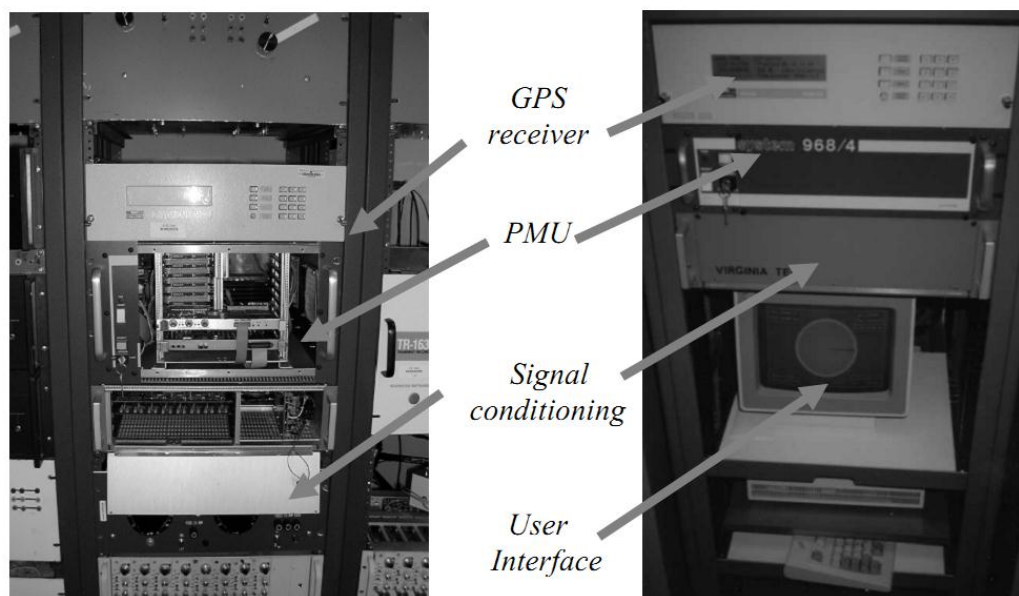


Fig. 1.1 The First phasor measurement units (PMUs)

Fig. 1.1 shown the first phasor measurement units (PMUs) built at the Power Systems Research Laboratory at Virginia Tech. The GPS receiver clock was external to the PMU, and with the small number of GPS satellites deployed at that time, the clock had to be equipped with a precision internal oscillator which maintained accurate time in the absence of visible satellites.

The prototype PMU units built at Virginia Tech were deployed at a few substations of the Bonneville Power Administration, the American Electric Power Service Corporation, and the

New York Power Authority. The first commercial manufacture of PMUs with Virginia Tech collaboration was started by Macrodyne in 1991. At present, a number of manufacturers offer PMUs as a commercial product, and deployment of PMUs on power systems is being carried out in earnest in many countries around the world. IEEE published a standard in 1991 governing the format of data files created and transmitted by the PMU. A revised version of the standard was issued in 2005. Concurrently with the development of PMUs as measurement tools, research was ongoing on applications of the measurements provided by the PMUs. It can be said now that finally the technology of synchronized phasor measurements has come of age, and most modern power systems around the world are in the process of installing wide-area measurement systems consisting of the phasor measurement units. For example a Croatia company named WAMSTER is installing PMU technology in many countries, with precise time base (1micro sec) with 50/60 samples per second allowing absolute phasor angles. Some of the installed PMUs are:-

i.e. .Installations 2010-2014 by WAMSTER

Year of installation	Project Name
2010-2011, 9 PMUs for University of Rijeka	CARWAMS, Croatia
2011-2012, 6 PMUs for HEP (Croatian TPO)	ZAGREB Ring, Croatia
2012, 4 PMUs for HEP (Croatian TPO)	HEP PP Sjever, Croatia
2012-2013, 6 PMUs for Energynautics Germany	UMEME 24/7, Kenya
	Infrastructure Agency Facility (NIAF) and Bauchi State University, Nigeria,NIAF Abuja demonstration project, Nigeria
2013, Bridging SCADA demonstration for KU Leuven (Belgium), Nigeria	
2014 (ongoing), 3 PMUs for Energynautics Germany	Renewable Energy Systems assessment, Seychelles
2014 (ongoing), 20 PMUs for TRANSCO/ADWEA United Arab Emirates	Dynamic React. Compensation Strategy project, UAE

The current prototypes of the modern “phasor measurement units” (PMUs) that were being installed this days are shown in Fig. 1.2

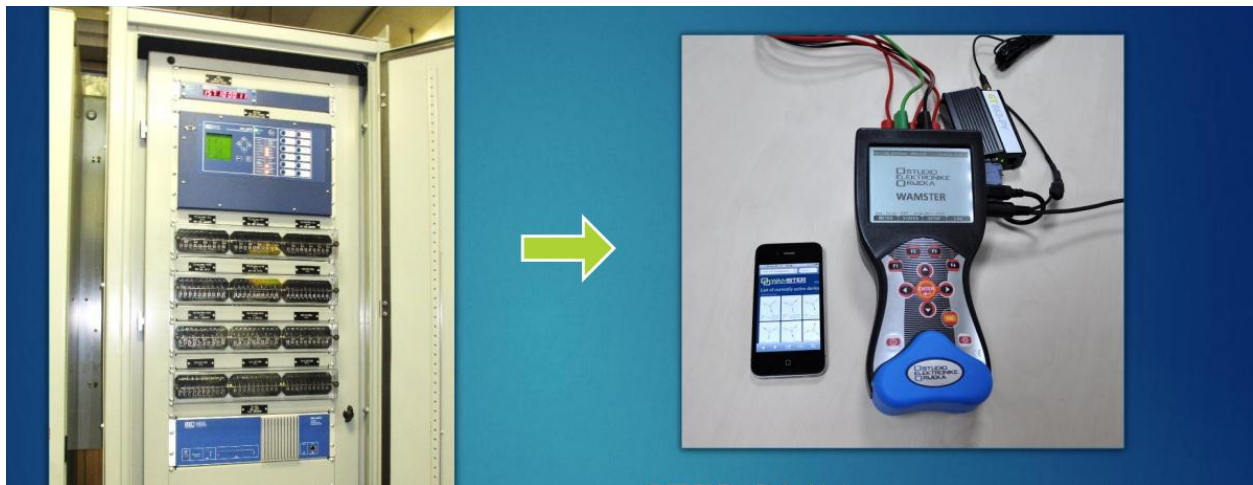


Fig.1.2 Mainstream PMU Switchboard and PMU As a Wireless Handled Device

1.2. OVERVIEW

Power grids throughout the world have evolved as huge interconnected networks spread over large geographical areas. This arrangement has worked satisfactorily for a long time but in recent years it has been experienced that the conventional model of power grid has reached its limit. It is becoming increasingly difficult to maintain a secure operation of the grid. Therefore a paradigm shift is required in the way power grid has been monitored and controlled. It necessitates utilization of modern sensing, communication and computation tools at all the stages in the grid. Smart grid is the name given to such a modern and futuristic power grid and it is expected to operate with enhanced efficiency, stability, and flexibility.

However a smart grid will not be a new structure altogether rather it will gradually emerge from the existing infrastructure with the introduction of new technologies at different levels of the grid. Therefore the utilities should start deploying and managing the existing and emerging technologies in a better way on the way to a smart grid.

An upcoming technology is the synchrophasor measurement of voltage and current in the power grid by phasor measurement units (PMUs). These measurements are synchronized via the Global positioning system (GPS). In fact some utilities in the world have already installed PMUs in their networks on a limited scale. PMUs provide dynamic visibility into the operation of power grid at a much faster rate as compared to the arrangement in conventional grid [90].

High cost of PMU as well as technical and economic requirements of the associated communication circuitry limits their widespread installation. However it is still possible to derive the benefits of PMU installation if some of the buses are monitored directly by installation of PMU and some are monitored indirectly, by the PMUs installed on connected buses. Determining a minimum number of PMUs to make the system observable is often referred to as Optimal PMU Placement (OPP) problem.

Traditionally, the available data are redundant measurements commonly provided by SCADA, including active and reactive power flows and injections and bus voltage magnitudes and statuses of switching devices. With the advent of global positioning system (GPS), the measurement set is enlarged to include the available synchronized phasor measurements provided by the phasor measurement units (PMUs), which are measurement devices providing the precise knowledge necessary to minimize and control power outages and avoid problems such as cascading blackouts. With the increasing use of PMUs in recent years, the accuracy and reliability of state estimates and bad data detection can be enhanced [91]. A PMU installed at a bus can be used to measure the voltage phasor of the bus and the current phasors of some or all the branches incident to that bus, assuming that the PMU has sufficient number of channels. In spite of their enormous potential for improving the operation and real-time control of large transmission systems, PMUs will not make state estimation obsolete but will help improving its performance. When rectangular coordinates are used for phasor measurements and state variables in a PMU only measured system, their relationship are linear and the states can be obtained by a linear non-iterative algorithm. However, since the majority of measurements in existing estimators are of the traditional type, it is hard to be fully replaced by PMUs in the near future and state estimators including both phasor and traditional measurements will be still necessary.

Here proposed an optimal placement of PMUs for state estimation improvement. State estimation (SE) is a key function of energy management system and all the downstream applications of contingency analysis; steady-state, voltage, and dynamic as well as the optimal power flow are dependent on the accuracy of the SE results. Therefore any improvement in the SE solution will be helpful in making the power grid more aware of its operating state, a fundamental requirement of the smart grid.

1.3. PROBLEM STATEMENT (MOTIVATION)

The present-day power systems are dynamic in nature, with network topology changing frequently with load demand. With increase in load demand, the power network is loaded to its limits thus making it susceptible to partial or total blackout under minor or major disturbances. Ethiopia also plagued by both scheduled and unscheduled power outage during the day which is a great nuisance to individuals, industry and the public sector ,especially critical institution like hospitals .Hence to overcome this problem a way must be developed that aim to reduce the number of unscheduled power outages and optimize the use of the Ethiopian electric grid. A special attention should be given to the country plan on providing power to the east Africa on finishing the construction of HIDASE dam where this grid network should be smart or closer to smart in order to provide a secure and reliable operation internationally. This is to be achieved by gathering synchrophasor data which will provide the best estate estimate and valuable insight in to the grid condition and ensuring that power disconnection or consumption reduction takes place only when they cause the least inconvenience and when the consumer understands and accept them. The above practical problem has become a motivation to do optimal PMU placement in the southern region power system as a model for hidase dam outgoings by setting a criterion enhanced state estimation result.

1.4. OBJECTIVES

The main objective of this study is to develop a model method to place PMU at optimal location that will improve power supply and stability for consumer by providing enhanced state estimation.

The specific objectives include:

- ❖ Assess the effect of phasor measurement on power system state estimation.
- ❖ Formulation of algorithm for hybrid state estimation in the presence of both conventional and phasor measurements
- ❖ Develop a criteria for optimal Placement of PMU (A PMU will be placed at which the state estimation error is minimum.
- ❖ Make necessary recommendation for state estimation enhancement

1.5. SIGNIFICANT OF THE STUDY

The significance of this study is concerned on the optimal placement of phasor measurement unit (PMU) for the standard and southern region power system for better state estimation performance. This project can be used for

- observability of the power system at lower cost
- for system monitoring based on absolute phasor angle at lower cost
- Best Model for future Hidase dam expansion control
- As a reference for further study

1.6. SCOPE OF THE STUDY

The scope and limitation of the study are as follow:

- The placement of PMU will be assessed only on the IEEE 14 bus system and on a southern region 21-bus power system to be a model for hidase dam and the placement of PMU will be assessed on the substation level which is connected to this grid only.
- Since PMU device is new and difficult to find at lab level, we can only simulate and suggest its practical application.

2. LITERATURE REVIEW

This chapter reviews the basics of phasor measurement unit and the research work and studies that have been done in the area of optimal placement of phasor measurement units (PMUs) that are presented to solve the typical optimal placement problem of PMUs. The basic concept of phasor measurement unit, the standards and its application on state estimation has been discussed. Moreover the heuristic methods to solve the OPP problem and the mathematical method are discussed in this chapter.

2.1. BASICS OF PHASOR MEASUREMENT UNIT (PMU)

A phasor provides a means of representing a power system voltage or current sine wave in the complex plane. This is shown in Fig. 2.1

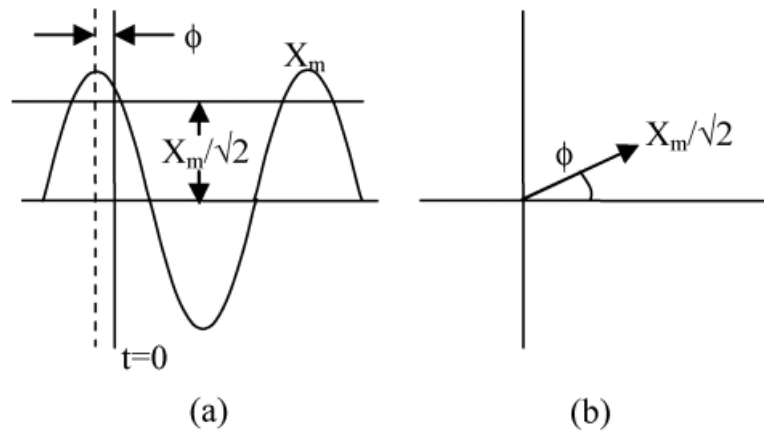


Fig. 2.1. Definition of a phasor

The mathematical formulation of this pure sinusoidal waveform is

$$x(t) = X_m \cos(\omega t + \phi) \quad \dots\dots\dots (2.1)$$

This is represented as the phasor

$$X = X_r + jX_i \dots\dots\dots (2.2)$$

$$= \frac{X_m}{\sqrt{2}} e^{j\phi} \dots\dots\dots (2.3)$$

Where X_m is the amplitude of the sinusoidal wave X_r and X_i are the real and imaginary part of the sinusoidal and ϕ and ω are the phase shift angle and the angular speed of the wave form at a time t respectively.

This is a complex variable and represents the position of the sine wave with respect to the absolute time reference chosen.

PMU is an advanced measurement device installed at a bus (or node) of the system, measuring the voltage and current phasors of the line (or branch) it is associated with. Fig 2.2 shows PMUs purely isolated to form a Wide Area Monitoring System (WAMS). GPS time stamped measurement signals are fed to a Phasor Data Concentrator (PDC) by using PMUs. The PDC collects and sorts the phasor measurements and signal processor converts data of PMUs into useful information which is visible on Human Machine Interface (HMI). The operator can easily access the critical information of the power system state

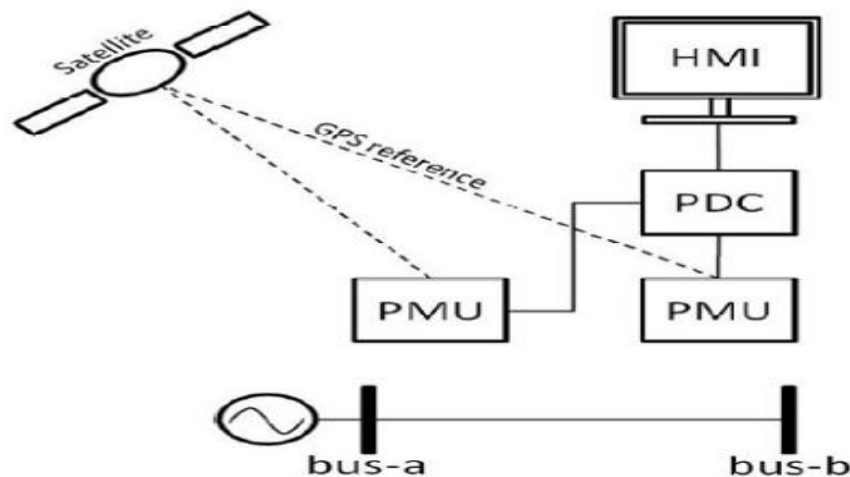


Fig. 2.2. Layout of PMU along with GPS time stamped signals

Accuracy of the phasor measurement in steady state is measured by the Total Vector Error (TVE) defined in equation 2.4.

$$TVE = \sqrt{(X_r(n) - X_r)^2 + (X_i(n) - X_i)^2} / (X_r^2 + X_i^2) \dots\dots\dots(2.4)$$

Where $X_r(n)$ and X_r are the real part of the steady state phasor and the theoretical values

$X_i(n)$ and X_i are the imaginary part of the steady state phasor and the theoretical values

And TVE is the magnitude of the vector difference between the theoretical phasor defined in 2.1 and the measured phasor defined in 2.3. The compliance level is that TVE is 1 at the most. The TVE variable blends three possible sources of error: magnitude, angle and time-synchronization [87]. The theoretical phasor is obtained by a signal generator. This signal is then measured to obtain the phasor measured by the PMU. Compliance tests compare phasor estimates under steady-state condition to the corresponding theoretical values of X_r and X_i and by calculating the TVE. Steady state conditions are those for which X_m , ω and ω in equation 2.3 are fixed for the period of measurement.

2.2. Synchrophasor standard IEEE C37.118

Synchronized phasor measurements or synchrophasors refer to the use of a common time reference for all measured phasors. A detailed definition of a time-synchronized phasor has been codified by the IEEE C37.118 standard synchrophasor for all measured phasors. The standard defines the synchronized phasor measurements used in power systems, the methods to verify compliance with the standard and the message formats for communication with a phasor measurement unit (PMU). The phasors are tagged with a Universal Time Coordinate (UTC) time corresponding to the time of measurement. This UTC time consists of three numbers: a second-of-century count (SOC), a fraction-of-second (FRACSEC) count and a time status value (TIME_BASE). The SOC count is a 4-byte binary count in seconds starting from midnight on January 1, 1970. The total measurement time is obtained by adding to the SOC count a fractional time which is obtained by dividing FRACSEC by the chosen TIME_BASE so that

$$\text{Time} = \text{SOC} + \frac{\text{FRACSEC}}{\text{TIME_BASE}} \quad (2.5)$$

Synchrophasors are measured by Phasor Measurement Units (PMU), which provide GPS synchronization to the UTC time for all measurements. The accuracy of the synchronization must be sufficient to meet the accuracy requirements ($TVE \leq 1\%$). The synchronization source should have continuous and uninterrupted availability. The best timing system is satellite broadcast timing which has a wide area of coverage. What is more, the 1 pulse per second (PPS) signal is not affected much by atmospheric conditions or seasonal variations. However, timing dissemination is not the primary purpose of satellite broadcast systems and during crises the primary purposes take priority which means the timing signal is unavailable. This UTC time synchronization can also be supplied by a local clock using a standard time code such as the IRIG-B amplitude modulated format supplemented by a 1 PPS pulse train to achieve the required accuracy. Other time dissemination system use IEEE Standard 1588 and ensures time synchronization via Ethernet.

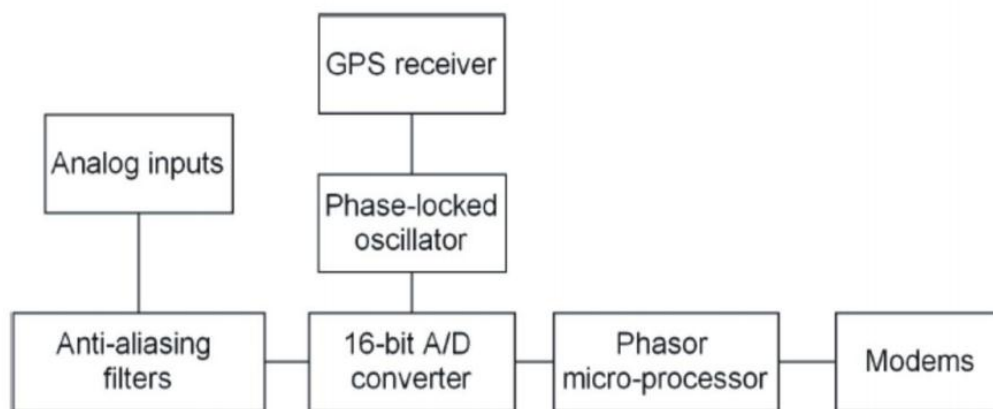


Fig. 2.3 PMU block diagram.

The one pulse per second signal is provided by the accurate clocks of the GPS satellites. Synchronization of the time signals for each receiver is accurate within 1 microsecond, so every GPS receiver provides the UTC (Universal Time coordinated) clock time. The signal is then captured by the GPS receiver and then divided by a phase-locked oscillator into the required number of pulses per second for sampling of the analog signal. Digital signal processing techniques are used to compute the positive sequence of voltage and current phasors. The resultant time-tagged phasors can then be transmitted to a local or remote receiver at rates up to 50 samples per second, as fixed by the Synchrophasor standard.

A PMU block diagram is shown in Fig 2.3. Each block is detailed below. i.e, a voltage or current level matched with the requirements of the converters is obtained by using appropriate measurement transformers. The analog AC waveforms are digitized by an Analog-to-Digital converter. The role of the anti-aliasing filter placed before the signal sampler is to remove the components of the signal whose frequency is equal or greater than one-half the Nyquist (sample) rate. The sampling is done using the Phasor Measurement Algorithm based on the Discrete Fourier Transform (DFT). Let us assume that the sampling frequency is $F_s = Nf$ where f is the signal frequency, so this means that there are N samples per cycle of the signal and the sampling of the signal over the window is denoted by the set. The variable N is also called the window-length, equal to the number of samples per cycle. The sampling period is therefore $T = \frac{1}{f}$. The process of computing this phasor estimate requires sampling the waveform over some interval of time, which is the measurement window. The time at which the phasor is estimated is the phasor time-tag [88]. The DFT-based phasor calculation at a particular instant of time is

$$X = \frac{\sqrt{2}}{N} \sum_{k=0}^{N-1} x_k e^{-j\omega T k} \dots\dots\dots (2.6)$$

Where $x_k \{k = 0, 1, \dots, N - 1\}$ are the N samples of the input signal taken over one period. Estimating the phasor requires sampling the waveform over some interval of time. An important point discussed in the standard is the choice of the time-tag within the window as shown in Fig 2.4. The standard fixes the time-tag as being near the center of the window.

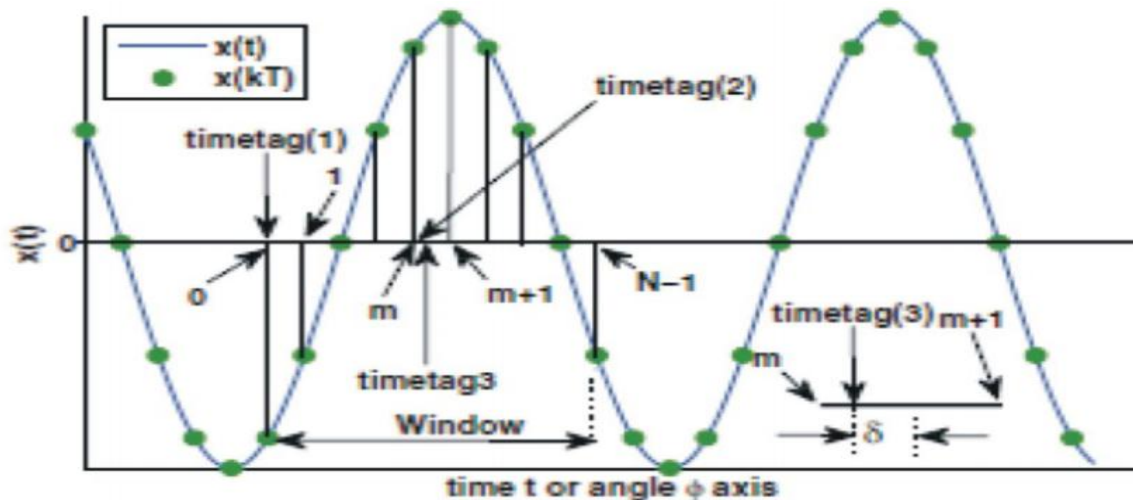


Fig 2.4. Signal sampling using DFT

It is important to standardize the format of messages to and from a PMU for use in real-time data transmission; taking place concurrently with the measurement process. This message protocol is used for communication with a single PMU or a secondary system that receives messages from several PMUs, like a Phasor Data Concentrator (PDC) as shown in Figure 2.5. Data concentrators ensure that phasor measurements are used effectively, and is a system that will retrieve files recorded at the measurement site and correlate files from different sites by the time stamp associated with each measurement.

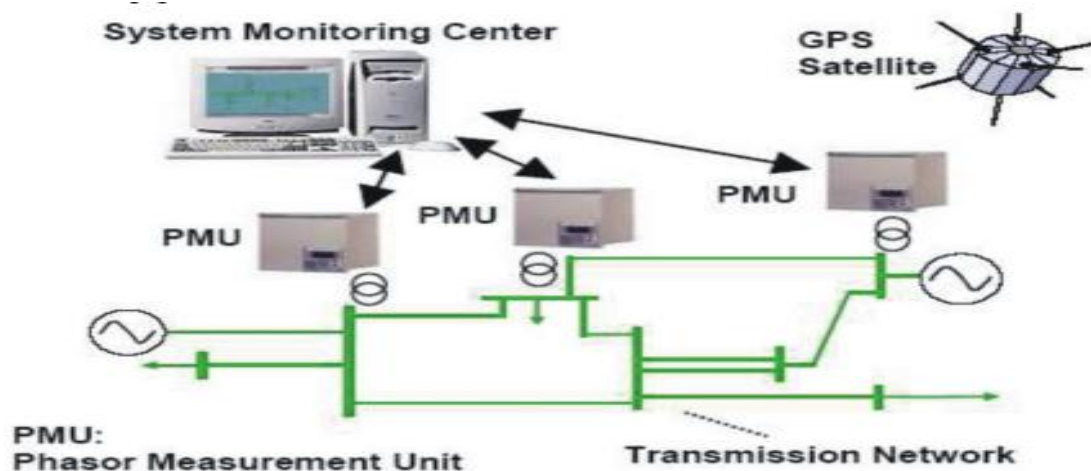


Fig. 2.5. Phasor Data Concentrator

The main role of a PDC is to gather data from the PMUs, reject bad data, align the time-stamps and create a coherent record of simultaneously recorded data from the power system parts monitored by the PMUs [88]. To use this type of infrastructure, communication facilities are essential. The two important aspects to be considered are channel capacity and latency. For certain applications, latency or the time lag between the instant the data is created and when it is available for the desired application, must be small. Typical times are given in Table 2.1.

Table 2.1.Associated Delays with various communication links

Communication link	Associated delay(millisecons)
Fiber-optic cables (50 Mbps - 1Gbps)	100-150
Digital microwave links	100-150
Power line (PLC) (up to 4 Mbps)	150-350
Telephone lines (up to 56 kbps)	200-300
Satellite link	500-700

Communication protocols used are serial communications using RS-232 to send byte by byte or Standard Internet Protocol (IP) which can be carried over Ethernet. There are four message types: data, configuration, header and command [87]. Data messages give information about the measurements made by a PMU, configuration is a machine-readable message describing the data and providing calibration factors and header is human-readable descriptive information sent from the PMU but provided by the user. Command messages are machine-readable codes sent to the PMU for control or configuration.

Compliance tests compare phasor estimates under steady-state conditions to the corresponding theoretical values of and by calculating the TVE. These tests are done in the laboratory before commercialization of the PMUs where the measured phasor is compared to the phasor generated by a signal generator. The standard thus ensures PMU interoperability under steady state conditions for PMUs of different brands. However, all testing is restricted to steady-state conditions and so PMUs from different brands can yield different results for the same phasor measurement in a transient state. The quantities influenced are the signal frequency, signal

magnitude, phase angle and harmonic distortion. For all tests the TVE must be smaller or equal to 1%.

2.3. CAPABILITIES PROVIDED BY SYNCHROPHASOR SYSTEMS

Monitoring devices called phasor measurement units (PMUs) measure the instantaneous voltage and current, and calculate frequency, at specific locations in an electricity transmission system. These parameters represent the “heart-beat” and thus the power system’s health. Voltage and current are parameters characterizing the delivery of electric power from generation plants to end-user loads, while frequency is the key indicator of the balance between electric load and generation.

PMUs sample power grid conditions at a rate of several hundred times per second and convert the measured parameters into phasor values, typically 30 or more values per second. A phasor is a complex number (mathematical representation) that represents the magnitude and phase angle of the sinusoidal waveforms of voltage or current at a specific point in time. The cutout at the top of Fig 2.6 depicts the magnitude and phase angle in the time based waveform and the same values in vector form. The PMUs also add a precise time stamp to the phasors based on the Global Positioning System (GPS) time signal. This turns phasor values into synchrophasors. The synchrophasor values provided by PMUs in different locations and across different power industry organizations are sent through communications networks to phasor data concentrators (PDCs) which collect and time-align the data.

The resulting PMU data provide transmission grid planners, operators, and engineers with a high-resolution “picture” of conditions throughout the grid, enabling capabilities such as wide area situational awareness, oscillation detection and monitoring, and detection of equipment problems not previously observable.

Fig.2.7 shows a way of installation of PMU on the substation in order to obtain the benefits of these devices.

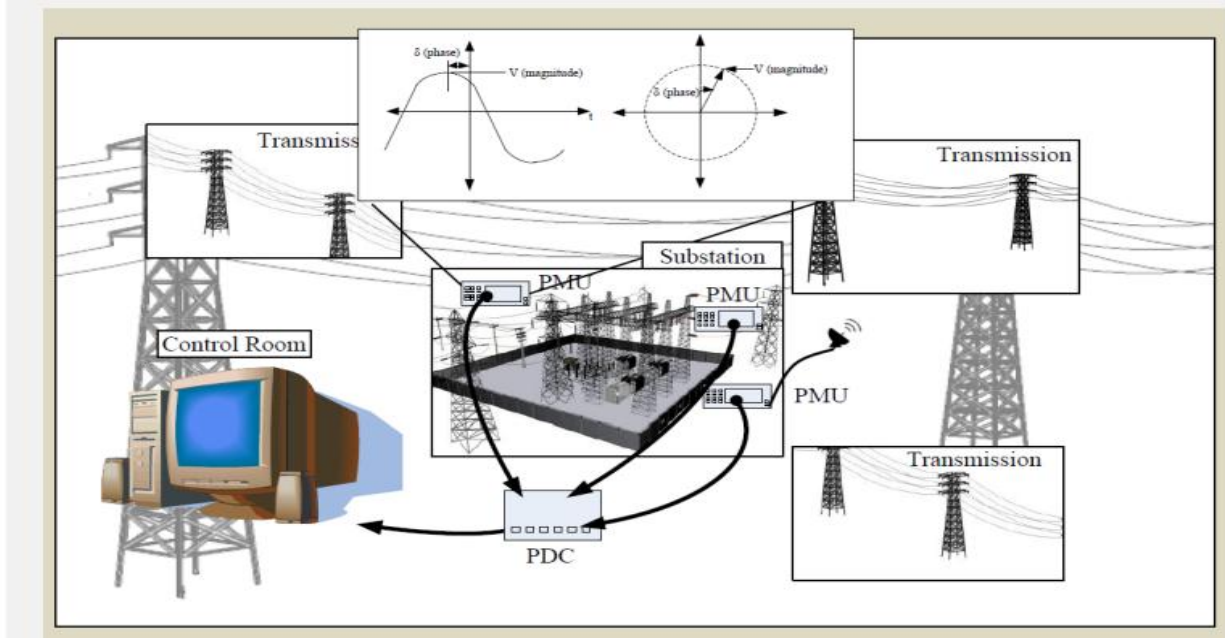


Fig 2.6 Collection and Flow of Synchrophasor Data

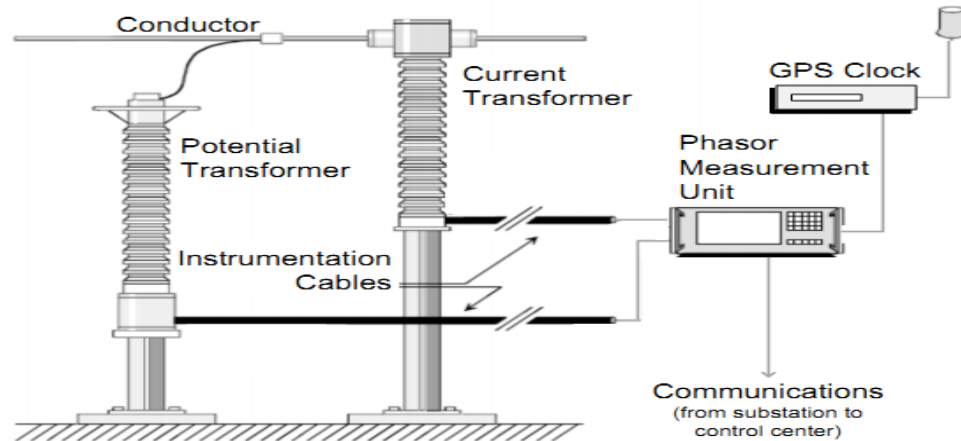


Fig 2.7. Representative PMU Installation within a Substation

In general in electric transmission systems, capabilities are generally specific to one of two operational scenarios:

- **Real-time capabilities:** These capabilities are employed in the minute-by-minute operation of the electric grid. These are generally carried out by power system operators or operational engineers.
- **Study mode/planning capabilities:** These capabilities are employed in planning for transmission system expansions, generation additions and retirements, system protection, and studying unexpected events or system behaviors. These are generally carried out by power system engineers.

2.3.1. Real Time Capabilities

Phase angle monitoring: The angular (phase angle) differences in voltages between two locations on a transmission grid provide information on power system stress and stability.

The grid is said to be “stressed” when power transfers are high and critical components in the grid operate close to their physical (thermal) limit. In this state, the ability of the power system to adjust to an equipment outage or other failure is reduced.

System stability is the ability of the system to move to a secure and steady operating condition following a disturbance (equipment outage or other failure). Some systems have stability limits on certain transmission paths. In these systems, high power transfers across those paths cause instability somewhere on the system. Power transfers across those paths are therefore limited (to the “stability limit”) in order to avoid putting the power system into an unstable configuration.

Knowing the extent to which the system is stressed and the proximity of the system to its stability limit lets operators know how much discretion they have in adjusting operating conditions and how much time they have to make the adjustments.

The phase angle difference is also a decisive factor for reclosing transmission lines. Traditionally, phase angles are calculated off-line with simulations and state estimation based on line flows. Now PMUs can measure phase angles directly, making them immediately available to system operators who can then monitor and remediate stressed power system conditions as they develop.

Oscillation detection and monitoring: When the electrical system is physically disturbed, the electrical characteristics (voltages, currents, and other parameters derived from these) of the

electrical system are also disturbed. Because these voltages, currents, frequencies, etc. tend to cycle quickly (analogous to physical vibrations), the disturbances are referred to as “oscillations.” Oscillations become a concern when they grow over time instead of waning. Oscillations also occur due to problems with control systems – most significantly generator controls. A system’s ability to damp oscillations is an important indicator of system stability. PMU data provide a way to detect the presence of oscillations, determine the extent to which an oscillation poses a threat to the electric grid, and monitor the oscillation to determine if conditions worsen.

Voltage stability monitoring: Maintaining voltage at levels that remain stable, even as demands from loads change and outages occur, is of paramount importance to a power system operator. PMU-enabled tools provide the system operator with insights as to the proximity of the system to its voltage stability limit. Voltage problems can develop almost instantly, so the ability to quickly detect and diagnose voltage problems and develop remedial actions is extremely valuable.

Event detection, management, and restoration: When an event that perturbs the normal operation of the power system, such as a tripped transmission line or tripped generator occurs, operators must take action to mitigate its effects. Good event detection, management, and restoration allow system operators to understand the event, minimize its impact, and restore service as quickly as possible. PMU data provide early indications of grid stress, including abnormal voltages, phase angles, frequencies, and power flows. As the event occurs, PMU data provide high-resolution graphic displays that show the system operator how the system is reacting. These displays provide insights into the type of event taking place and its extent. The sooner an operator understands the event, the more options are available for mitigating the event.

Islanding detection, management, and restoration: Islanding is the separation of a part of the power system from the larger power grid. Islands can occur when multiple lines are forced out of service thus isolating a region of the grid. This usually happens as a result of a major storm. The synchrophasor technology provides participants with the ability to detect, monitor, and better manage islands when they occur. Synchrophasor technology can quickly identify an island, allowing operators to synchronize the island to the larger power grid and restore its connection to the rest of the grid.

Equipment problem detection: Detecting equipment problems early can prevent unplanned outages due to equipment failures. Equipment failures can cause customer outages, costly equipment replacement, and could even damage other equipment. Synchrophasor technology has provided the capability to detect and diagnose failing equipment (for example, being able to detect intermittent off normal voltages undetectable by SCADA), allowing for replacement during planned outages.

Wide area situational awareness: Data from PMUs provide a high-resolution view of localized electrical phenomena. If PMU data are widely shared, conditions that develop across an entire interconnection can be detected and monitored. Wide area situational awareness is the capability for grid operators to see key parameters of the bulk electric power system beyond their service territory across an entire interconnection with enough shared data. Wide area displays allow all operators in an interconnection to have a common understanding of the condition of the overall electric grid, resulting in better collaboration and better solutions to emerging problems.

2.3.2. Study Mode/Planning Capabilities

Model validation and calibration: Models are mathematical representations of power system components. Models are used in all aspects of power system planning and operations. Typical components that are modeled in the power industry include:

- Conventional generating units (nuclear, hydro, steam)
 - ❖ Generator dynamics
 - ❖ Exciters
 - ❖ Turbines
 - ❖ Governors
- Motors
- Wind generating units
- HVDC (high-voltage direct current) devices
- Electronically-coupled (i.e., inverter-driven) loads
- Power system stabilizers
- Power system components (transformers, circuit breakers, transmission lines, etc.)

The accuracy of these models drives the efficiency and effectiveness of long-term capital investments and real-time system operations. Therefore, the quality of grid operation is strongly dependent on the quality of these models. Model validation improves the mathematical representation of the various components of the electric grid, thus allowing analytical engines to provide more accurate insights. Synchrophasor data enable engineers to evaluate, validate, and improve their models more quickly, accurately, and frequently, and with less effort. With more accurate models, analysis tools provide better insights to decision makers.

Post-event analysis: After a system disturbance, utilities study the sequence of events that led to the problem in order to prevent it from reoccurring. The insights gained by these analyses allow operators and planners to learn what initiated the system event, how the system responded to the incident, and most importantly, what can be done to prevent similar events from occurring. PMUs provide high-resolution, time-stamped data so that an accurate, temporally aligned record of events can be constructed quickly. Synchrophasor technology has significantly reduced the labor and cost requirements for such analyses, allowing engineers to investigate many events that they previously would not have had time to analyze.

Renewable resource integration: Large-scale renewable generation plants are relatively new to the bulk electric system, so their plant characteristics and control algorithms need to be better understood in the context of the operation of the grid as a whole. This is especially true as they become a greater portion of the generation. Renewables can be challenging to manage in the grid due to the as-available nature of their production and the vendor specific characteristics of their control systems.

Synchrophasor systems are particularly useful for monitoring, modeling, managing, and integrating distributed generation and renewable energy into the bulk power system. PMU data show in high-resolution how the renewable generation facilities affect the grid and respond to changes in grid conditions.

2.4. REVIEW OF TECHNIQUES OF OPTIMAL PLACEMENT OF PMU UNIT

A proper methodology is required to find the optimum number of the PMUs which will fully observe the power network. To solve the Optimal PMUs Placement (OPP) problem a number of mathematical methods have been employed. Among these methods linear Programming, non-linear programming, dynamic programming are the common optimization methodologies are proposed to solve this problem. Problems such as difficulties of obtaining local minima and handling constraints in conventional techniques are overcome by other optimization methodology like Artificial techniques, such as depth first search, iterated local search, simulated annealing, tabu search, genetic algorithms, differential evaluation, imperialistic, particle swarm optimization and ant colony optimization etc.

This section reviews the research work and studies that have been done in the area of optimal placement of phasor measurement units (PMUs) that are presented to solve the typical optimal placement problem of PMUs.

2.4.1. MATHEMATICAL METHODS

These methods are classified into three broad categories: Exhaustive search, Integer Programming, Integer Quadratic Programming (IQP).

2.4.1.1. Integer Programming (IP)

Integer programming (IP) is a mathematical programming method of solving an optimization problem having integer design variables, while the objective function and the constraints are linear, nonlinear, or quadratic, thus leading to integer linear programming (ILP), integer nonlinear programming (INLP) and integer quadratic programming (IQP) algorithms, respectively[49,80,84].

In [40] an IP was used to achieve optimal PMU placement for observability in a manner that can also detect and eliminate bad data in critical and critical measurement pairs, which otherwise can render the state estimation problem unsolvable or bad data undetectable.

In [32], a new method is proposed for optimal PMU placement in power systems suffering from random component outages. The original ILP method is valid to get a PMU placement solution which can make the whole system topologically observable with minimum number of PMU.

Reference [41] proposed a new algorithm to determine optimum number of PMUs for fully observability of the system under any contingency. The suggested algorithm operates in four stages; the essential measurement for totally known condition is found out in first stage. In next stage, it selects redundant measurement set under any contingencies. Above both stages uses least condition number of normalized measurement matrix to select the proper measurement location. Further, minimum redundant measurement set is chosen from these measurements using the binary integer programming technique. Finally these measurements i.e. essential and minimum redundant measurements are arranged such that locations of PMUs are minimum. For this purpose heuristic method is used. The IEEE test systems results show proposed placement method which ensures the system observability under the given contingency. Also it provides the reliable measurement to the system.

The contributions of reference [42] by using ILP are: new OPP model is devised in which dc lines are modeled for observability analyses. The new OPP model seeks the minimal cost of the PMU placement while the cost of each PMU is a function of its measurement channels. This practical assumption is emphasized in our optimization model by the allocation of more than one PMU to a single node because a PMU would not have many channels due to technical limitations. The performance of the proposed model is examined on the IEEE test systems and a real-size power system.

An Integer Linear programming is used to determine the optimal number and location of PMUs to make the system measurement model observable and thereby it can be used for power system state estimation. When installing PMUs in a power system that already has SCADA measurement units, it was suggested that the minimum amount of PMUs first be calculated and then install those PMUs in phases, where in each phase a certain number of PMUs are installed in locations providing the highest possible level of observability chosen from the optimal locations[43,44].

In [45], OPP problem is solved using Integer Linear Programming considering sensitivity analysis. As the cost of one PMU is very high, so it is required to place minimum number of PMU's without neglecting the safety and security of the system. So the most sensitive buses in given power system based on load changes, are found and PMU's are optimally placed such that all the sensitive buses are observed.

A technique is proposed in [46] for placing the PMUs in multiple stages over a given time period that ensures complete power system observability even under a branch outage or a PMU failure. The proposed technique is based on integer linear programming (ILP) and a multi-criteria decision-making (MCDM) approach. Integer Linear Programming method is used in first stage to find the optimum number of PMUs for the complete observability of the system. These PMU placements include the contingency cases such as single branch outage and single PMU outage. A multi-criteria decision-making (MCDM) subsequently developed to prioritizing the optimal PMU locations. This criteria include three indices, viz. tie-line oscillations observability index (TOI), voltage control area observability index (VOI) and bus voltage observability index (BOI). Finally, select the PMU locations in each stage by ranking the optimal locations using the proposed indices in the MCDM process. The numbers of PMU's are further reduced and hence reducing the system cost.

The authors in [47] addresses the various aspects of optimal PMU placement using binary integer linear programming(BILP). Cases with and without zero injection buses have also been considered. Minimum PMU placement problem may have multiple solutions, therefore, two indices viz. BOI (Bus Observability Index) and SORI (System Observability Redundancy Index) have been proposed to further rank these solutions.

An integer-linear programming (ILP) model is developed in [48] to find the minimum number and location of synchrophasors subject to the feasibility of proposed backup protection scheme.

Reference [50] represents an optimization model to determine optimum number of PMUs in power network. The proposed model consists of observability necessities based on a set of probabilistic criteria. The nonlinear mathematical expression related with the probability of observability index is changed to a linear model by a useful linearization technique. This linear

model further described as a Mixed Integer Programming (MIP) problem. This proposed mathematical model is therefore well-suited with the Mixed Integer Programming solution tools and also it tested on large scale power systems.

A different formulation which is numerical and uses integer programming allows easy analysis of network observability [51]. An efficient and comprehensive formulation based on mixed integer linear programming method for the optimal placement of PMUs taking into account the channel availability and different contingency conditions associated with power systems, i.e. single line outage and single PMU loss were also considered in [52].

In [53] mixed integer linear programming (MILP) framework was formulated for the proposed model that simultaneously optimizes two objectives functions: (i) minimizes the number of PMUs and (ii) maximizes the expected value of system's redundancy (or minimizes the unobservability of the system). Incorporating the impact of zero-injection buses, the formulation is used to evaluate the probability of unobservability of buses resulted from line outages and PMU loss.

An Integer linear programming based method for PMU placement has been considered to ensure complete system observability under critical contingencies. In case of optimum placement difficulties two indices have been proposed to choose the optimal solutions; namely Bus Observability Index (BOI) which gives the total number of PMUs observing a given bus and to get the sum of all BOI for the system, System Observability Redundancy Index (SORI) is used. An optimal PMU placement scheme is also be proposed considering different PMU channel capacities. This has been also indicated the to help the power system planner to greater extent from application-cost point of view for PMU- PDC installation in the WAMS based power network at the planning stage. In addition to the cost factor, different criteria are suggested for the proper allocation of PMUs in a given system. Network observability, state estimation accuracy and robustness present samples of such criteria [54-60].

Reference [61] proposed Integer Linear Programming with a fast and novel practical model to determine the optimum number of PMU. It consist power system contingency conditions i.e. line outages and measurement losses. The proposed model is flexible than existing model because

other situation can be included separately or all together in power systems. Also, model consist measurement limitations i.e. communication constraints of power networks. The main importance of this method; it take low time getting the result also it gives local optimal solution for large-scale power.

The problem of PMU placement is formulated and solved as an IP problem [62]. The solution will provide the minimum number of strategically located PMUs that will eliminate measurement criticality in the entire system. This implies that any bad data appearing on any single measurement will be detectable. It is shown that depending on the measurement configuration and the system topology, this goal can be achieved by using only a few extra PMUs to transform several critical measurements into redundant ones in the system.

A new multi-stage PMU placement approach to maximize the network observability during time horizons was proposed. Linear Integer Programming (LIP) is widely used to solve the OPP problems. An optimal placement of synchronized phasor measurement units (PMUs) to ensure complete power system observability in the presence of non-synchronous conventional measurements and zero injections is also presented.[63, 64, 65].

A Binary Integer Programming technique for instantaneous PMUs placement and conventional power flow measurements for fault observability in power systems proposed in [66]. The formulation is initially based on a non-linear programming problem is unsolvable therefore initial formulation results in a binary non-linear integer programming problem that is transformed into a corresponding ILP problem through Boolean implications. The transformation provides a high scale of scalability.

In [67] using the Binary linear Integer Programming method, local redundancy of the existing measurement is considered in the PMU placement strategy .Besides, considering the number of PMUs implemented yearly limited, a multistage placement strategy is proposed according to the improvements on local redundancy.

A new method has been proposed in [68] that approached the PMU placement problem with purpose to ensure that all WAMPAC applications attain complete measurement observability. It presented an optimal method of placing PMUs for three categories of WAMPAC applications.

The author weighted the application by all three criteria and categorize by the type of measurement shared. Category 1 consists of applications that require phasor measurement relative to a network's stability. Category 2 requires a bus' normal voltage and 3 requires measurement under faulted conditions. The phasing strategy ensures that initial PMU placements enable Category 1 and subsequently 2 and 3.

Reference [69] propose a new PMU placement algorithm, which can minimize the number of PMU, considering network connectivity, ZIB, N-1 line contingency, system reliability and voltage stability level. By introducing the concept of system reliability, the proposed method can effectively reduce the number of PMU while ensuring the observability for major contingencies. The proposed method also employs the Voltage Stability Index (VSI) to weigh the priority level of bus, which is used to screen the severer buses during contingencies.

Based on the integer linear programming formulation proposed for optimal PMU placement, the authors in [70] presents a generalized integer linear programming formulation for cases including redundant PMU placement, full observability and incomplete observability.

The authors in [71] developed an optimal placement algorithm for PMUs by using integer programming. The proposed formulation improved the non-linearity with and without conventional power flow and power injection measurements.

In Reference [73] an optimal phasor measurement unit (PMU) placement model considering power system controlled islanding so that the power network remains observable under controlled islanding condition as well as normal operation condition has been proposed using ILP.

An ILP (Integer Linear Programing) model for the optimal multistage placement of PMUs is proposed in [74]. The approach finds the number of PMUs and its placement in separate stages, while maximizing the system observability at each period of time. The model takes into account: the available budget per stage, the power system expansion along with the multistage PMU placement, redundancy in the PMU placement against the failure of a PMU or its communication links, user defined time constraints for PMU allocation, and the zero-injection effect.

The authors in [75] present an integer linear programming (ILP) framework for the optimal placement of phasor measurement units (PMUs), in the presence of conventional measurements. Furthermore, by the proposed method, the power system is to be remaining completely observable during all possible single contingencies for lines and measurement devices.

The reference in [76] proposes a technique based on singular value decomposition (SVD) to define the role of each placement in dynamic stability assessment and find the best PMU places to achieve this goal. Since observability is still a very important factor, the integer linear programming method is used to solve the optimal PMU placement problem for full system observability.

The contribution of the reference in [77] is to present a PMU placement methodology with observability redundancy; that is, to get more observability times of buses with the same number of PMUs. The method employs MILP, to make it possible to explore the global optimal solution unlike nonlinear programming.

The aim of the authors in [78] is to recognize the effect of channel capacity of PMUs on their optimal placement for complete power system observability. Initially, the conventional full observability of power networks is formulated. Next, a modified algorithm based on integer linear programming model for the optimal placement of these types of PMUs is presented.

Reference [79] presents a novel integer linear programming approach, which is shown to be equivalent to the ES-based PMU placement. The linearity of circuit equations ensures that there is a one-to-one relationship between the solutions of the proposed equivalent integer linear programming method (EILPM) and those of ES. Therefore, impractical ES is translated into EILPM, which can be solved in a few seconds or minutes. Furthermore, additional constraints for observability preservation following single PMU or line outages can easily be developed in the proposed EILPM. Moreover, the limitation of communication channels is also handled by rendering the nonlinear terms into linear one

2.4.1.2. Exhaustive Search

Exhaustive search is a general optimization technique that systematically enumerates all possible candidates for the solution and selects the candidate that satisfies the constraints at the optimum value of the objective function. Its main advantage is that it guarantees the finding of the global optimum. However, it is not suitable for large-scale systems with huge search space [84].

An exhaustive binary search method is implemented in [38] to determine the minimum number of PMUs needed to make the system observable. In case there is more than one placement set having the same minimum number of PMUs, a method is proposed to select the one resulting in the most preferred pattern of measurement redundancy. Due to its exhaustive nature, the method gives the global optimal solution, and hence the results for a number of standard test systems are reported to provide benchmark solutions for researcher's investigating various methods of optimal PMU placement.

A contribution in [27] is the development of an exhaustive binary search algorithm to determine the optimal placement of PMUs as well as calculate the channels availability, the availability of measurement system of each PMU placed, and the global availability of measurement and maximize the average identification capability of multiple line outages in [39].

2.4.1.3. Branch and Bound Method

In [86] authors presented the optimal positioning of the phasor measuring units for complete monitoring of power network in Khuzestan province in Iran. The (ILP) integer linear programming based branch and bound method is used to find the optimal number and position for PMUs. In [87] authors proposed an optimal phasor measurement units (PMUs) placement algorithm for power system observability and also increasing the performance of secondary voltage control scheme. The OPP is formulated to minimize the number of PMU installations subject to full network observability and monitoring pilot buses of the system to improve secondary voltage control performance. The branch and bound optimization method is used to solve the OPP problem which is suitable for problems with integer and Boolean variables[72].

2.4.1.4. Integer Quadratic Programming (IQP)

Quadratic Programming (QP) is solving the optimization problem which has quadratic objective function and linear constraint. In this technique all design variables take only an integer value [86].

An integer-quadratic-programming-based method in [81] is used to determine the minimum number and the optimal locations of the PMUs to ensure complete topological observability of the system.

In [82] an integer quadratic programming approach is used to determine the optimal locations of PMUs. The optimization process tries to attain dual objectives: 1) to minimize the number of PMUs needed to maintain complete observability of the system for normal operating conditions as well as for the outage of a transmission line or PMU and 2) to maximize the measurement redundancy at all buses in the system. The method was applied on IEEE test systems considering the outage of a single transmission line or a single PMU. The proposed method can be used to determine PMU locations when conventional measurements, such as line flows and power injection measurements, are available

While the authors in [83] presents a new method for optimal placement of phasor measurement units (PMU) for complete power system observability by formulating the optimal PMU placement (OPP) problem as a quadratic minimization problem with continuous decision variables subject to nonlinear observability constraints. The optimal solution is obtained by an unconstrained nonlinear weighted least squares (WLS) approach.

2.4.2. Artificial Intelligence Techniques

2.4.2.1. Simulated annealing (SA)

Simulated annealing (SA) is a procedure for solving complicated combinatorial optimization in which the current solution is randomly altered. The new solution is the worse alteration with the probability that is reduced as the computation proceeds. An optimal solution for a large combinatorial optimization problem needs a fit perturbation mechanism, cost function, solution space, and cooling schedule to be solved by SA. Sufficiency of SA can be found by searching a

large-scale system and obtaining good speed in terms of finding an optimal or near-optimal solution [85].

In [34] an algorithm dealing with communication constrained PMU placement is modeled within the framework of simulated annealing. Here the pragmatic constraint of not placing PMUs at buses without communication support is recognized.

The proposed method in [35] considers project practical view. It used the concept of indirect observation, the MST and SA method. And it also improved the state estimation with SCADA/EMS measurement and PMU measurement. The method has been declared as effective, and it also have some advanced points such as strong maneuverability and small investment.

2.4.2.2. Imperialistic Competition Algorithm (ICA)

ICA is an evolutionary optimization algorithm which is inspired by socio-political process of imperialistic competition. This algorithm has proven its superior ability to achieve faster convergence and better solution compared to the other evolutionary algorithms.

In Reference [1], the optimal placement of PMUs and minimization of the cost of associated CI (communication infrastructure) are carried out simultaneously by using ICA. That is to say the optimal placement of PMUs, the optimal location of the PDC and minimal communication paths between PMUs and the PDC are obtained simultaneously in order to make the network as fully observable as possible in terms of the state estimation.

2.4.2.3. Cellular learning automata (CLA)

Cellular learning automata (CLA) algorithm is based on the usage of learning automata (LA) to the state transition probability adjustment of cellular automata (CA). This method starts by specifying the internal state of every cell. Then, the reinforcement signal for each LA is determined on the basis of the rule of CLA. Finally, as for supplied reinforcement signal and action chosen by the cell, each LA updates its action probability vector. The desired state will be reached by continuing this process [85].

Reference [2] proposed a CLA algorithm to tackle the optimization problem, introducing new CLA local rules to enhance the optimization process in the formulation of a modified multi-objective PMU placement method by first calculating worth of observability for a set of PMUs in contingency, by ranking the prospective contingencies.

2.4.2.4. Chemical reaction optimization (CRO)

Chemical reaction optimization (CRO) is a heuristic algorithm based on population processed by obtaining a lower energy stable state by simulating the action and reaction of molecules in a chemical reaction, which aimed to reach the minimum state of free energy.

Authors in [3] proposed chemical reaction optimization (CRO) method to solve the problem of maximizing the power system observability by placing phasor measurement units (PMUs) in a multistage manner where the problem is constrained by the financial budgets available at each installation stage.

2.4.2.5. The fruit fly optimization algorithm (FOA)

The fruit fly optimization algorithm (FOA) is a new method for finding global optimization based on the food finding behavior of the fruit fly. The fruit fly itself is superior to other species in sensing and perception, especially in osphresis and vision. The osphresis organs of fruit flies can find all kinds of scents floating in the air; it can even smell food source from 40 km away. Then, after it gets close to the food location, it can also use its sensitive vision to find food and the company's flocking location, and fly towards that direction too. Fruit flies are a kind of insect, which live in the temperate and tropical climate zones and eat rotten fruit. The fruit fly is superior to other species in vision and osphresis. The food finding process of fruit fly is as follows: it smells the food source first with its osphresis organ, and flies towards that location; after it gets close to the food location, its sensitive vision is also used for finding food and other fruit flies' flocking location, and then it flies towards that direction [4]. Fig. 2.8 shows the food finding iterative process of a fruit fly swarm.

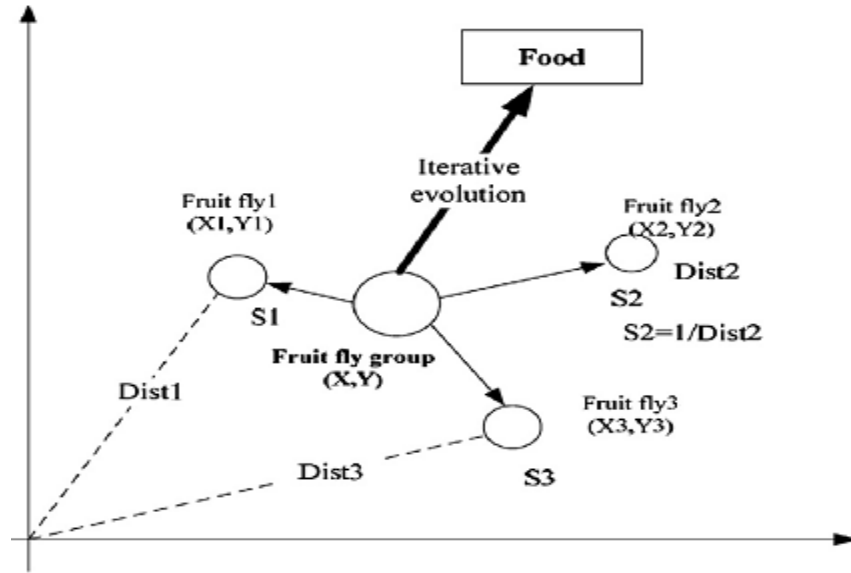


Fig.2.8 the food finding iterative process of a fruit fly swarm

The authors in [4] develop a modified fruit fly method for solving PMU placement problems. This method is found to speed up the convergence.

2.4.2.6. Genetic algorithm (GA)

Genetic algorithm (GA) is adaptive heuristic search algorithm that repeats the process of natural evolution. This process is used to generate solutions to optimization and also search problems. Modeling natural selection is the base of genetic algorithm (GA) which does not need any secondary functions such as derivatives computation. Some positive characteristics of GA which make it more usable in optimization problems are as follows : (a) probability of local minimum trapping is decreased, (b) computations of going from one state to another is declined, and (c) evaluation of the fitness of each string guides the search.

A genetic algorithm is proposed in [5] in order to minimize the number of PMU placement sites. The proposed method is numerical observability by using the minimum condition number of the normalized measurement matrix as criteria. The sequential elimination is used to find the essential measurements for the completely determined condition. The sequential addition is used to select the redundancy measurements under the contingency.

Reference [6] investigates the application of immunity genetic algorithm (IGA) for the problem of optimal placement of phasor measurement units (PMUs) in an electric power network. The problem is to determine the placement sites of the minimal set of PMUs which makes the system observable.

The authors in [7] presents a non-dominated sorting genetic-based algorithm (NSGA), which can successfully solve PMU placement problem with two competing objectives, such as minimization of the number of PMUs and maximization of the measurement redundancy. The optimization is performed without any preference information given with respect to the objectives.

In [8] genetic algorithms to find the optimal placement of PMUs has been generated using a MATLAB program .The genetic algorithm method was tested on three network-tests, and the results were compared with other algorithms to prove that an optimal placement of PMUs ensures a total observability of the network and thus confirming the efficiency of genetic algorithm.

The genetic algorithm in [9] was used to calculate the best PMU location that meets the prespecified threshold of the LODE index for system observability reliability. In this work multiple solutions for optimal PMU placement in the normal system state are generated and the solutions are ranked using the observability reliability indices. Then, the best solution is compared with the optimal PMU placement solution considering the full observability under N-1 conditions of transmission branch failures.

Reference [10] presents a genetic algorithm based method for optimal meter placement problem by which measurement system can be optimally determined and upgraded to maintain network observability. The proposed algorithm yields a measurement configuration that withstands any single branch outage and/or loss of single measurement, without losing network observability.

In [11, 12] genetic algorithm (GA) is proposed for PMU and SCADA placement for state estimation identification. The results obtained in [11] are indicative of the fact that the optimal placement of PMU increases the accuracy of the obtained estimates and efficiency of the bad data detection algorithms.

In [13] Genetic algorithm was developed to solve the proposed new model that utilizes network observability rules and determine the optimal investment decision for the placement of PMUs in the power grid. Reference [14] proposes an optimal PMU allocation method for substations based on a specialized Genetic Algorithm (GA) while in [15] the same method has been used for complete and incomplete observability.

The authors in [16] presented a new optimization algorithm for optimal PMU configuration based on combination of graph theory and genetic algorithm. The method for PMU allocation has shown a decent result, obtaining the same or smaller number of PMUs to fully observe the power system compared with the references. What is more important is that the computation cost is greatly reduced, which mainly attributes to the topology constraint analysis.

2.4.2.7. Greedy Algorithm

The greedy algorithm is an optimization methodology that follows the problem solving heuristic of making the locally optimal choice at each stage with the hope of finding the global optimum.

In [17], Optimal PMU placement problem was considered in the power system to improve the state estimation results. The PMU placement problem has been formulated as an optimal experiment design problem, with a class of well-known optimality criteria. It has been showed that the design criteria are usually related to a key property, sub modularity, which often allows efficient greedy algorithms. A greedy PMU placement algorithm that has been showed achieving an approximation ratio of $(1-1/e)$ for any PMU budget K , which is the best guarantee among polynomial-time algorithms has been presented in [18].

Reference [19] a greedy algorithm was proposed to find the unique optimal PMU deployment with the best state estimation performance in the three-phase network. A demonstration that the greedy algorithm is efficient and taking PMU phase mismatch into account will help in the improvement of the PMU placement was presented in [20].

2.4.2.8. Differential evolution (DE)

Differential evolution (DE) concept employs N -dimensional element vectors to minimize ongoing space functions. Mutation, crossover, and selection are the principle operators utilized to

carry out the global optimization. This heuristic method could be widely used in different cost function problems such as non-differentiable, non-linear and, multi-modal functions. Parallel computations, easy usage, and good convergence properties are other benefits of this approach [85].

Reference [21] presents a multi-objective optimal model of PMU placement with the objectives of minimization of the number of PMU and maximization of the N-1 measurement reliability by using DE algorithm.

2.4.2.9. Particle Swarm Optimization (PSO)

Particle swarm optimization (PSO) is a population-based stochastic optimization technique in which each potential solution (called a particle) is assigned a randomized velocity and then flows through the problem hyperspace. PSO has been found to be extremely effective in solving a wide range of engineering optimization problems [2, 84].

A modified discrete binary version of particle swarm algorithm was used in [21, 25] as an optimization tool in finding the minimal number of the required PMUs for the complete system observability. At first, the PMU placement was solved with the goal of minimizing the total number of required PMUs for the complete system observability. Then, the effect of PMU loss or a branch outage was taken into consideration, and a placement scheme was obtained which maintains complete system observability during the contingency conditions. In [23, 24] PSO optimization is also used for observability with the objective of minimum cost of PMU.

2.4.2.10. Iterated local search (ILS)

Searching a smaller subspace, which is defined as local optima, instead of the whole solution space, is the main viewpoint of iterated local search (ILS) method. By utilizing an embedded heuristic, a sequence of solutions is provided in which the best solution is obtained if one were to utilize repeated random trials of that heuristic.

The Iterated Local Search (ILS) meta-heuristic in [26, 28, and 31] is used to minimize the size of the PMU configuration needed to observe the network. The algorithm is tested on IEEE test

networks with 14, 57 and 118 nodes and compared to the results obtained in previous publications.

Reference [29] uses an iterated local search heuristic method to derive the minimum number of PMUs. In addition to the practical model, this study also considers the ideal model, in which all load nodes are assumed to be zero injection. The numerical studies on various IEEE power test systems demonstrated the superior performance of the proposed algorithm in both the models in regard to computational time and solution quality.

In [30] the empirical observability Gramian is applied to quantify the degree of observability of a power system under a specific PMU configuration and an optimal PMU placement method for dynamic state estimation is proposed by maximizing the determinant of the empirical observability Gramian. It is effectively and efficiently solved by the NOMAD solver which implements the Mesh Adaptive Direct Search algorithm.

2.4.2.11. Intelligent search technique

An intelligent-search technique called best first search (BFS) method utilizes information about each node by computing an evaluation function at each of them while moving towards the goal state. It helps the BFS algorithm to search and explore only the most promising nodes, thereby making the search process, simpler and faster one (Luger & Stubblefield, 2002). The most promising nodes are identified as those having least/highest score of an evaluation function. While searching towards the least optimal cost path, the BFS algorithm has the capability to change its search path from the current search path to the most promising path (Kumar, 2008). This characteristic makes the BFS method superior over other graph theoretical methods for optimal PMU placement problem.

The authors in [33] utilize this method for the optimal placement of PMUs to make the power system topologically observable. The proposed method determines the optimal locations of PMUs in two stages. Stage I determines the sub-optimal locations of PMUs using best first search, which is an intelligent search algorithm with an ability to change its search path from the current node to the most promising node. From the results of stage I, the redundant PMU locations are identified and eliminated in stage II using pruning. The proposed method is

extended to take into account the presence of conventional flow measurements in the system. It is also found to be effective in handling both single as well as multiple flow measurements connected to a bus.

2.4.2.12. *Tabu search (TS)*

Tabu search (TS) is an adaptive algorithm that utilizes many other methods such as linear programming algorithms and heuristic concepts. This procedure is presented to solve the combinational optimization problems in scheduling and covering. Tabu list which is one of the main elements of TS consists of the number of recently visited states plus a number of unwanted states. Other main elements of TS are aspiration, diversification, and definition of a state and the surrounding area. There is a reset in TS when it is not converging [85].

A TS method to obtain a solution for the OPP problem by checking network observability with a numerical algorithm is introduced in [36, 37]. The impact of three different TS initialization schemes, together with the two TS parameters (TL and TSI), on the optimal solution is investigated. The experimental results verify the superiority of the proposed RTS method over the MTS method. The effectiveness and flexibility of the proposed scheme is demonstrated by the simulation results tested on the four different IEEE test systems.

2.5. SUMMARY

Under smart grid environment the placement of phasor measurement units is very critical because accurate measurement of active power transferred is required. Hence in this section a comprehensive review of PMU placement methods, its basics, capabilities and standards is presented. Optimal PMU placement (OPP) problem has been solved by utilizing numerous optimization methods. To obtain solutions for the OPP problem, two major techniques including mathematical methods and artificial techniques exist. The correct methodology is required to find the optimal placement of PMUs in power network. The presented review of heuristic/artificial and conventional/mathematical optimization techniques could largely help researchers in terms of employing new concepts to solve the OPP problem. Further economic point can also be added to make optimal PMU placement problem more practical to the industry stakeholders. This review can be helpful to my thesis work for making improvement in already

existing methods and develop new mathematical optimization approaches for optimal PMU placement considering linear integer programming for example. Additionally security assessment with PMU's is also an important issue which could also be incorporated.

3. POWER SYSTEM STATE ESTIMATION

This chapter aims at presenting the concepts of the Traditional formulation of state estimation (TSE) in which the network is modeled by the bus-branch model. In addition to this, the chapter also discusses and portrays two different approaches for the type of measurement processed by the state estimator. In the first instance, TSE is formulated considering that the set of available measurements is composed only of conventional measurements, which are: power flow, power injection, and voltage magnitude measurements. In the second instance, the modifications required to include synchronized phasor measurements units (PMU) are deliberated and described.

3.1. APPLICATION OF PMU FOR STATE ESTIMATION

Since the 1965 Northeast blackout, state estimation has become a critical application function at energy control centers. Existing state estimation algorithms [92] use measurements of line flows and injections, both real and reactive power, to estimate all bus voltage angles and magnitudes. The complex bus voltages are the state of the system. The complex power flows in lines and the complex power injections at buses can be determined from the bus voltages and an accurate model of the network. Prior to synchronized phasor measurements, the state could not be measured directly but only inferred from the unsynchronized power flow measurements. This fact and the process of getting large numbers of measurements into the control center (a scan) forced early state estimators to make compromises that persist today and have an influence on how phasor measurements are integrated into existing state estimation algorithms. The most significant of these assumptions is that the system did not change during the scan—that the system was static. Another view of the resulting estimate of the state is that it is the state of a hypothetical system which could support the complete set of measurements. Depending on how long the scan takes and the changes in the system that took place during the scan, the hypothetical system may not exist or may be quite different from the real system. While scans have become quicker, the introduction of phasor measurements forces reconsideration of the static assumption.

The direct integration of a few synchronized phasor measurements into an existing nonlinear estimator as shown in Fig.3.1 is straightforward but results in an estimator that has most of the

limitations of the original conventional estimator. The new estimator is still recursive and still has the assumptions required for the static state estimator. The issue of how to place the precisely timed PMU data into the scanned data can also introduce a skew.

If an estimate could be formed with only PMU data, then the issues of data scan and time skew could be eliminated. The PMU data would be time tagged and the static assumption removed. Since the voltage and current measurements are linear functions of the system state, the estimation problem is simple a linear weighted least squares problem which requires no iterations. Given PMU data can be reported at rates as high as 60 times a second, a truly dynamic estimate would be available. An estimate of a dynamic system at an instant in time corresponding to the measuring instant would be obtained. Issues that must be addressed include the need for redundancy to eliminate bad data and determination of how many PMUs is required.

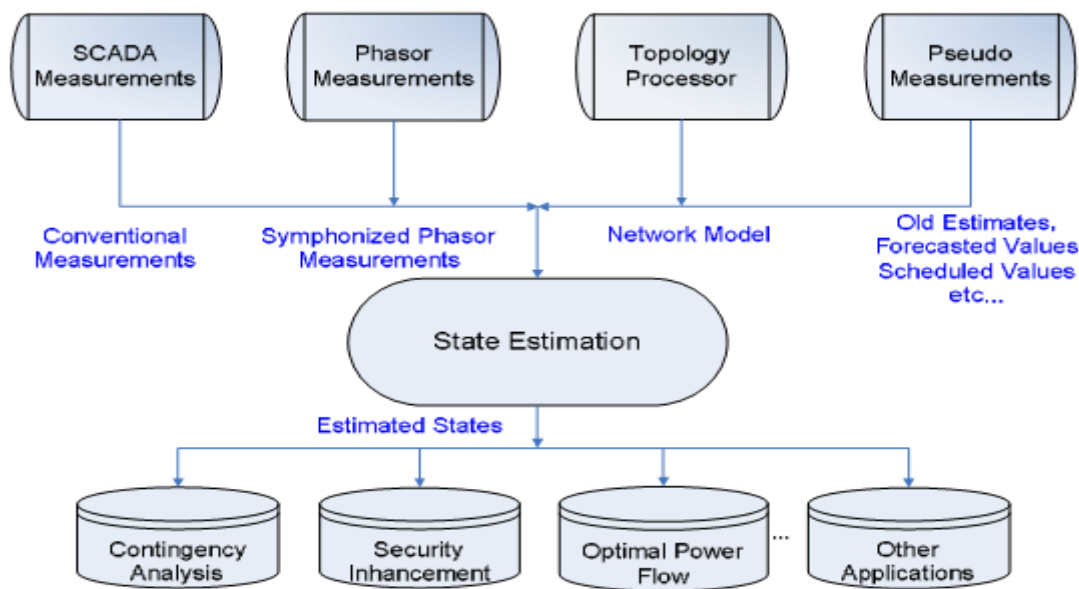


Fig.3.1.Role of state estimation on power system control and operation using different measuring techniques

Recognizing that a PMU in a substation would have access to line currents in addition to the bus voltage reduces the number of PMUs needed. Measuring line currents can extend the voltage measurements to buses where no PMU is installed. Using a model of the transmission line, the line current can be used to compute the voltage at the other end of the line. With a large number of PMUs the redundancy issue is addressed. On the other hand, the smallest number of PMUs needed to indirectly measure all the bus voltages and the optimum PMU location to achieve this

has been a subject of a number of papers [93]. With different approaches, treatment of zero injection buses, and differing assumptions about types of PMU used, the consensus is that the minimum number of PMUs is approximately one-third the total number of buses. A compromise between the full nonlinear and linear formulations is described in [92]. An interesting sub problem is the issue of sequentially adding PMUs to a system. With limited annual investments it would be desirable to add a limited number of PMUs until a final goal was achieved. Initially there will not be enough PMUs to have a linear estimator. The attempt is to place the PMUs so that at each stage the selection satisfies some design criteria. One criterion is the degree of unobservability [94], which measures how far away a bus, is from a PMU. The solution to this problem is available using the technique from [95]

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}. \quad \dots\dots\dots (3.1)$$

The bus–bus incidence matrix A is a square matrix with the dimension of the number of buses. There is a one on each diagonal and a one in the ij th position if bus i is connected to bus j . The A matrix for the IEEE 14 bus system is given in (3.1). If we imagine placing a PMU at bus 2, for example, we would learn the voltages at buses 1, 2, 3, 4, and 5, which are the nonzero entries in column 2 of A.

The placement problem for complete observability is then given by the integer programming problem formulation as shown in (3.2) below [96]. It is interesting to note that the powers of the

incidence matrix have an interpretation that helps extend the integer programming problem to the partial observability problem

$$\min f^T x$$

$$\text{Subject to } Ax > 0, x_i = 1 \text{ or } 0 \quad \dots\dots\dots (3.2)$$

$$f^T = [1 \ 1 \ 1 \ . \ . \ 1]$$

Theorem: The ij entry in the n th power of the incidence matrix for any graph or digraph is exactly the number of different paths of length n , beginning at vertex i and ending at vertex j [97].

$$\text{sign}(A^2) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \end{bmatrix} \quad \dots\dots\dots (3.3)$$

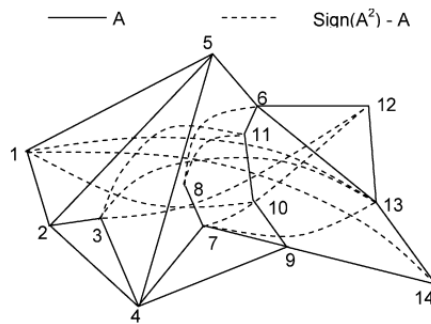


Fig.3.2. the incidence matrix for the IEEE 14-bus system along with the added branches

The proof is by induction. The signum of A^2 is the incidence matrix of another graph that has branches added to the graph (the dotted line) in Fig.3.2. The dashed branches in Fig. 3.2 represent connections between nodes that pass through another node, i.e., those nodes are one step removed. If we imagine the complete observability problem is solved [x_1 is the solution to (3.4)] the one degree of unobservability problem is the solution of

$$\begin{aligned} \min f^T x_2 \\ \text{Subject to } \text{sign} (A^2)x_2 > 0, x_{2i} = 1 \text{ or } 0 \end{aligned} \quad \dots\dots\dots (3.4)$$

$$f^T = [1 \ 1 \ 1 \ . \ . \ . \ 1]$$

And the addition of the constraint $(f - x_1)^T x_2 = 0$ forces the solution to be chosen from the PMU location in x_1 [98]. So in general the sequence of integer programming problems in (3.5)

$$\begin{aligned} \min f^T x_n \\ \text{Subject to } \text{sign} (A^{n+1})x_n > 0 \end{aligned} \quad \dots\dots\dots (3.5)$$

$$(f^T - x_{n-1})^T x_n = 0$$

Produce a nested sequence of PMU locations.

3.2. TRADITIONAL STATE ESTIMATION

This section presents the formulation of Traditional State Estimation in which conventional measurements, such as power flow, power injections, and voltage magnitudes, are pondered.

3.2.1. SE with Conventional Measurements

Traditional State Estimation (TSE) refers to a procedure for obtaining all the voltage phasors of a given power network. The system is modeled by the bus-branch model; buses represent the substations, and PI models indicate the transmission lines and transformers. Fig. 3.3 portrays a power system at the bus-branch model.

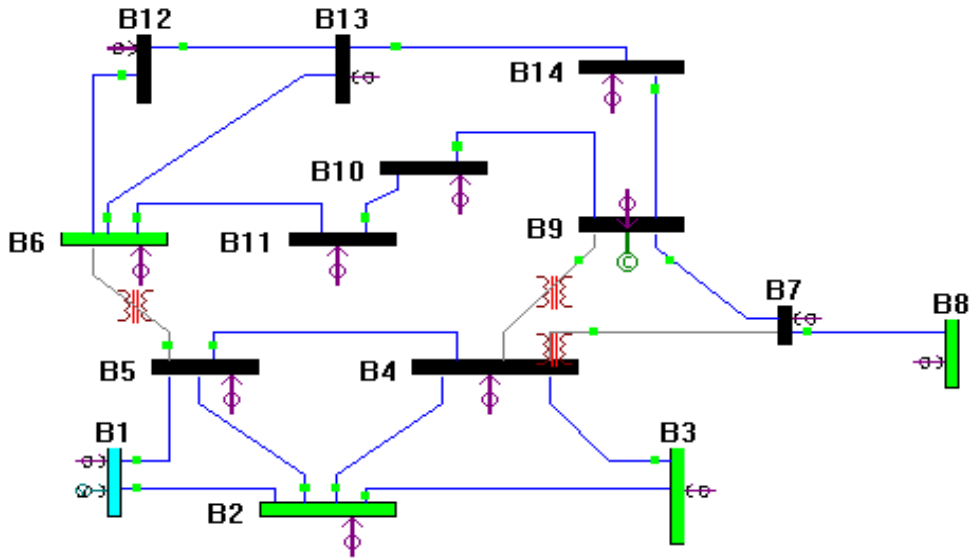


Fig.3.3. IEEE 14-Bus system

In the TSE, SCADA system gathers all the real-time measurements spread all over the power system, and it processes them in the control center. SE algorithms will provide the most likely estimated states of the entire network if the system is observable. Since real time measurements contain errors, and such errors have a Gaussian (Normal) distribution, its variance depends on the measuring device precision. Thus, the procedure for obtaining the estimated states uses a statistical approach.

The classical and established SE method is the Weighted Least Squares (WLS), which relies on the following measurement model, shown in Eq. (3.6).

$$z = h(x) + e \quad \dots\dots\dots (3.6)$$

Where:

z : is the measurement vector, with size m ;

x : is the state vector, with size n ;

$h(x)$: is the nonlinear function relating the measurements to the system states;

e: is the vector of measurements errors;

m: is the number of measurements;

n: is the number of states.

The state vector x has a dimension of $2N - 1$, where N is the number of the buses, and is given by Eq. (3.7).

$$x = [\theta_2 \ \theta_3 \ \dots \ \theta_n \ V_1 \ V_2 \ V_3 \ \dots \ V_n]^T \dots\dots\dots (3.7)$$

In Eq. (3.7) the bus 1 is chosen as the slack bus, with the phase angle set to zero, what provides a system reference. Eq. (3.8) represents the measurement function $h(x)$ vector, which relates each measurement to the state variables:

$$h(x) = \begin{bmatrix} P_{km} \\ P_k \\ Q_{km} \\ Q_k \\ V_k \end{bmatrix} \dots\dots\dots (3.8)$$

where:

P_{km} : refers to active power flow measurements from a generic bus k to bus m ;

P_k : refers to active power injection measurements at a generic bus k ;

Q_{km} : refers to reactive power flow measurements from a generic bus k to bus m ;

Q_k : refers to reactive power injection measurements at a generic bus k ;

V_k : refers to voltage magnitude measurements at a generic bus k ;

The purpose of the WLS SE is to obtain the state variables, which minimizes the following objective function, presented in the Eq. (3.9)

$$J(x) = \sum_{i=1}^m (z_i - h_i(x))^2 / R_{ii} = [z - h(x)]^T R^{-1} [z - h(x)]$$

..... (3.9)
 where:

R: is a diagonal covariance matrix, given by $R = \text{diag}\{\sigma_1^2, \sigma_2^2, \dots, \sigma_m^2\}$.

In others words, the WLS SE aims at minimizing the sum of weighted measurement residues.

A minimum is found once the first-order optimality conditions are satisfied, and it is achieved by a first-order approximation, given in the following Eq. (3.10):

$$g(x) = \frac{\partial J(x)}{\partial x} = -H^T(x)R^{-1}[z_i - h_i(x)] = 0 \dots\dots\dots(3.10)$$

Where: $H(x)$: is the Jacobean matrix, given by Eq. (3.11).

$$H(x) = \frac{\partial h(x)}{\partial x} = \begin{bmatrix} \frac{\partial P_{km}}{\partial \theta} & \frac{\partial P_{km}}{\partial V} \\ \frac{\partial P_k}{\partial \theta} & \frac{\partial P_k}{\partial V} \\ \frac{\partial Q_{km}}{\partial \theta} & \frac{\partial Q_{km}}{\partial V} \\ \frac{\partial Q_k}{\partial \theta} & \frac{\partial Q_k}{\partial V} \\ \frac{\partial V_k}{\partial \theta} & \frac{\partial V_k}{\partial V} \end{bmatrix} \dots\dots\dots(3.11)$$

By expanding the nonlinear function $g(x)$ into its Taylor series around the state vector x^k , and by neglecting the higher order terms, it can lead to an iterative solution scheme, given in Eq. (3.12):

$$G(x^k)\Delta x^{k+1} = H^T(x^k)W[z - h(x^k)] \dots\dots\dots(3.12)$$

where:

$G(x^k) = H^T(x^k)WH(x^k)$: is the Gain matrix;

$W = R^{-1}$: is the weighting matrix;

$\Delta x^k = x^{k+1} - x^k$: being k the iteration index.

In [89] it has been presented a step by step algorithm to solve the traditional state estimation problem, which is summarized as follows:

1. Set iteration index k equal to zero;
2. Initialize the state vector x^k in a flat start (voltage magnitudes equal to one and Voltage angle equal to zero);
3. Calculate the Gain matrix $G(x^k)$;
4. Calculate the equation $H^T(x)R^{-1}[Z - h_i(x^k)]$;
5. Determine Δx^k ;
6. Test for convergence, i.e. $\max|\Delta x^k| \leq \varepsilon$, where ε is the tolerance;
7. If the tolerance is attained, stop. If not, update $x^{k+1} = x^k + \Delta x^k$, $k=k+1$ and go back to step 3.

3.2.2. SE with Synchronized Phasor Measurements

The PMU advent recalls as a distance relay with symmetrical components, developed in the 70's at Virginia Tech Laboratory. The capability of obtaining synchronized measurements with GPS time stamp has developed important advances, for instance: PMU provides the magnitude and the angle of the voltage at the bus where it is connected; it is usable as a measurement in the state estimation equations and it does not required to set a slack bus, as reported in (ZHU; ABUR, 2007). Hence, the state vector must include all the voltage angles and magnitudes, with a dimension $2N$, as presented in Eq below.

$$x = [\theta_1 \ \theta_2 \ \theta_3 \ \dots \ \theta_n \ V_1 \ V_2 \ V_3 \ \dots \ V_n]^T.$$

PMU not only provides the voltage phasors but it also can provide current phasor measurements of adjacent power lines, depending upon the number of channels (KORKALI; ABUR, 2009). This way, the current phasors may also be included as measurements in state estimation equations.

Considering the current flow from a bus to another, one can say that it is usually measured and transmitted in the polar form, i.e. current magnitude (I_{km}) and angle (δ_{km}). It is preferable, however, to use them in the rectangular form, due to numerical problems in case of either lightly load systems or flat start initialization [90]. Not with standing, the rectangular coordinates present disadvantage as it will amplify errors of PMU measurements. Such conversion is simple to apply, as it can be seen at the set of Eq. (3.13).

$$I_{km}^{Re} = I_{km} \cos(\delta_{km}) \dots\dots\dots (3.13)$$

$$I_{km}^{Im} = I_{km} \sin(\delta_{km})$$

Where:

I_{km}^{Re} and I_{km}^{Im} : refers to the real and imaginary parts of phasor current measurement;

I_{km} : refers to the phasor current magnitude, measured by PMU;

δ_{km} refers to the phasor current angle, measured by PMU.

Thus, the measurement function equations and Jacobean matrix elements must be properly adapted to accommodate the new measurements, as follows:

$$h(x) = \begin{bmatrix} P_{km} \\ P_k \\ Q_{km} \\ Q_k \\ V_k \\ \theta_k \\ I_{km}^{Re} \\ I_{km}^{Im} \end{bmatrix} \dots\dots\dots (3.14)$$

$$H(x) = \partial h(x) / \partial x = \begin{bmatrix} \partial P_{km} / \partial \theta & \partial P_{km} / \partial V \\ \partial P_k / \partial \theta & \partial P_k / \partial V \\ \partial Q_{km} / \partial \theta & \partial Q_{km} / \partial V \\ \partial Q_k / \partial \theta & \partial Q_k / \partial V \\ \partial V_k / \partial \theta & \partial V_k / \partial V \\ \partial \theta_k / \partial \theta & \partial \theta_k / \partial V \\ \partial I_{km}^{Re} / \partial \theta & \partial I_{km}^{Re} / \partial V \\ \partial I_{km}^{Im} / \partial \theta & \partial I_{km}^{Im} / \partial V \end{bmatrix} \dots\dots\dots (3.15)$$

By deriving the current phasor flow equations as a function of the power flows, it is possible to determine the partial derivatives of the Jacobean matrix, as follows:

$$I_{km}^{Re} = \frac{P_{km} \cos(\theta_k) + Q_{km} \sin(\theta_k)}{V_k} \dots\dots\dots (3.16)$$

$$I_{flow}^{Im} = \frac{P_{km} \sin(\theta_k) - Q_{km} \cos(\theta_k)}{V_k}$$

where:

V_k : refers to the voltage magnitude at the sending bus k, measured by the PMU;

θ_k : refers to the voltage angle at the sending bus k, measured by the PMU.

Having the above mentioned in mind, the partial derivatives of real part of the current phasor will be:

$$\begin{aligned} \frac{\partial I_{km}^{Re}}{\partial \theta_k} &= \frac{1}{V_k} \left[\cos(\theta_k) \left(\frac{\partial P_{km}}{\partial \theta_k} + Q_{km} \right) - \sin(\theta_k) \left(\frac{\partial Q_{km}}{\partial \theta_k} - P_{km} \right) \right] \\ \frac{\partial I_{km}^{Re}}{\partial \theta_m} &= \frac{1}{V_k} \left[\frac{\partial P_{km}}{\partial \theta_m} \cos(\theta_k) + \frac{\partial Q_{km}}{\partial \theta_m} \sin(\theta_k) \right] \end{aligned} \dots (3.17)$$

$$\frac{\partial I_{km}^{Re}}{\partial V_k} = \frac{1}{V_k} \left[\frac{\partial P_{km}}{\partial V_k} \cos(\theta_k) + \frac{\partial Q_{km}}{\partial V_k} \sin(\theta_k) \right] - \frac{1}{V_k^2} [P_{km} \cos(\theta_k) + Q_{km} \sin(\theta_k)]$$

$$\frac{\partial I_{km}^{Re}}{\partial V_m} = \frac{1}{V_k} \left[\frac{\partial P_{km}}{\partial V_m} \cos(\theta_k) + \frac{\partial Q_{km}}{\partial V_m} \sin(\theta_k) \right].$$

considering partial derivatives of imaginary part of the current phasor:

$$\begin{aligned}
\frac{\partial I_{km}^{Im}}{\partial \theta_k} &= \frac{1}{V_k} \left[\cos(\theta_k) \left(P_{km} - \frac{\partial Q_{km}}{\partial \theta_k} \right) + \sin(\theta_k) \left(Q_{km} + \frac{\partial P_{km}}{\partial \theta_k} \right) \right] \\
\frac{\partial I_{km}^{Im}}{\partial \theta_m} &= \frac{1}{V_k} \left[\frac{\partial P_{km}}{\partial \theta_m} \sin(\theta_k) - \frac{\partial Q_{km}}{\partial \theta_m} \cos(\theta_k) \right] \dots\dots\dots (3.18) \\
\frac{\partial I_{km}^{Im}}{\partial V_k} &= \frac{1}{V_k} \left[\frac{\partial P_{km}}{\partial V_k} \sin(\theta_k) - \frac{\partial Q_{km}}{\partial V_k} \cos(\theta_k) \right] - \frac{1}{V_k^2} [P_{km} \sin(\theta_k) - Q_{km} \cos(\theta_k)] \\
\frac{\partial I_{km}^{Im}}{\partial V_m} &= \frac{1}{V_k} \left[\frac{\partial P_{km}}{\partial V_m} \sin(\theta_k) - \frac{\partial Q_{km}}{\partial V_m} \cos(\theta_k) \right].
\end{aligned}$$

Moreover, there are the partial derivatives of voltage angle measurements, which are linearly, related to the states, so that:

$$\frac{\partial \theta_k}{\partial \theta_k} = 1, \frac{\partial \theta_k}{\partial \theta_m} = 0, \frac{\partial \theta_k}{\partial V_k} = 0, \frac{\partial \theta_k}{\partial V_m} = 0 \dots\dots\dots (3.19)$$

after such modifications, the process for obtaining the states is the same as that one from the previous section, applying the normal equation and performing the previous algorithm.

The General state estimation algorithm Proposed, i.e. the Weighted Least Square algorithm, has been generalized in the following flow chart.

3.3. SIMULATION SOFTWARE

In this thesis, a well-known computing language MATLAB and the network analysis tool PET “POWER EDUCATION TOOL” is used for simulation purposes. When starting state estimation simulation, it carries out the active power flow and injection measurements, the reactive power flow and injection measurements, the voltage magnitude and the current magnitude measurements. The in-built programming language, MATLAB optimization tool box has been used for the optimization to solve the linear integer programming problem and PET for observability analysis. And at last a written state estimation code developed by me has been used for state estimation problem. This section briefly discusses about the soft wares. i.e.

MATLAB(Matrix Laboratory): is a multi-paradigm numerical computing environment and fourth- generation programming language .A proprietary programming language developed by math

works, MATLAB allows matrix manipulations ,plotting of functions and data, implementation of algorithms, creation of user interfaces, and interfacing with programs written in other language.

Although MATLAB is intended primarily for numerical computing, an optional toolbox uses the MuPAD symbolic engine; allowing access to symbolic computing abilities .additional packages, Simulink adds graphical multi domain simulation and model based design for dynamic and embedded system.

P.E.T (Power Education Toolbox): is an interactive software package that has been developed primarily as an educational tool in studying state estimation ,bad data and network observability analysis algorithm. In addition to these functions, the software package also supports a power flow function .power flow function is useful in creation of consistent measurement data to be used by the state estimator.

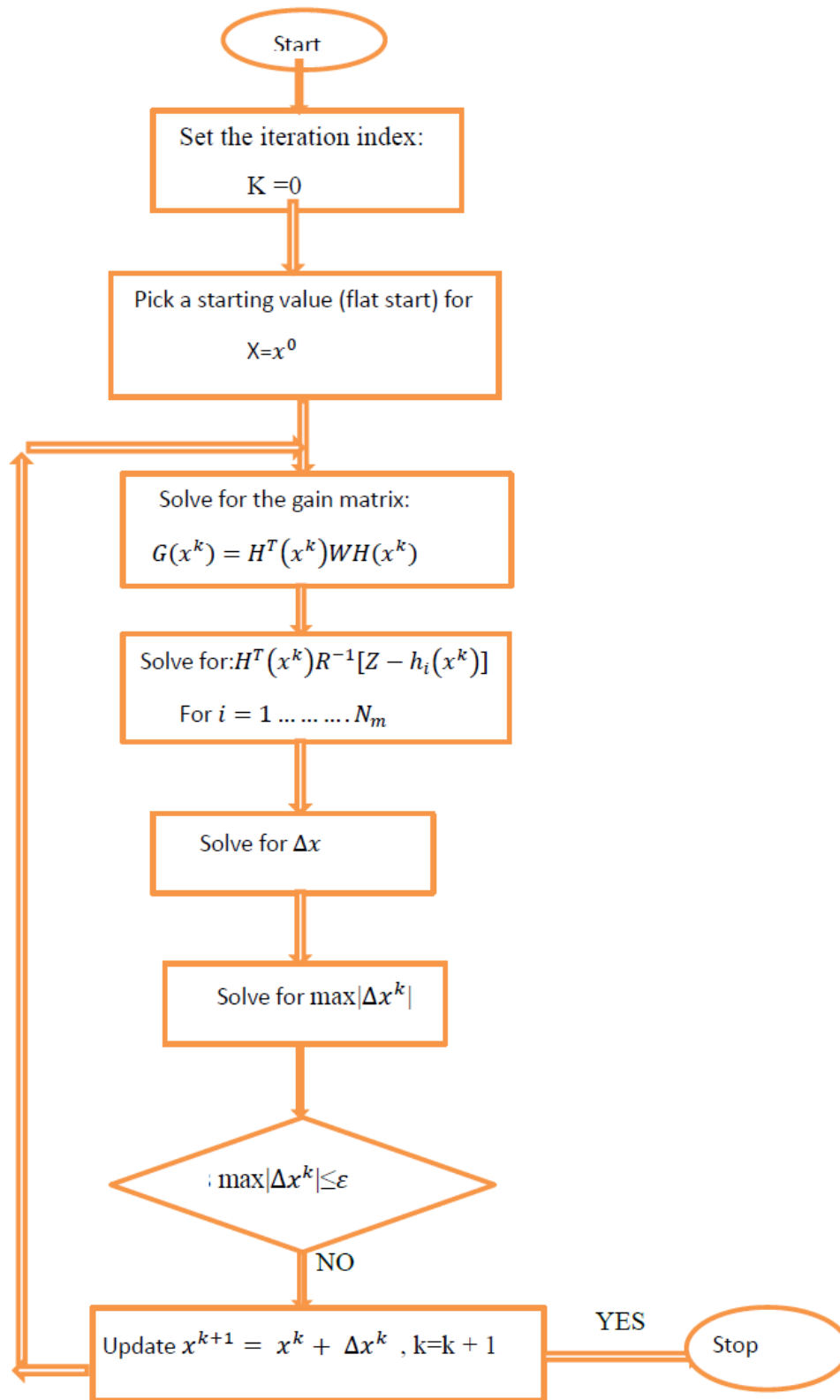


Diagram1: General WLS algorithm Flow chart

4. OPTIMAL PLACEMENT OF PMU FOR STATE ESTIMATION

4.1. INTRODUCTION

STATE estimation plays a critical role for secure power system operation. State estimated solutions aid operators in security assessment, in initiating appropriate control measures to effectively manage power system operations and enabling pricing calculations for real-time electricity markets. Therefore, robustness of the state estimator is extremely important. Since the robustness of the state estimator is influenced by the measurement configuration, measurements should be placed such that: 1) a certain level of redundancy is present, 2) the capabilities for bad data detection are improved, and 3) unobservability due to the loss of measurements is prevented. Conventional measurements commonly provided by the remote terminal units (RTU) at substations normally include real/reactive power flows, power injections, and magnitudes of bus voltages and branch currents. Most recently, with the advent of global positioning system (GPS), synchronized phasor measurements provided by phasor measurement units (PMU) are becoming available at selected substations in the system. Because of the PMU installation cost, only a few PMUs are currently being incorporated in power systems. So a natural question is how to place PMUs in an optimal (i.e., cost effective) way to improve the performance of state estimator in the presence of conventional measurements in power systems.

The central motivation for this Section is to achieve optimal PMU placement for observability in a manner that can also detect and eliminate bad data in critical and critical measurement pairs, which otherwise can render the state estimation problem unsolvable or bad data undetectable. Thus, system observability with PMU measurements is crucial from the point of state estimation. Since bad data detection techniques fail for critical measurements and critical measurement pairs [40], it is important to eliminate their occurrence whenever possible to improve the robustness of state estimators.

4.2. OPTIMALITY BY OBSERVABILITY ANALYSIS TYPE 1

The effectiveness of performing state estimation on a power network depends on the availability of enough and well distributed measurements throughout the system (MONTICELLI, 1999).

Conventionally, the observability analysis is carried out prior to the state estimation execution, and once the system is observable, it enables to perform further analysis. However, if the system is unobservable, it is yet useful to determine what portions are observable, as well as which parts are unobservable. As a consequence, it is possible to perform a partial state estimation, or use pseudo measurements to restate the system observability.

This Section addresses the concepts of Observability analysis in power networks modeled at the bus-branch levels. It discusses the numerical and topological approaches to determine the system observability along with further numerical methods to find unobservable branches and observable islands and how to place PMU optimally to make the system observable

4.2.1. TRADITIONAL OBSERVABILITY ANALYSIS: BUS-BRANCH ANALYSIS

In real time modeling, topology processor reduces the network from the bus section level into the bus-branch model, as it processes the status of switches and circuit breakers inside the substations. After that, the Traditional State Estimation is performed.

One of the first papers addressing observability issues calls for a topological approach by making usage of an iterative algorithm whose aim is to find a spanning tree of full rank. Conversely, the approach proposed by Monticelli and Wu (1985a, 1985b) claims for a numerical approach to determine the system observability by applying the triangular factorization in the Gain matrix.

For observability purposes, it is convenient to use a simplified linearized model that represents only the active part, and assuming 1.0 p.u. reactance. The reactive part should also be tested, but since the measurements usually come in pairs, the second part is seldom necessary

When it comes to Traditional Observability Analysis (TOA), the first step to be taken is the modeling of the measurements. There are three types of measurements: (i) analog, consisted by power flows, power injections, bus voltage magnitude, current magnitude, and also synchronized voltage and current phasor measurements; (ii) logical, composed by the status of switches and circuit breakers and (iii) pseudo measurements, consisted by forecasted bus loads and generations

Logical information is used for topology processing and it is performed prior the observability analysis. Pseudo measurements become relevant once the system is found unobservable, and are used to restore observability. Therefore, only the first type of measurements is considered for observability analysis and their modeling is addressed as follows.

(a) Power flow measurements

Figure 4.1 presents a line model connecting buses k-m, with a power flow measurement on it.

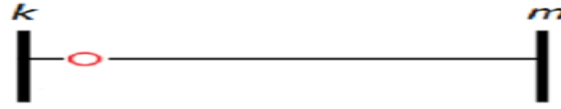


Fig 4.1. Power flow measurement

Assuming the line reactance (x_{k-m}) equal to 1.0 p.u., the following equation represents the linear power flow measurement.

$$P_{k-m}^m = \frac{1}{x_{k-m}} (\theta_k - \theta_m) = \theta_k - \theta_m \quad \dots\dots\dots (4.1)$$

(b) Power injection measurements

A power injection measurement at a bus is modeled as a summation of all power flows from all adjacent lines. Taking for instance the power injection measurement set at the generic bus t of the 3 bus system in Figure 4.2, its linear model is given by Eq. (4.2).

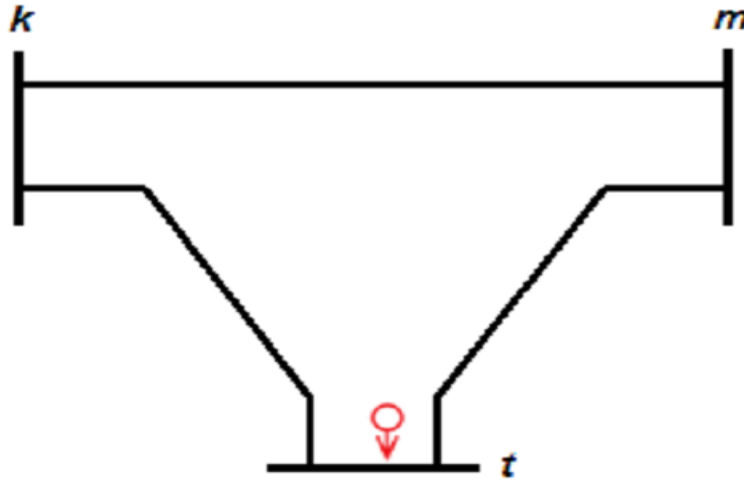


Fig.4.2. power injection measurement

$$P_t^m = \sum_i^{n_t} P_i = \frac{1}{x_{t-k}} (\theta_t - \theta_k) + \frac{1}{x_{t-m}} (\theta_t - \theta_m) = 2\theta_t - \theta_k - \theta_m \quad \dots\dots\dots (4.2)$$

Where:

n_t : refers to a set of all branches connected to bus t.

The measurements can be grouped in a matrix form to facilitate numerical analysis, such as the one presented ahead. Taking the system in Figure 4.3 as an example, with the given measurement design, the measurement matrix is formed as follows.

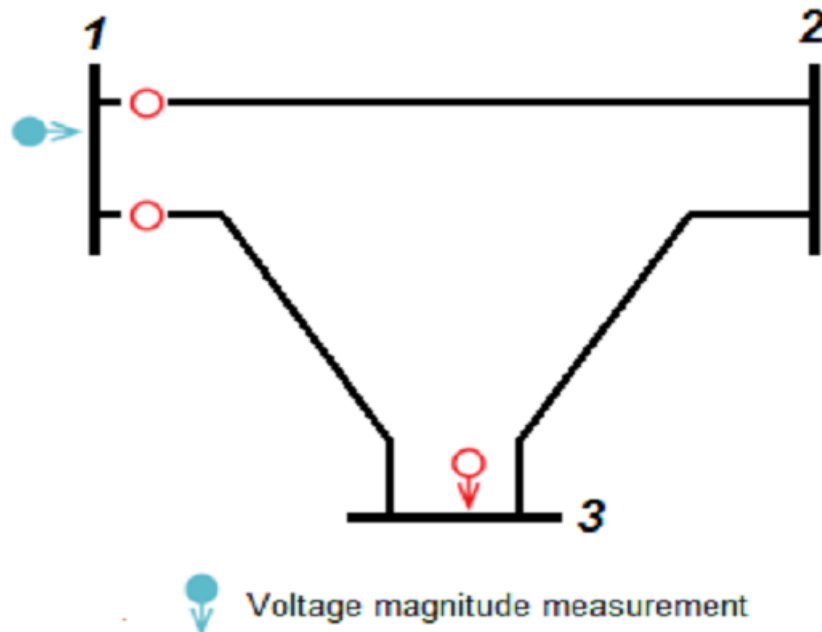


Fig 4.3. 3 bus network

4.2.1.1. Basic Numerical Method

Eq. (4.3) Represents the Jacobean matrix (measurement matrix) of the given measurement design.

$$H_{AA} = \begin{matrix} & \theta_1 & \theta_2 & \theta_3 \\ \begin{matrix} P_{1-2} \\ P_{1-3} \\ P_3 \end{matrix} & \begin{bmatrix} 1 & -1 & \\ 1 & & -1 \\ -1 & -1 & 2 \end{bmatrix} \end{matrix} \dots\dots\dots (4.3)$$

The reactive model requires an additional measurement, that is, the voltage magnitude. The voltage angle, in its turn, corresponds to the same in the active model. Computing the rank of the Jacobean matrix in Eq. (4.3) it is possible to determine whether the network is observable or not, regarding the available measurements. If the Jacobean matrix has a full rank (i.e. Rank $(H_{AA}) = n = N_b - 1$, where n is the number of states and N_b the number of buses) the system is

said observable. In the example of Figure 4.3, the corresponding rank is full ($\text{rank}(H_{AA}) = N_b - 1 = 2$), rendering the system observable.

Another way to determine the system observability is by computing the Gain matrix and performing the triangular factorization, as in Eq. (4.4) and (4.5) (MONTICELLI; WU, 1985b).

$$G = H_{AA}^T H_{AA} = \begin{matrix} & \theta_1 & \theta_2 & \theta_3 \\ \begin{matrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{matrix} & \begin{bmatrix} 3 & -3 \\ 2 & -2 \\ -3 & -2 & 5 \end{bmatrix} \end{matrix} \dots\dots\dots (4.4)$$

$$U = \begin{matrix} & \theta_1 & \theta_2 & \theta_3 \\ \begin{matrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{matrix} & \begin{bmatrix} 1.7 & -1.7 \\ & 1.4 & -1.4 \\ & & 0 \end{bmatrix} \end{matrix} \dots\dots\dots (4.5)$$

The existence of only one zero pivots, i.e. a zero in the diagonal of U matrix, render the system observable. Such fact indicates that only one angular reference is required, that is, an angle constraint ($\theta_3 = 0$ for instance), and it provides an angular reference. Having more zero pivots, the system is unobservable.

4.2.1.2. Basic Topological Method

In the topological approach, the power flow and injection measurements are processed as edges connecting the vertices (representing network buses) so as to form an observable spanning tree (KRUMPHOLZ; CLEMENTS; DAVIS, 1980). Basically, a power flow measurement between buses k and m, such as presented in Figure 5.1, is processed as an edge connecting the related vertices k and m, as shown in Figure 4.4.



Fig 4.4. Edge of a power flow measurement

Power injection measurements, on the other hand, can form edges with all the adjacent vertices. In Figure 4.5, for instance, the injection measurement at bus t forms edges connecting vertices k and m . However, only one edge can be used to ensure the network observability. In Figure 4.5, only one of the two edges can be picked.



Fig 4.5. Edges of a power injection measurement

Observability analysis is carried out considering the $P - \theta / Q - V$ decoupling principle, since the measurements come in pairs. To accomplish the analysis, an angular reference must be provided for $P - \theta$ observability, and at least one voltage magnitude measurement is required to ensure $Q - V$ observability. They are treated as a fictitious flow, which connects a vertex to an extra (ground) node (CLEMENTS; DAVIS; KRUMPHOLZ, 1981). Figure 4.6. Portrays the measurement graph of the network shown in Figure 4.3, which forms an Observable Spanning Tree (OST).

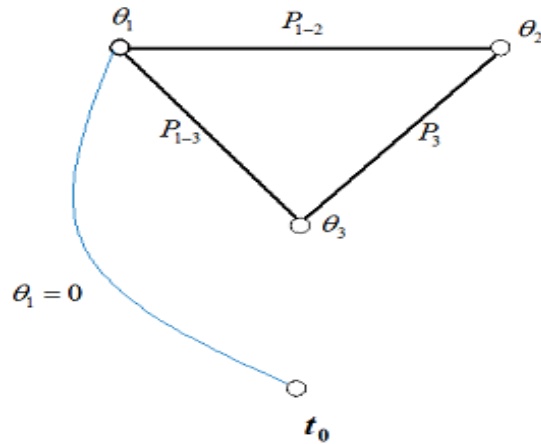


Fig 4.6. Measurement graph of 3 bus network

4.2.1.3. Identification of Observable Islands

Further analysis can be carried out, for instance, measurement criticality, state estimation, bad data and so forth, if a network is observable. On the other hand, if the network is unobservable, it is desirable to find the observable islands and unobservable branches.

The first step consists of identifying and removing the irrelevant branches with the absence of flow measurements, and injection measurements on its adjacent buses. Taking for instance the example of Monticelli and Wu (1985b) presented in Figure 4.7, the branch 2-3 is found irrelevant and its corresponding row is eliminated from incidence matrix A, in Eq. (4.6).

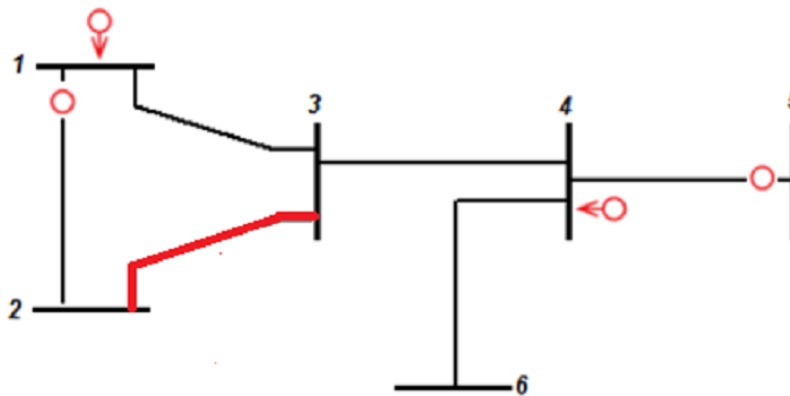


Fig .4.7. 6 bus network example

$$A = \begin{matrix} & \theta_1 & \theta_2 & \theta_3 & \theta_4 & \theta_5 & \theta_6 \\ \begin{matrix} b_{1-2} \\ b_{1-3} \\ b_{3-4} \\ b_{4-5} \\ b_{6-4} \end{matrix} & \begin{bmatrix} 1 & -1 & & & & \\ 1 & & -1 & & & \\ & & 1 & -1 & & \\ & & & 1 & -1 & \\ & & & -1 & & 1 \end{bmatrix} \end{matrix} \quad \dots\dots\dots(4.6)$$

In the next step, the measurements are processed forming the Jacobean matrix H and the corresponding Gain matrix G, as presented in Eq. (4.7) and (4.8), respectively.

$$H_{AA} = \begin{matrix} & \theta_1 & \theta_2 & \theta_3 & \theta_4 & \theta_5 & \theta_6 \\ \begin{matrix} P_1 \\ P_4 \\ P_{1-2} \\ P_{4-5} \end{matrix} & \begin{bmatrix} 2 & -1 & -1 & & & \\ & -1 & -1 & 3 & -1 & -1 \\ 1 & -1 & & & & \\ & & & 1 & -1 & \end{bmatrix} \end{matrix} \quad \dots\dots\dots(4.7)$$

$$G = \begin{matrix} & \theta_1 & \theta_2 & \theta_3 & \theta_4 & \theta_5 & \theta_6 \\ \begin{matrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \end{matrix} & \begin{bmatrix} 5 & -3 & -2 & & & \\ -3 & 2 & 1 & & & \\ -2 & 1 & 2 & -3 & 1 & 1 \\ & & -3 & 10 & -4 & -3 \\ & & 1 & -4 & 2 & 1 \\ & & 1 & -3 & 1 & 1 \end{bmatrix} \end{matrix} \quad \dots\dots\dots(4.8)$$

At this point, is possible to evaluate the rank of matrix H and to certify the system unobservability. Performing the triangular factorization of the Gain matrix G, two zero pivots are found, as follows.

$$U = \begin{matrix} & \theta_1 & \theta_2 & \theta_3 & \theta_4 & \theta_5 & \theta_6 \\ \begin{matrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \end{matrix} & \begin{bmatrix} 2.24 & -1.34 & -0.89 & & & \\ & 0.45 & -0.45 & & & \\ & & 1 & -3 & 1 & 1 \\ & & & 1 & -1 & \\ & & & & 0 & \\ & & & & & 0 \end{bmatrix} \end{matrix} \dots\dots\dots (4.9)$$

Those two zero pivots indicate that the system is divided into two observable islands (MONTICELLI; WU, 1985b), and two angular references must be provided, on buses 5 and 6. Replacing the zero pivots by “1”s, at positions (5, 5) and (6, 6) of U matrix, and changing the corresponding number in the right vector t_A by random numbers, one can determine the unobservable state, as in Eq. (4.10).

$$\hat{\theta} = (U^T U)^{-1} t_A \dots\dots\dots(4.10)$$

Where:

$$t_A = [0 \ 0 \ 0 \ 0 \ 0 \ 1]^T$$

The vector t_A is a modified vector, with zeros in the positions related to nonzero pivots, and random numbers on the positions related to zero pivots. Integer numbers such as 0, 1, 2, etc. are usually used.

We will find the estimated state $\hat{\theta}$ as $[-0.996, -1, -1, 0, 0, 1]^T$ and put 1 at zero place and 0 at non zero positions and then multiplying the estimated state $\hat{\theta}$ by the incidence matrix A, it obtains the power flows through the branches, as follows.

$$P_b = A\hat{\theta} = \begin{matrix} & \theta_1 & \theta_2 & \theta_3 & \theta_4 & \theta_5 & \theta_6 \\ \begin{matrix} b_{1-2} \\ b_{1-3} \\ b_{3-4} \\ b_{4-5} \\ b_{6-4} \end{matrix} & \begin{bmatrix} 1 & -1 & & & & \\ 1 & & -1 & & & \\ & & 1 & -1 & & \\ & & & 1 & -1 & \\ & & & -1 & & 1 \end{bmatrix} \end{matrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ -1 \end{bmatrix}$$

where:

bk-m: refers to a branch from generic bus k to m.

The non-zero flows indicate the unobservable branches. In such case, branches 3-4 and 6-4 are unobservable and the corresponding row is removed from matrix A.

After that, the irrelevant injection measurements must be identified and removed from the set of interest. The HA A and A matrices are updated by removing the rows related to the irrelevant injection measurements in HA A, and unobservable branches in A. Therefore, the power flows are computed again, unobservable branches are removed, as well as irrelevant measurements, until no non-zero flows are found. In the example of Figure 4.7, the injection measurement at bus 4 is irrelevant, since it has at least one adjacent unobservable branch.

By adapting the matrix HA A, processing the new Gain matrix and performing the triangular factorization, the new U matrix is obtained, as follows:

$$U = \begin{matrix} & \theta_1 & \theta_2 & \theta_3 & \theta_4 & \theta_5 & \theta_6 \\ \begin{matrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \end{matrix} & \begin{bmatrix} 2.24 & -1.34 & -0.89 & & & \\ & 0.45 & -0.45 & & & \\ & & 0 & & & \\ & & & 1 & -1 & \\ & & & & 0 & \\ & & & & & 0 \end{bmatrix} \end{matrix} \dots\dots\dots (4.11)$$

Differently, three zero pivots are found and by changing them into one, and applying the Eq. (4.10) with the following vector $tA = [0 \ 0 \ 0 \ 0 \ 1 \ 2]$, gives $\hat{\theta} = [0 \ 0 \ 0 \ 1 \ 1 \ 2]$ so that the power flows are evaluated as follows.

$$P_b = A\hat{\theta} = \begin{matrix} b_{1-2} \\ b_{1-3} \\ b_{4-5} \end{matrix} \begin{bmatrix} 1 & -1 & & & \\ 1 & & -1 & & \\ & & & 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \dots\dots\dots (4.12)$$

As a consequence, the process stops as all the power flows are null.

Although the process has stopped, it is necessary to determine if the irrelevant branches that were removed at the beginning of the process, are observable or not. In this example, only the branch 2-3 was labeled irrelevant. Updating the A matrixes, taking into account the irrelevant branch and removing the already labeled unobservable branches, we have the following matrix, shown in Eq. (4.13):

$$P_b = A\hat{\theta} = \begin{matrix} b_{1-2} \\ b_{1-3} \\ b_{4-5} \end{matrix} \begin{bmatrix} 1 & -1 & & & \\ 1 & & -1 & & \\ & & & 1 & -1 \end{bmatrix} \begin{array}{c} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{array} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \dots\dots\dots (4.13)$$

Since all the power flows are null, all those branches are labeled observable. The islands are identified by selecting the appropriate buses from the estimated vector $\hat{\theta}$. In this case, θ_1, θ_2

and $\theta 3$ form an island, $\theta 4$ and $\theta 5$ form another, and bus $\theta 6$ forms the last one, as depicted in Figure 4.8 a&b.

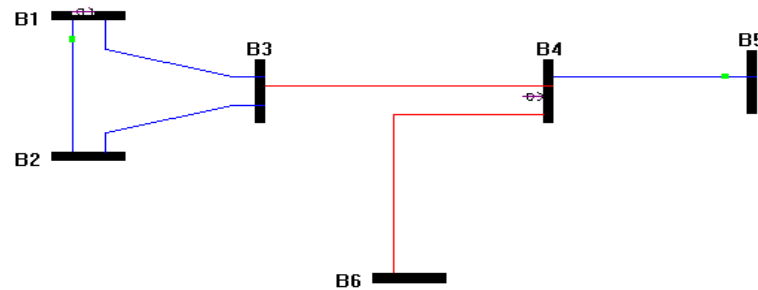


Fig.4.8.(a), 6 bus network example indicating non observable branches

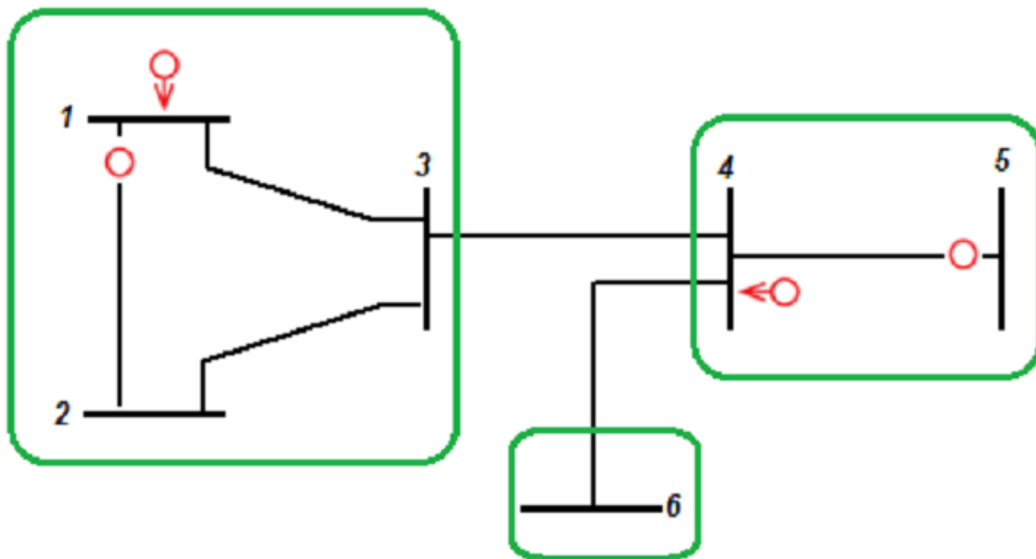


Fig.4.8.(b). 6 bus network example indicating islands.

Hence in the above analysis it is observed that the super node positions i.e. the nodes where having more branches, are the optimal position to place PMU to make the system fully observable

As depicted in the fig above there are two super nodes, bus 3 and bus 4 having three branches.

Now to choose one optimal position of PMU among the two buses where placing the pmu at either bus will make the system observable and we will use the sensitivity analysis method.

The criticality analysis can be carried out using the traditional method, as described in [40]. The Sensitivity matrix (S) is computed as described in Eqn. 4.14.

$$S = I - H_{AA}(H_{AA}^T H_{AA})^{-1} H_{AA}^T \dots\dots\dots (4.14)$$

By computing the measurement function by using eq.4.1 &4.2, the Jacobean (H) matrix can be formed as:

$$H_{AA} = \begin{matrix} & \theta_1 & \theta_2 & \theta_3 & \theta_4 & \theta_5 & \theta_6 \\ \begin{matrix} P_1 \\ P_4 \\ P_{1-2} \\ P_{4-5} \end{matrix} & \begin{bmatrix} 2 & -1 & -1 & & & \\ & -1 & -1 & 3 & -1 & -1 \\ 1 & -1 & & & & \\ & & & 1 & -1 & \end{bmatrix} \end{matrix}$$

And computing the sensitivity matrix using eq 4.14 becomes

$$S = \begin{bmatrix} -0.7500 & -0.3750 & -0.2500 & 0 \\ -0.7500 & -0.5000 & -0.5000 & 0 \\ -0.2500 & -0.2500 & 0 & 0 \\ 0 & 0.0625 & 0 & 0 \end{bmatrix}$$

Diag, S= {-0.75,-0.5, 0, 0}

Depending on the diagonal of matrix S, the following column vector is defined:

$$b = [0 \ 0 \ 1 \ 1]^T$$

If the diagonal element in S is non-zero, the corresponding element in b is 0; if the diagonal element in S is zero, the corresponding element in b is 1, which implies that this position can convert critical measurement to non-critical [40].

The zero diagonals entries in matrix S represent critical measurements. As seen from the sensitivity matrix S the 3rd and 4th measurement (the 3rd & 4th diagonal entries is null) which implies that the 3rd and 4th measurement in the Jacobean H matrix is the critical measurements which make either the state estimation problem unsolvable or bad data undetectable. Hence placing a PMU unit at bus 3 and at bus 4 or one of bus 3's and bus 4's neighboring buses is essential to convert this critical measurement into a non-critical measurement. In this example, we know buses 3 and 4 corresponding to null diagonal entries and buses 1, 2, 5 and 6 are associate buses to buses 3 and 4. To add a PMU unit at any one of buses 1, 2, 3, 4, 5, and 6 can convert the critical measurement to non-critical. For example, if a PMU is placed at bus 3, the angles at buses 1, 2, 3, and 4 are measured. Hence the following row vector is defined:

$$T_3 = \begin{matrix} \theta_1 & \theta_2 & \theta_3 & \theta_4 & \theta_5 & \theta_6 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{matrix}$$

Each element in T3 corresponds to a bus angle. If its value is 1, then it means the corresponding bus angle is obtained by the installed PMU unit; and 0 otherwise. Considering all the possible buses (1, 2, 3, 4, 5, 6) to place a PMU, one has a T matrix of:

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

Then the integer linear programming formulation is

$$\text{Min } f = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$$

$$\text{St. TX} = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6]^T$$

$$\geq \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$X = \{x_1, x_2, \dots, x_N\}$$

$$x_i \in \{0, 1\}$$

It is easy to see that the first two and the last two inequalities in the above constraints are always true and become unnecessary. After removing those rows in matrix T and zero elements in the inequalities (b) (and remove zero columns in T and their corresponding variables if any), and one has

$$\text{Min } f = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$$

$$\text{St. TX} = \begin{vmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{vmatrix} [x_1 + x_2 + x_3 + x_4 + x_5 + x_6]^T \geq [1 \ 1]^T$$

The solution of the above ILP is

$$x = [0 \ 0 \ 0 \ 1 \ 0 \ 0]^T$$

This indicates that placing a PMU unit at bus 4 can convert critical measurement to non-critical. It is optimal place.

4.3. SYSTEM OBSERVABILITY IN STATE ESTIMATION TYPE 2

Consider the addition of a new measurement to the existing set of measurements. This will modify the existing gain matrix G , as follows:

$$G' = G + h_k^T h_k \dots\dots\dots(4.15)$$

where h_k , represents the row of H corresponding to the new measurement. Using the triangular factors of G , as derived in [monticelii] the above equation can be re-written as:

$$L^{-1}G'L^{-T} = D + L^{-1}h_k^T h_k L^{-T}$$

Denoting $M = L^{-1}h_k$ yields the following result [40]

Theorem 3.1: $\text{Rank}(G') = \text{Rank}(G) + 1$ if and only if, $M(i) \neq 0$ for an i such that $D(i,i) = 0$.

Theorem 3.1 implies an algorithm for PMU placement. That is, using the triangular factors of G , one can find a minimal set of PMUs such that, for all i $M(i) \neq 0$.

A PMU can provide the measurement of voltage phasor at the installed bus and measurements of current phasors of the lines associated to this bus. Usually, these voltage and current phasor measurements are used in the formulation of observability analysis and state estimation. In the proposed algorithm, only measured or computed voltage phasors related to PMU measurements are used. In other words, the angle measurements from a PMU bus and the angles at the buses adjacent to a PMU bus using the corresponding current phasor measurements are utilized. For instance, the measurement vector of a PMU installed at bus i can be expressed as

$$T_i = [0 \quad \dots \quad 1 \quad \dots \quad 0 \quad \dots \quad 1 \quad \dots].$$

In the measurement vector T_i , elements 1 correspond to the buses having angle measurements; while elements 0 correspond to buses not adjacent to bus i .

An ILP formulation for optimal PMU placement is then given by:

$$\min C^T X_m$$

$$S.T. \quad TX_m \geq b_m$$

where $C = [1 \ 1 \ \dots \ 1 \ 1]^T$, $X_m = [x_1 x_2 \ \dots \ x_m]^T$ is an integer variable vector, corresponding to the candidate buses for PMU placement. b_m is a unitary column vector with a length of m .

Algorithm: Step 1) Obtain the matrix D from the conventional measurements using the method proposed in [40]. Form vector b_m according to D .

Step 2) Using vector b_m , find all possible buses that can increase the rank of D if a PMU is placed at the bus.

Step 3) Use the proposed ILP formulation to solve for X . where by the state estimation result will be improved.

Note: The number of rows in matrix T is decided by the number of zero diagonals in D , while the number of columns in T is decided by the number of buses associated with buses corresponding to zero diagonals in D . It is obvious that a reduced ILP formulation is thus obtained.

Now after analysing the observability of the system and increasing the a certain level of redundancy we can then compute the state estimation of the system using the proposed weighted least square algorithm.

4.4. Forming Least-Squares Equations

One is often confronted with problems wherein data have been obtained by making measurements or taking samples on a process. Furthermore, the quantities being measured are themselves functions of other variables that we wish to estimate. These other variables will be called the state variables and designated x , where the number of state variables is N_s . The measurement values will be called z . We will assume here that the process we are interested in can be modeled using a linear model. Then we say that each measurement z_i is a linear function of the states x_i ; that is,

$$z = h(x) = h_{i1}x_1 + h_{i2}x_2 + \cdots + h_{iN_s}x_{N_s} \dots\dots\dots(4.16)$$

We can also write this equation as a vector equation if we place the h_{ij} coefficients into a vector h ; that is,

$$h = \begin{bmatrix} h_{i1} \\ h_{i2} \\ \vdots \\ h_{iN_s} \end{bmatrix} \dots\dots\dots(4.17)$$

Hence z_i becomes

$$z = h_i^T x \dots\dots\dots(4.18)$$

Where

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N_s} \end{bmatrix} \dots\dots\dots(4.19)$$

Finally, we can write all the measurement equations in a compact form

$$z = H[X] \dots\dots\dots(4.20)$$

Where

$$Z = \begin{bmatrix} Z_1 \\ Z_2 \\ \vdots \\ Z_{N_m} \end{bmatrix} \dots\dots\dots(4.21)$$

$$[H] = \begin{bmatrix} h_{11} & h_{12} \cdots & h_{1N_s} \\ h_{21} & h_{22} \cdots & \\ \vdots & \vdots & \vdots \\ h_{N_m1} & \cdots & h_{N_mN_s} \end{bmatrix} \dots\dots\dots(4.22)$$

Where row i of [H] is equal to vector h_i^T

With N_m measurements greater than the number of states N_s , One way to estimate the x_i values is to minimize the sum of the squares of difference between the measurement values z_i and the estimate of z_i that is, in turn, a function of the estimates of x_i . That is, we wish to

$$J(X) = \sum_{i=1}^{N_M} [Z_i - h_i(x_1, x_2, \dots, x_{N_s})]^2 \dots\dots\dots(4.23)$$

Implies

$$J(X) = \sum_{i=1}^{N_M} (Z_i - h_i^T X)^2 \dots\dots\dots(4.24)$$

And this can be written in a still more compact form as

$$J(X) = (Z - [H] X)^T (Z - [H] x) \dots \dots \dots (4.25)$$

If we wish to find the value of x that minimizes $J(x)$, we can take the first derivative of $J(x)$ with respect to each x_j ($j = 1, \dots, N_s$) and set these derivatives to zero. That is,

$$\frac{\partial J(X)}{\partial x_i} = 0 \quad \text{for } j = 1 \dots N_s \dots \dots \dots (4.26)$$

If we place these derivatives into a vector, we have what is called the gradient of $J(x)$, which is written $\nabla_x J(x)$. Then,

$$\nabla_x J(X) = \begin{bmatrix} \frac{\partial J(X)}{\partial x_1} \\ \frac{\partial J(X)}{\partial x_2} \\ \vdots \end{bmatrix} \dots \dots \dots (4.27)$$

Then the goal of forcing each derivative to zero can be written as

$$\nabla_x J(x) = 0 \dots \dots \dots (4.28)$$

Where $\mathbf{0}$ is a vector of N_s elements, each of which is zero. To solve this problem, we will first expand Eq. $J(x)$.

$$\begin{aligned} J(X) &= (Z - [H]X)^T (Z - [H]X) \\ &= Z^T Z - X^T [H]^T + Z^T [H]X + X^T [H]^T [H] \dots \dots \dots (4.29) \end{aligned}$$

The second and third term in the above Equation is identical, so that we can write

$$J(X) = Z^T Z - 2Z^T [H]X + X^T [H]^T [H]X \dots \dots \dots (4.30)$$

Before proceeding, we will derive a few simple relationships.

The gradient is always a vector of first derivatives of a scalar function that is itself a function of a vector. Thus, if we define $F(y)$ to be a scalar function, then its gradient $\nabla_y F$ is:

$$\nabla_y F = \begin{bmatrix} \frac{\partial F}{\partial y_1} \\ \frac{\partial F}{\partial y_2} \\ \vdots \end{bmatrix} \dots\dots\dots (4.31)$$

Then, if we define F as follows:

$$F = y^T b = [y_1 \ y_2 \ \dots] \begin{bmatrix} b_1 \\ b_2 \\ \vdots \end{bmatrix} \dots\dots\dots (4.32)$$

Where b is a vector of constants b_i , $i = 1, \dots, n$, then, F can be expanded as

$$F = y_1 b_1 + y_2 b_2 + y_3 b_3 + \dots \dots\dots (4.33)$$

And the gradient of F is

$$\nabla_y F = \begin{bmatrix} \frac{\partial F}{\partial y_1} \\ \frac{\partial F}{\partial y_2} \\ \vdots \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = b \dots\dots\dots (4.34)$$

It ought to be obvious that writing F with y and b reversed makes no difference. That is,

$$F = b^T y = y^T b \dots\dots\dots (4.35)$$

$$\text{And, therefore, } \nabla_y (b^T y) = b \dots\dots\dots (4.36)$$

Suppose we now write the vector b as the product of a matrix $[A]$ and a vector u .

$$b = [A]u \dots\dots\dots (4.37)$$

Then, if we take F as shown above

$$F = y^T b = y^T [A]u \dots\dots\dots(4.38)$$

We can say that

$$\nabla_y F = [A]u \dots\dots\dots(4.39)$$

Similarly we can define

$$b^T = u^T [A] \dots\dots\dots(4.40)$$

If we can take F as the other expression stated above

$$F = b^T y = u^T [A] y \dots\dots\dots(4.41)$$

Then

$$\nabla_y F = [A]^T u \dots\dots\dots(4.42)$$

Finally, we will look at a scalar function F that is quadratic, namely ,

Then

$$\begin{aligned} F &= y^T [A]y \\ &= [y_1 \ y_2 \ \dots \ y_n] \begin{bmatrix} a_{11} & a_{12} & \cdots \\ a_{21} & a_{22} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \\ &= \sum_{i=1}^n \sum_{j=1}^n y_i a_{ij} y_j \dots\dots\dots (4.42) \end{aligned}$$

Then

$$\nabla_y F = \begin{bmatrix} \frac{\partial F}{\partial y_1} \\ \frac{\partial F}{\partial y_2} \\ \vdots \end{bmatrix} = \begin{bmatrix} 2a_{11}y_1 + 2a_{12}y_2 + \dots \\ 2a_{21}y_1 + 2a_{22}y_2 + \dots \\ \vdots \end{bmatrix}$$

$$= 2[A]y \dots\dots\dots (4.43)$$

Then, in summary:

- | | | |
|----|----------------|----------------------|
| 1. | $F = y^T$ | $\nabla_F = b$ |
| 2. | $F = y^T$ | $\nabla_F = b$ |
| 3. | $F = b^T$ | $\nabla_F = b$ |
| 4. | $F = u^T [A]u$ | $\nabla_F = [A]^T u$ |
| 5. | $F = y^T [A]y$ | $\nabla_F = 2[A]y$ |

We will now use the above expression to derive the gradient of $J(x)$, that is $\nabla_x J$. The first term, $z^T z$ is not a function of x , so we can discard it. The second term is of the same form as (4) in the above expression, so that,

$$\nabla_x (-2Z^T [H]X) = -2[H]^T Z \dots\dots\dots (4.44)$$

The third term is the same as (5) in the above expressions with $[H]^T[H]$ replacing $[A]$; then,

$$\nabla_x (X^T [H]^T [H]X) = 2[H]^T [H]X$$

Then combining the two

$$\nabla_x J = -2[H]^T Z + 2[H]^T [H]X \dots\dots\dots (4.45)$$

But, as stated before we wish to force $\nabla_x J$ to zero. Then

$$-2[H]^T Z + 2[H]^T [H]X = 0 \dots\dots\dots (4.46)$$

Or

$$X = \left[\begin{bmatrix} H \end{bmatrix}^T \begin{bmatrix} H \end{bmatrix} \right]^{-1} \begin{bmatrix} H \end{bmatrix}^T Z \dots\dots\dots (4.47)$$

If we had wanted to put a different weight, w_i , on each measurement, we could have written $J(x)$ as

$$J(X) = \sum_{i=1}^{n_m} w_i \left(z_i - h_i^T X \right)^2 \dots\dots\dots (4.48)$$

This can be written as

$$J(X) = (Z - [H]X)^T [W] (Z - [H]X) \dots\dots\dots (4.49)$$

Where $[W]$ is a diagonal matrix. Then

$$J(X) = Z^T [W] Z - X^T [H]^T [W] Z - Z^T [W] [H] X + X^T [H]^T [W] [H] X \dots\dots\dots (4.50)$$

If we once again use the expression summarized above, we would obtain

$$\nabla_X J = -2 [H]^T [W] Z + 2 [H]^T [W] [H] X \dots\dots\dots (4.51)$$

Gives

$$X = \{ \{ [H]^T [W] [H] \}^{-1} [H]^T [W] Z \} \dots\dots\dots (4.52)$$

This is the final weighted least square algorithm formulation in which it be implemented in this thesis.

5. RESULTS, ANALYSIS AND DISCUSSION

This chapter presents the results of the developed algorithm, which was implemented in MATLAB. The Power Education Toolbox (PET) software was also used as a means to simplify the way the system is modelled, and to generate the system data and measurements. The well-known IEEE 14 bus benchmark system was used as a base case, and the 21 bus southern region power system network is modeled in detail.

5.1. BASE CASE

A IEEE 14-bus power system is shown in Fig. 5.1. The conventional measurements are given in Fig.5.1. This optimal problem is modeled based on the algorithm proposed in section 4.3.

Table 5.1 bus data for IEEE 14-bus system

Bus No	Bus type	Vsp	Angle	Pgi	Qgi	Pli	Qli	Qmin	Qmax
1	1	1.06	0	0	0	0	0	0	0
2	2	1.045	0	40	42.4	21.7	12.7	-40	50
3	2	1.01	0	0	23.4	94.2	19	0	40
4	3	1	0	0	0	47.8	-3.9	0	0
5	3	1	0	0	0	7.6	1.6	0	0
6	2	1.07	0	0	12.2	11.2	7.5	-6	24
7	3	1	0	0	0	0	0	0	0
8	2	1.09	0	0	17.4	0	0	-6	24
9	3	1	0	0	0	29.5	16.6	0	0
10	3	1	0	0	0	9	5.8	0	0
11	3	1	0	0	0	3.5	1.8	0	0
12	3	1	0	0	0	6.1	1.6	0	0
13	3	1	0	0	0	13.5	5.8	0	0
14	3	1	0	0	0	14.9	5	0	0

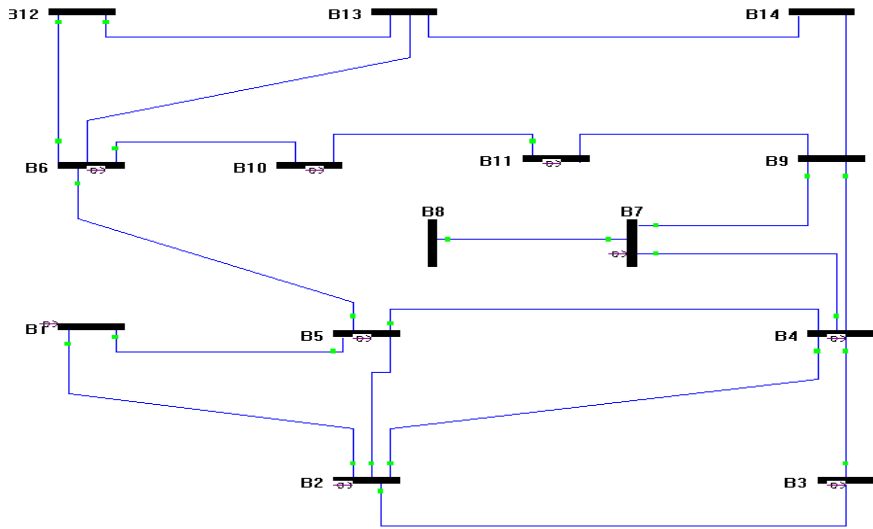


Fig 5.1. IEEE 14-bus power system

The Jacobian matrix was formed and its rank was found to be 12 which is less than 13 implying a non observable system and the state estimation is unsolvable. Two zeros are found at the 13th and 14th diagonal elements of matrix D in the factorization $G = LDL^T$.

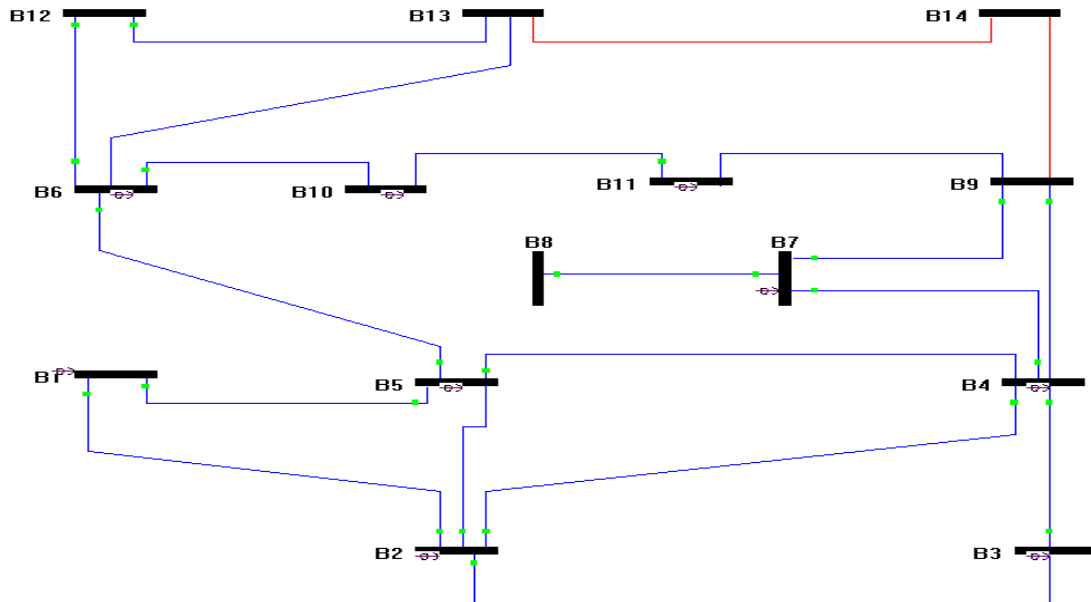


Fig.5.2 14 bus observability analysis on PET software

- A. If we simply include injection measurement at bus 14 and flow measurement from bus 6 to 13 (based on the IEEE 14 bus standard) and running the state estimation on Matlab results:

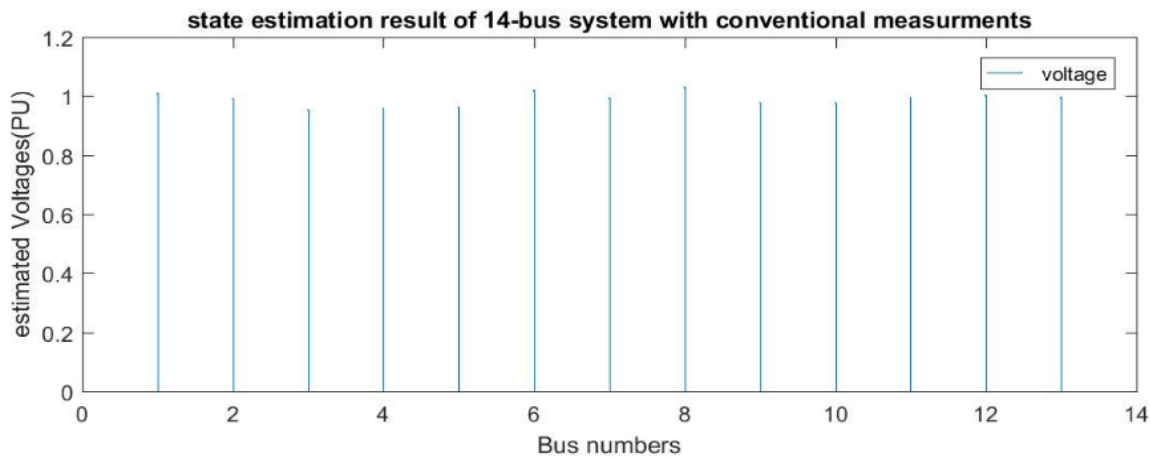
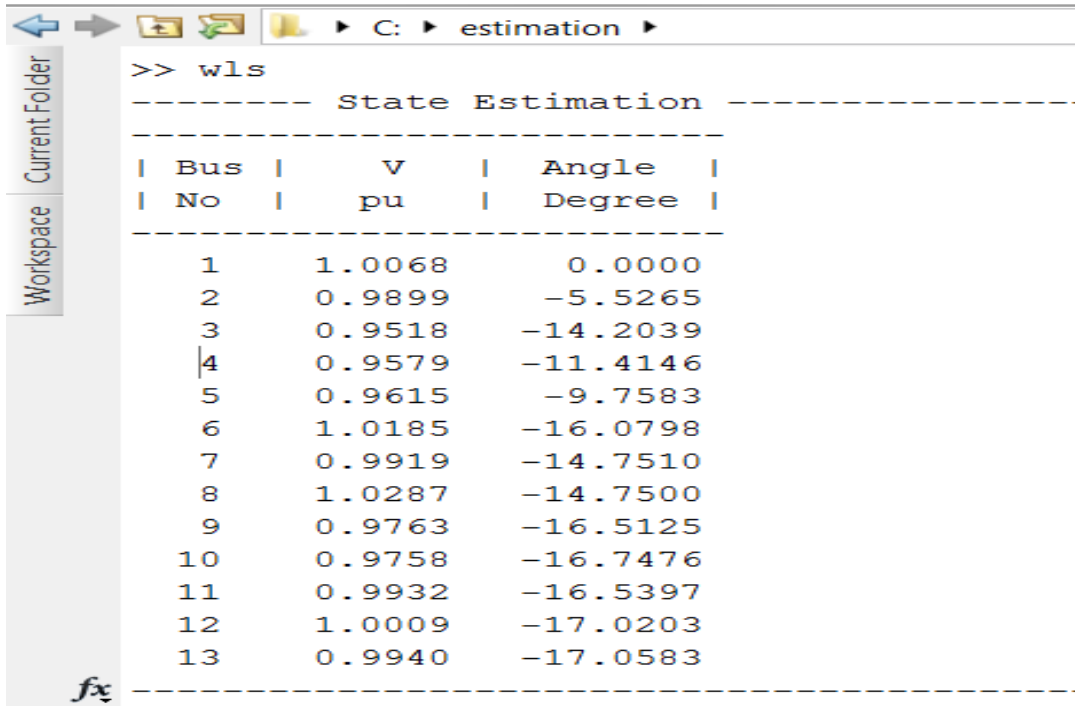


Fig. 5.3 MATLAB State estimation output and its Voltage profile of a 14-bus system before PMU installation

- B. And if we applying Algorithm (proposed in Section 4.3), results in the following ILP model:

$$\text{Min } f = x_6 + x_9 + x_{12} + x_{13} + x_{14}$$

$$\text{St. TX} = \begin{vmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{vmatrix} [x_6 + x_9 + x_{12} + x_{13} + x_{14}]^T \geq [1 \ 1]^T$$

$$X = [x_6 + x_9 + x_{12} + x_{13} + x_{14}]$$

$$X_i \in \{0, 1\}.$$

$$\text{The solution is } X = [0 \ 0 \ 0 \ 0 \ 1 \ 0]^T$$

The solution of the above ILP problem shows that a PMU installed at bus 13 can make the whole system observable. It is obvious that the best bus to place a PMU is bus 13 which connects bus 6 and bus 14. It is the optimal solution. Notice that the proposed integer linear programming formulation changes the original formulation from a dimension of 5 X 14 to a dimension of 2 X 5, size of the variable vector changing from 14 to 5. These reductions will become more substantial with increasing system sizes.

Now from section 3.2.2 by converting the current measurement obtained from the PMU into power flows and running the state estimation in matlab we get a result of .

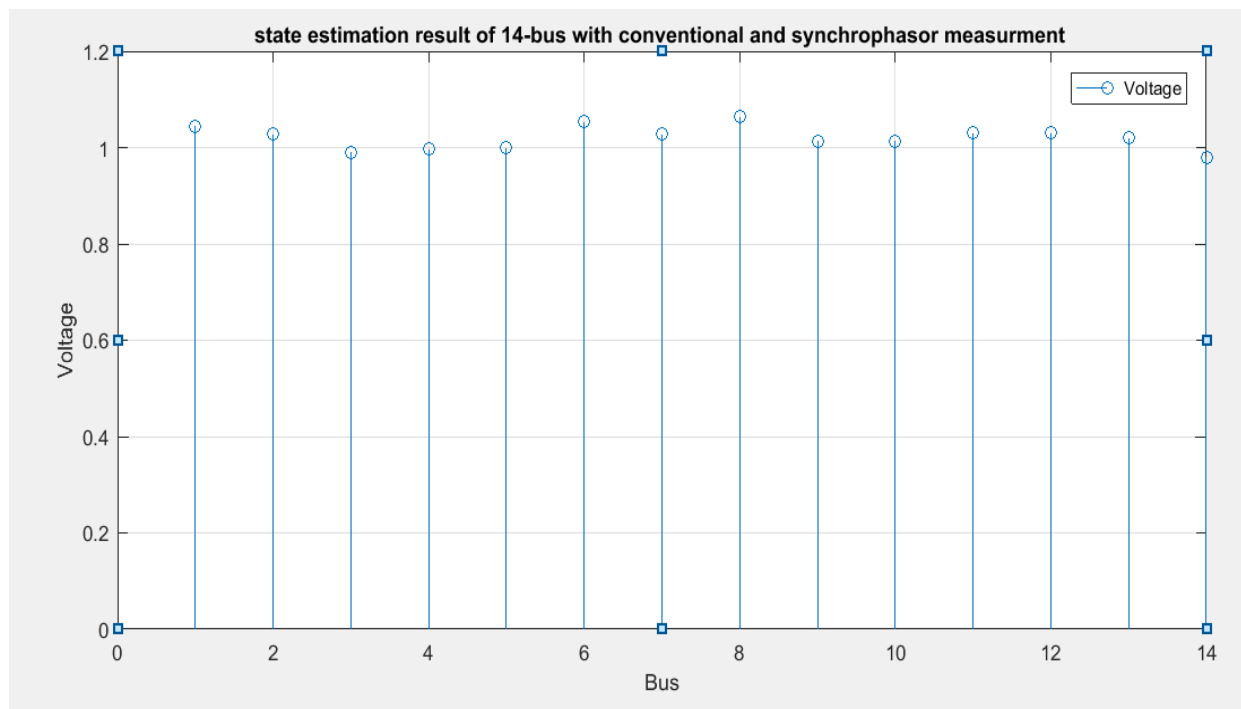
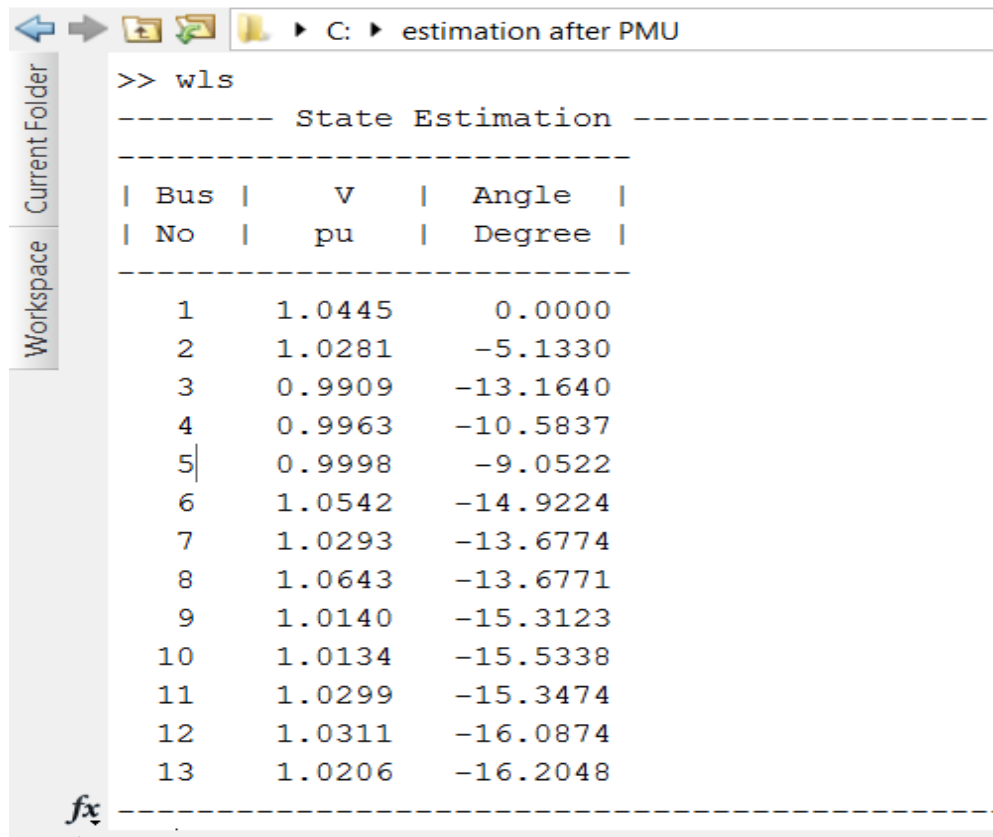


Fig. 5.4 MATLAB State estimation output voltage profile of a 14-bus after PMU and

As we can see from the above obtained result installing one PMU at optimal position will improve the state estimation performance. It actually enhance the state estimation mean square error from 0.00144 to 0.00038 which is around 73% improvement .The following table shows the over all performance comparison.

Table 5.2 : Mean square error comparison for state estimation performance

Bus	Actual value	estimated before PMU		estimated after PMU	
	V [p.u.]	V [p.u.]	error square	V [p.u.]	error square
Bus 01	1.06	1.0068	0.00283	1.044	0.00024
Bus 02	1.045	0.9899	0.00303	1.028	0.00029
Bus 03	1.01	0.9518	0.00338	0.991	0.00036
Bus 04	1	0.9579	0.00177	0.996	1.4E-05
Bus 05	1	0.9615	0.00148	1.000	4.7E-08
Bus 06	1.07	1.0185	0.00265	1.054	0.00025
Bus 07	1	0.9919	6.5E-05	1.029	0.00086
Bus 08	1.09	1.0287	0.00376	1.064	0.00066
Bus 09	1	0.9763	0.00056	1.014	0.00019
Bus 10	1	0.9758	0.00058	1.013	0.00018
Bus 11	1	0.9932	4.6E-05	1.030	0.0009
Bus 12	1	1.0009	8.3E-07	1.031	0.00097
Bus 13	1	0.9940	3.6E-05	1.021	0.00042
Bus 14	1				
Mean square error			0.00144		0.00038

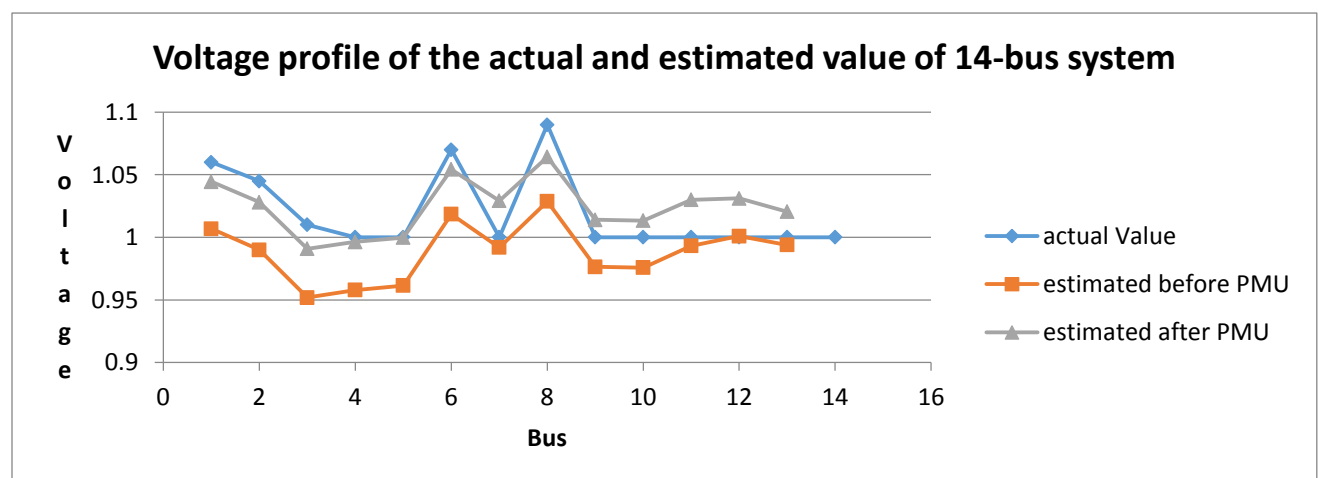


Fig.5.5 state estimation result comparison with the actual

5.2. 21-BUS, Southern Region network, case study

Consider a 21 bus network system of southern region shown in fig 5.2. below . The conventional measurements are given in Fig.5.3 and modeling the optimal problem is based on the algorithm proposed in section 4.3 .

Table 5.3 21-bus southern region bus data

NAME OF BUS	Bus No.	V (base KV)	V(pu)	angle	PD MW	QD mvar	PG mw	QG mvar	Qmax mvar	Qmin mvar	Qshint mvar
Melka Wakena	1	230	0.9591	0	0	0	80	20	0	0	0
Gilgel Gibe(omo)	2	400	0.9788	0	0	0	75	15	0	0	0
Gibe-3	3	400	0.971	0	0	0	40	15	0	0	0
RAMO	4	230	0.9591	0	0.5	20	0	0	0	0	20
Yadot	5	132	0.9661	0	1	2	0	0	0	0	0
Robe	6	66	0.9334	0	10		0	0	0	0	0
Shashemene	7	132	0.9062	0	12.672	5.524	0	0	0	0	0
Alaba	8	132	0.9148	0	3.57	1.556	0	0	0	0	0
Hawassa	9	132	0.8951	0	33	6.071	0	0	0	0	0
Yirgalem	10	132	0.8844	0	13.283	5.79	0	0	0	0	0
Shakisso	11	132	0.8637	0	11	4.438	0	0	0	0	0
Dilla	12	132	0.887	0	8	2.603	0	0	0	0	0
Hagere Mariam	13	132	0.887	0	10	1.297	0	0	0	0	10
Bukulu Luguma	14	132	0.9145	0	3.5	2	0	0	0	0	0
Negele Borana	15	132	0.7861	0	3		0	0	0	0	0
Hossana	16	132	0.9287	0	8.5	2.39	0	0	0	0	0
soddo- 1	17	400	0.971	0	75		0	0	0	0	0
soddo- 2	18	132	0.9029	0	24	2.786	0	0	0	0	0
Aribaminch	19	132	0.8902	0	9.2	3.362	0	0	0	0	0
Sawla	20	132	0.8902	0	3	0.821	0	0	0	0	0
Keyafer	21	132	0.8902	0	1.5	2	0	0	0	0	0

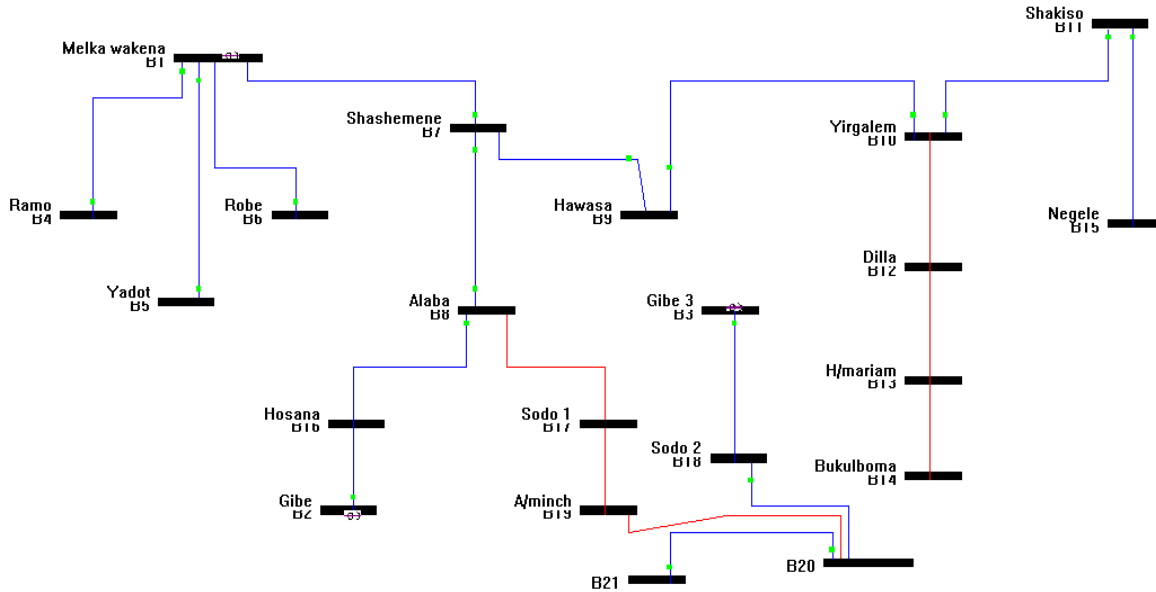


Fig 5.7. 21-Bus system developed in PET software/conventional measurement/.

The red line in the figure indicates that the system is not fully observable with the given conventional measurement that are indicated (3 injection measurement and 20 flow measurements)

Assuming zero injection measurement at bus no 12,13,14,17,19 and 20 to make the system state estimation solvable and running the state estimation program on matlab without any PMU we get

```
>> wls
----- State Estimation -----
-----
| Bus |      V      | Angle |
| No  |      pu     | Degree |
-----
    1    1.0000    0.0000
    2    1.0190   -4.8502
    3    1.0182   -4.2275
    4    0.9901    0.2217
    5    0.9992   -0.0091
    6    0.9992   -0.0962
    7    0.9920   -4.2745
    8    0.9989   -5.2162
    9    0.9892   -4.6757
   10    0.9858   -5.1790
   11    0.9845   -5.4414
   12    0.9861   -5.1866
   13    0.9867   -5.2016
   14    0.9873   -5.2161
   15    0.9845   -5.4877
   16    1.0087   -5.0554
   17    1.0002   -5.1157
   18    1.0052   -4.4960
   19    1.0022   -4.9343
   20    1.0028   -4.7291
-----
```

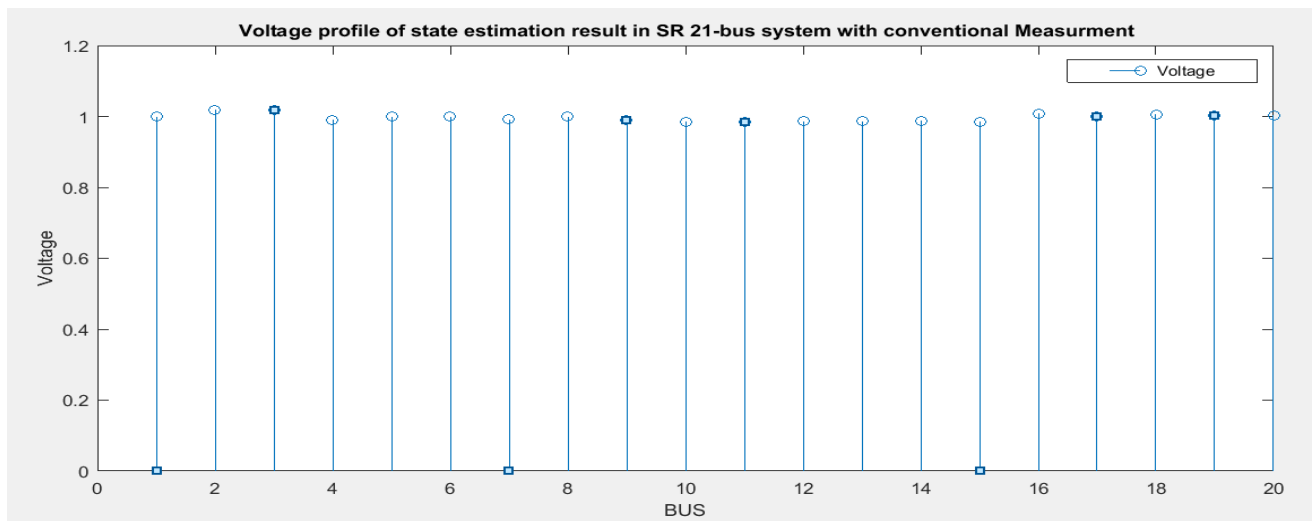


Fig.5.8 MATLAB state estimation output and Voltage Profile of SR 21-bus system from conventional measurments only.

Now by assuming the R to 1 and , all the reactance to 1 and DC flow for only analysis purpose, in the non observable system , the Jacobean matrix is formed using matlab as :

H =

```

4  0  0  -1  -1  -1  -1  0  0  0  0  0  0  0  0  0  0  0  0  0
0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  -1  0  0  0  0
0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  -1  0  0  0
1  0  0  -1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
1  0  0  0  -1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  -1  0  0  0  0  0  0  0  0  0  0  0  0  0  -1  0  0  0  0
0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  -1  0  0  0
-1  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
-1  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
-1  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0
-1  0  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0
0  0  0  0  0  0  1  -1  0  0  0  0  0  0  0  0  0  0  0  0
0  0  0  0  0  0  -1  1  0  0  0  0  0  0  0  0  0  0  0  0
0  0  0  0  0  0  0  1  0  0  0  0  0  0  0  -1  0  0  0  0
0  0  0  0  0  0  -1  0  1  0  0  0  0  0  0  0  0  0  0  0
0  0  0  0  0  0  0  0  1  -1  0  0  0  0  0  0  0  0  0  0
0  0  0  0  0  0  0  0  -1  1  0  0  0  0  0  0  0  0  0  0
0  0  0  0  0  0  0  0  0  1  -1  0  0  0  0  0  0  0  0  0
0  0  0  0  0  0  0  0  0  0  -1  1  0  0  0  0  0  0  0  0
0  0  0  0  0  0  0  0  0  0  0  1  0  0  0  -1  0  0  0  0
0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  -1
0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  -1
0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  -1  1

```

And the corresponding gain matrix is found to be :

G =

22	0	0	-6	-6	-5	-5	0	0	0	0	0	0	0	0	0	0	0	0	0
0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0
-6	0	0	3	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
-6	0	0	1	3	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
-5	0	0	1	1	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0
-5	0	0	1	1	1	5	-2	-1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	-2	3	0	0	0	0	0	0	0	-1	0	0	0	0
0	0	0	0	0	0	-1	0	3	-2	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	-2	4	-2	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	-2	3	0	0	0	-1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-1	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	3	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

```

0 0 -2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 3 0 -1 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -1 0 3 -2
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -2 2

```

Using the algorithm proposed in section 4.3 The D matrix developed in matlab and the diagonal of the D matrix is

Diag D=[22 , 2 ,2 ,1.36 ,1.07 , 0.5 , 3 , 1.67 ,2.4 ,2.33 ,1.29,0 ,0 ,0 ,2.22 ,2,0,1,0,2,0]

Implies to make the system observable and the state estimation solvable ,we must place PMU at bus no 12,13,14,17,19 and 21 and or at adjacent buses 8,10 and 20 and hence the rank of D will be increased .Considering all the possible buse (8, 10 ,12 ,13,14, 17, 19, 20 ,21)one has a T matrix of :

```

T= 0|0000001100000000110000
    |0000000011110000000000
    |0000000001011000000000
    |0000000000011100000000
    |0000000000001100000000
    |000000010000000010100
    |000000000000000010110
    |000000000000000001111
    |000000000000000000011

```

And bm=[0 0 1 1 1 1 1 0 1]

And if we applying Algorithm I proposed in Section 4.3 results in the following ILP model:

$$\text{Min } f = x_8 + x_{10} + x_{12} + x_{13} + x_{14} + x_{17} + x_{19} + x_{20} + x_{21}$$

$$\text{St} = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_8 + x_{10} + x_{12} + x_{13} + x_{14} + x_{17} + x_{19} + x_{20} + x_{21} \end{bmatrix}^T \geq [1 \ 1 \ 1 \ 1 \ 1 \ 1]^T$$

$$\text{The solution is } X = [0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0]^T$$

This indicates that placing a PMU unit at bus 13 and 19 can make the entire system observable. It is obvious that the best bus to place a PMU is bus 13(at Hagere Maryam substation) and 19(A/minch substation) it is the optimal solution. Notice that the proposed integer linear programming formulation changes the original formulation from a dimension of 9 x 21 to a dimension of 6 x 9.

Now solving the state estimation in MATLAB after placing the proposed PMU results:

```
>> wls
```

```
----- State Estimation -----
```

```
-----
```

Bus	V	Angle
No	pu	Degree
1	0.9593	0.0000
2	0.9788	-5.2775
3	0.9710	-5.5872
4	0.9491	0.2406
5	0.9587	-0.0113
6	0.9585	-0.1050
7	0.9506	-4.6501
8	0.9578	-5.6750
9	0.9477	-5.0870
10	0.9442	-5.6356
11	0.9428	-5.9221
12	0.9434	-5.7633
13	0.9422	-5.9545
14	0.9408	-6.0566
15	0.9429	-5.9726
16	0.9680	-5.5002
17	0.9536	-6.3311
18	0.9574	-5.8835
19	0.9543	-6.3188
20	0.9548	-6.1409

```
-----
```

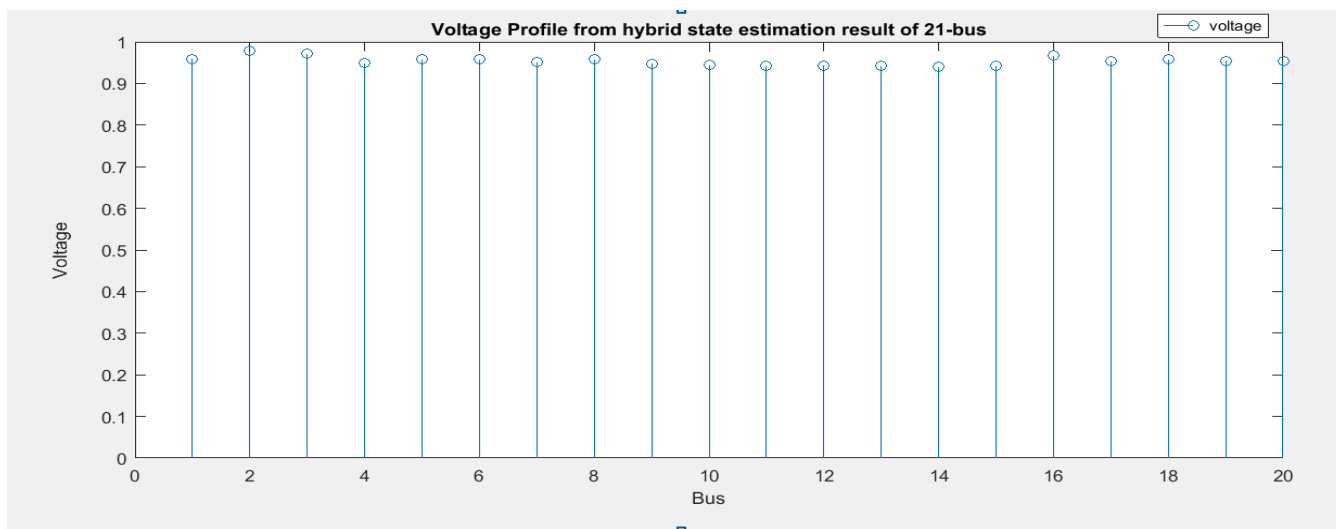


Fig.5.9. MATLAB output and its Voltage profile of the 21-bus system from hybrid state estimation result(after PMU placement).

As we can see from the above obtained result installing two PMU at optimal position will improve the state estimation performance. It actually enhance the state estimation mean square error of southern Region 21-bus system from 0.007972 to 0.002943, it is about 63% improvement. The following table shows the over all performance comparison.

Table 5.4: Mean square error comparison for state estimation PERFORMANCE, 21 BUSES

Bus	Actual value	estimated before PMU		estimated after PMU	
	V [p.u.]	V [p.u.]	error square	V [p.u.]	error square
1	0.9591	1	0.0016728	0.95929	3.5995E-08
2	0.9788	1.019047	0.00161978	0.97878	5.706E-10
3	0.971	1.018189	0.00222682	0.97097	1.0108E-09
4	0.9591	0.990085	0.00096008	0.94909	0.00010013
5	0.9661	0.999202	0.00109576	0.9587	5.4817E-05
6	0.9334	0.99918	0.00432707	0.95852	0.00063122
7	0.9062	0.991997	0.0073611	0.95065	0.00197542
8	0.9148	0.9989	0.00707123	0.958	0.00184948
9	0.8951	0.9892	0.00885007	0.948	0.00276649
10	0.8844	0.9858	0.0102766	0.944	0.00357031
11	0.8637	0.9845	0.01458155	0.943	0.00625571
12	0.887	0.9861	0.00981833	0.943	0.00317973
13	0.887	0.9867	0.00993785	0.942	0.00304241
14	0.9145	0.9873	0.00529677	0.941	0.0006908
15	0.7861	0.9845	0.03937621	0.943	0.02458305
16	0.9287	1.0087	0.00639271	0.968	0.00154218
17	0.971	1.0002	0.00085073	0.954	0.00030365
18	0.9029	1.0052	0.01046715	0.957	0.00296609
19	0.8902	1.0022	0.01254589	0.954	0.00410989
20	0.8902	1.0028	0.01267295	0.955	0.00417624
21	0.8902				
Mean square error			0.0079715		0.00294275

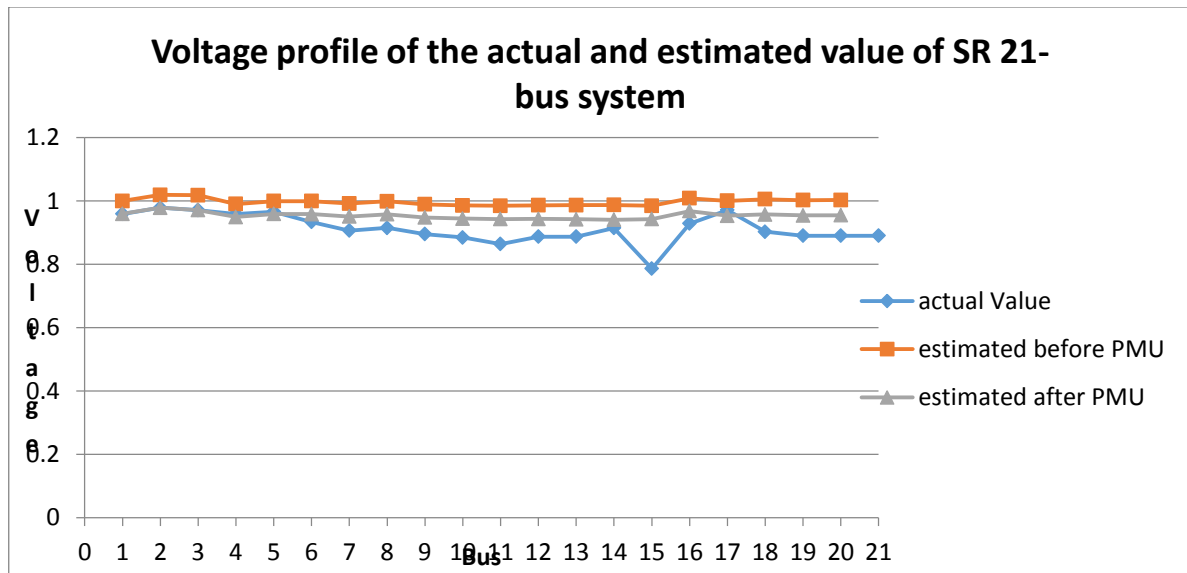


Fig.5.10. state estimation comparison with respect to the actual, 21-bus

6. CONCLUSION AND RECOMMENDATION

6.1. RECOMMENDATION

It is recommended to implement the gained result in the 21 bus network in order to make the system observable and hence can improve state estimation of the system. As we have seen from the result in the above section installation of a PMU in southern region in such a way that :

- A. with the inclusion of the existing measurement (conventional measurement) to improve the state estimation performance of southern region 21-bus system it is highly recommended that One PMU at Hageremariam substation and one PMU at A/minch substation should be installed.
- B. the above result has been evident that a PMU device are very effective device in improving state estimation result and hence the overall power system monitor and control capability. Therefore for the future the conventional measurement has to be replaced by this devices in order to make the system as smart as possible in achieving the vision of the Ethiopian electric power. To do this the above formulated OPP problem can easily be applied to the 21-bus system as a model i.e by using the PSAT (power system analysis toolbox) in matlab and using one of the above algorithm stated in section 2 one can have the OPP solution as : (PSAT generates the following result)

PMU PLACEMENT REPORT

P S A T 2.1.10

Author: Kinfu Negash

/PGengw/006/07

e-mail:

kinfemichael.negash@gmail.com

Advisor: Dr. Mengesha Mamo and

Dr. Baseem Kahn

File:

C:\psat\tests\S_021_pmu_121_md1

Date: 05-Feb-2017

17:43:47

Placement Method: Annealing
Method
Elapsed Time: 0h 0m
1.0634s

STATISTICS

Buses	21
Lines	20
PMUs	6
PMU Sets	1
Meas. Currents	15
Pseudo-Meas. Currents	5

PMU PLACEMENT

Bus Name	Set 1
Melka Wakena 01	1
Gilgel Gibe(omo) 02	1
Gibe-3 03	1
RAMO 04	0
Yadot 05	0
Robe 06	0
Shashemene 07	0
Alaba 08	1
Hawassa 09	0
Yirgalem 10	1
Shakisso 11	0
Dilla 12	0
Hagere mariam 13	0
Bukulu Luguma 14	0
Negele Borana 15	0
Hossana 16	0
soddo- 1 17	0
soddo- 2 18	0
Aribaminch 19	0
Sawla 20	1
Keyafer 21	0

Which implies that to make the whole system observable excluding any conventional measurments (only for southern region 21-bus)it is recommended to install 6 PMU(at the

position of PMU sets 1 i.e.at Melkawakena ,Gilgel Gibe ,Gibe 3,Alaba,yirgalem and Sawla substation .Thereby the power system controll capability will achieve its maxima and hence the network will become smart.

In General For Ethiopia there is an economical PMU installation facility which is being carrying on for example in kenya and Nigeria among Africa known as WAMSTER system .This system utilizes the existing mobile network for communication facility there by diminishing the cost for communication and provides a portable hand held pmu devices .

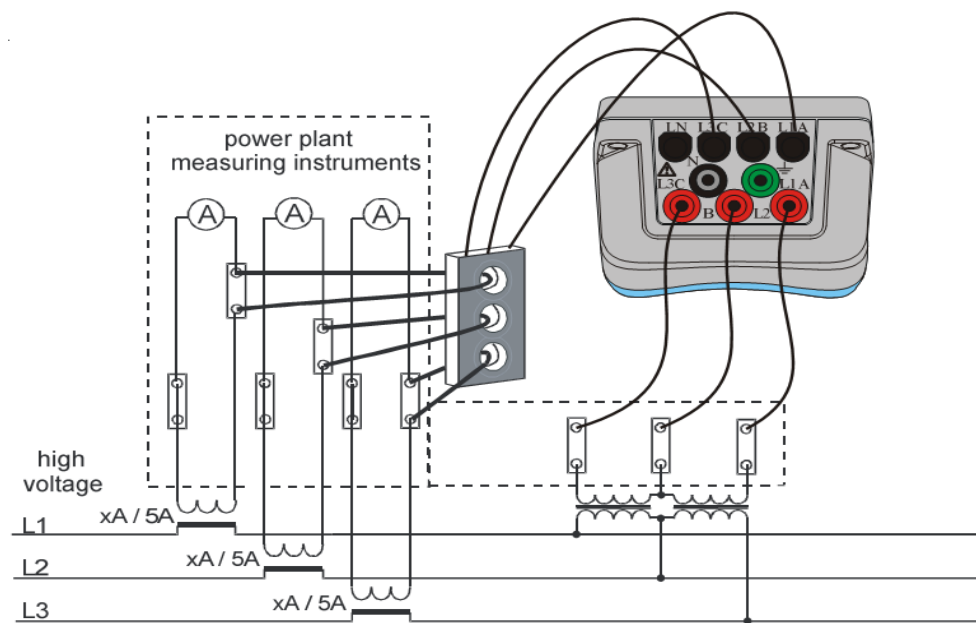


Fig.6.1 STER PMU connected to existing CTs using direct insertion,using open-delta voltage connection

The device can be installed as shown in Fig 6.1 at the above recommended bus location and can be applied for such as

- ✓ synchrophasor data base lining and disturbance events capturing
- ✓ for predicting black-outs or voltage disruption
- ✓ Data collected by WAMSTER can be used for model tuning in conjunction with data captured by the existing SCADA system. Several captured events(as experianced by

kenya and nigeria) revealed previously unknown dynamics of the system, and the use of WAMSTER allowed these new findings to be incorporated into the existing algorithm definition at an early design phase.

- ✓ To reveal new dynamics of the problematic generator set.
- ✓ To detect various fluctuations in the grid across the Region.

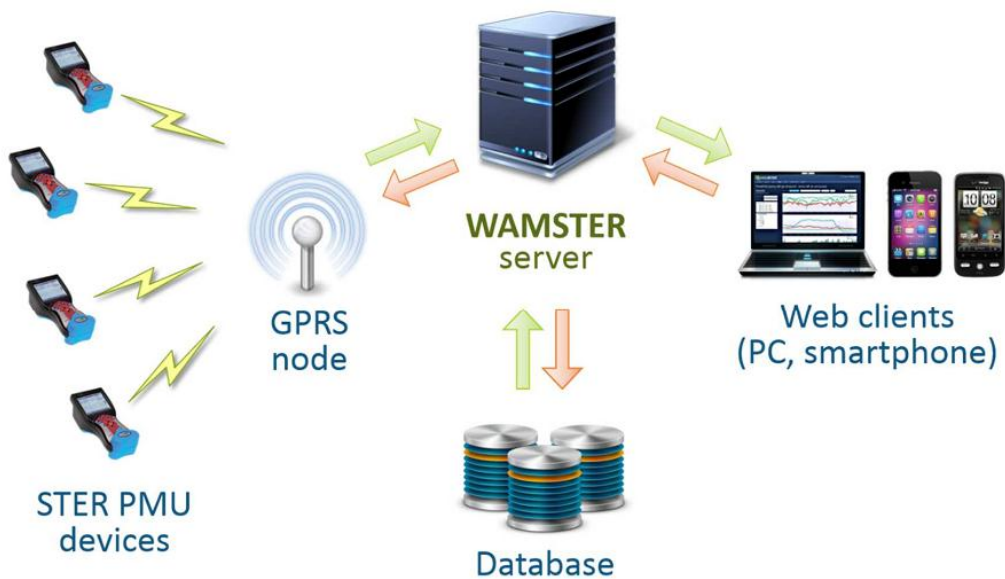


Fig.6.2 WAMSTER system overview.

In Fig 6.2 .Handheld STER PMU devices (shown on the left) communicate with the server over the mobile (GPRS) network. WAMSTER server processes received synchrophasors, drives them through the event triggering system, stores them into the database (shown at the bottom), sends notification messages to clients, and renders an online interface for web clients (right).

For the future the whole system of Ethiopian network can be studied concerning how to make the system completely observable dynamically by using these PMU devices some criteria can be included in a way that can obtain a unique opp solution .

6.2. CONCLUSION

This Master's thesis presented the theory of the well-known WLS Traditional State Estimation related to conventional measurements. The inclusion of synchronized phasor measurements was discussed, addressing the use of voltage and current phasor measurements in the bus-branch levels.

The theory of observability analysis, including the topological and numerical approaches were also presented. The numerical approach was deeply investigated as its use was paramount for the proposed method, since it was used to identify observable and unobservable portions of the system, forming the super-nodes in the proposed methods.

A reduced ILP formulation for the PMU placement algorithm for observability whose dimension is governed by the rank deficiency of the conventional measurements and methods of sensitivity analysis, suggested an approaches to place a PMU at optimal place in order to improve the state estimation results by eliminaitaing the critical measurments has been studied.

The proposed algorithms are illustrated on IEEE 14-bus system as a base case and Southern region 21-bus system as a study case using Matlab and PET soft ware. Numerical results shows that the proposed algorithms and results offer PMU placement solutions that improve robustness of traditional state estimation in such away that improve the state estimation result by minimizing the mean square error from 0.00144 to 0.00038 in the 14-bus analysis and from 0.007972 to 0.002943 in the 21-bus test system which is an improvement by 73% and 63% respectively .

Finally a PMU instalation at two buses along with conventional measurment for the southern region network has been recomemnded . 6 PMU with out any conventional measurment has also been recommended to be installed in southern region network for complete system observability and hence state estimation enhancement consequently .For this a cost effective device known as a STERPMU has been recommended by taking usufule experience from kenya and nigeria from africa.

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APPENDIX A

21 BUS SOUTHERN REGION SYSTEM DATA

Table A.1 Bus Data of 21 BUS SOUTHERN REGION

NAME OF BUS	Bus No.	V (base KV)	angle	PD MW	QD mvar	PG mw	QG mvar	Qmax mvar	Qmin mvar	Qshint mvar
Melka Wakena	1	230	0	0	0	80	20	0	0	0
Gilgel Gibe(omo)	2	400	0	0	0	75	15	0	0	0
Gibe-3	3	400	0	0	0	40	15	0	0	0
RAMO	4	230	0	0.5	20	0	0	0	0	20
Yadot	5	132	0	1	2	0	0	0	0	0
Robe	6	66	0	10		0	0	0	0	0
Shashemene	7	132	0	12.672	5.524	0	0	0	0	0
Alaba	8	132	0	3.57	1.556	0	0	0	0	0
Hawassa	9	132	0	33	6.071	0	0	0	0	0
Yirgalem	10	132	0	13.283	5.79	0	0	0	0	0
Shakisso	11	132	0	11	4.438	0	0	0	0	0
Dilla	12	132	0	8	2.603	0	0	0	0	0
Hagere Mariam	13	132	0	10	1.297	0	0	0	0	10
Bukulu Luguma	14	132	0	3.5	2	0	0	0	0	0
Negele Borana	15	66	0	3		0	0	0	0	0
Hossana	16	132	0	8.5	2.39	0	0	0	0	0
soddo- 1	17	400	0	75		0	0	0	0	0
soddo- 2	18	132	0	24	2.786	0	0	0	0	0
Aribaminch	19	132	0	9.2	3.362	0	0	0	0	0
Sawla	20	132	0	3	0.821	0	0	0	0	0
Keyafer	21	132	0	1.5	2	0	0	0	0	0



Table A.2 Line Data of 21 BUS SOUTHERN REGION

From Bus Name	To Bus Name	Lines	Length	Resistance (R)	Reactance (X)	Susceptance (B)
Melka Wakena(1)	RAMO(4)	1—4	22.2	0.0215	0.050588	0.011388
Melka Wakena(1)	yadot(5)	1—5	16.7	0.01626	0.038	0.00865
Melka Wakena(1)	robe(6)	1—6	7.278	0.0080966	0.01658	0.0037
Melka Wakena(1)	shashemene(7)	1—7	11.928	0.0115	0.02718	0.006127
Gilgel Gibe(omo)2	Hossana(16)	2—16	4.062	0.0039	0.00923	0.00208
Gibe-3	soddo- 2 (18)	3—18	5.8	0.032567	0.01322	0.00298
Shashemene(7)	Alaba(8)	7—8	6.323	0.006124	0.014409	0.003245
Shashemene(7)	Hawassa(9)	7—9	2.16	0.00209	0.0049	0.00112
Alaba(8)	Hossana(16)	8—16	3.96	0.003835	0.00902	0.00203
Alaba(8)	soddo- 1 (17)	8—17	6.181	0.005986	0.01404	0.00317
Hawassa(9)	Yirgalem(10)	9—10	3.509	0.003398	0.007996	0.0018
Yirgalem(10)	shakisso (11)	10—11	13.321	0.129	0.003035	0.0068
Yirgalem(10)	Dilla (12)	10—12	3.971	0.037876	0.009056	0.002037
shakisso (11)	Negele Borana (15)	11—15	11.39	0.00113	0.02595	0.005843
Dilla (12)	Hagere mariam(13)	12—13	9.468	0.00926	0.02166	0.00495
Hagere mariam(13)	Bukulu Luguma(14)	13—14	22.6	0.021887	0.05158	0.01159
soddo- 1 (17)	Aribaminch (19)	17—19	10.913	0.0105689	0.02468	0.005598
soddo- 2 (18)	sawla (20)	18—20	12.4	0.012009	0.0283	0.00636
sawla (20)	Keyafer (21)	20—21	11.1	0.01075	0.0253	0.00569

Table A.3 General Load flow Result of southern Region Network (Source Ethiopian Electric Power, EEP, LDC)

Node	Netw ork Level	U [kV]	U/Un [%]	phi [°]	U/Uref [%]	P [MW]	Q [MVA r]	S [MVA]	t [h]	State
GILGEL GIBE I 230	230	222.3 21	96.661 32	28.06 561	0	0	0	0	22	Ok
SHASHEMENE 132	132	119.6 153	90.617 68	8.985 527	0	0	0	0	22	Limit reached
SHAKISSO 132	132	114.0 079	86.369 62	4.231 347	0	0	0	0	22	Limit reached
MELKA WALKENA 132	132	126.9 781	96.195 57	11.94 51	0	0	0	0	22	Ok
MELKA WAKENA YOU 132	132	126.5 989	95.908 29	11.76 246	0	0	0	0	22	Ok
AWASA 132	132	118.1 566	89.512 59	7.781 827	0	0	0	0	22	Limit reached
YIRGA ALEM 132	132	116.7 451	88.443 28	6.297 753	0	0	0	0	22	Limit reached
WOLAYITA SODO 132	132	119.1 843	90.291 14	10.82 621	0	0	0	0	22	Limit reached
ARBAMINCH 132	132	117.5 055	89.019 32	9.473 848	0	0	0	0	22	Limit reached
GILGEL GIBE OLD 132 BB1	132	127.1 019	96.289 32	21.01 776	0	0	0	0	22	Ok
HOSAINA 132	132	122.5 865	92.868 6	15.28 578	0	0	0	0	22	Limit reached
ALABA 132	132	120.7 502	91.477 44	12.19 415	0	0	0	0	22	Limit reached
GIGEL GIBE I 132	132	127.2 225	96.380 66	21.17 622	0	0	0	0	22	Ok
GILGEL GIBE 15	15	14.71 441	98.096 07	19.35 492	0	- 2.064 1	- 0.6784 4	2.172 736	22	Ok
NEGELE BORENA 66	66	51.88 297	78.610 55	- 3.148 14	0	0	0	0	22	Limit reached
GILGEL GIBE 13.8 GR3	13.8	13.8	100	30.91 258	0	30	- 13.535 4	32.91 213	22	Ok
MELKA WAKENA 230	230	221.6 217	96.357 24	14.00 05	0	0	0	0	22	Ok
MELKA WAKENA 13.8 GR1	13.8	13.11	95	16.26 156	0	14	11.780 26	18.29 684	22	Ok
MELKA WAKENA YOU 66	66	64.46 243	97.670 35	7.320 007	0	0	0	0	22	Ok
ROBE 66	66	61.60 211	93.336 52	5.991 403	0	0	0	0	22	Limit reached
MELKA WAKENA YOU 15	15	14.58 467	97.231 14	6.524 737	0	- 3.650 15	- 1.1997 5	3.842 262	22	Ok
GILGEL GIBE 13.8 GR2	13.8	13.8	100	30.91 258	0	30	- 13.535	32.91 213	22	Ok

							4			
GILGEL GIBE 13.8 GR1	13.8	13.8	100	30.91 258	0	30	- 13.535 4	32.91 213	22	Ok
MELKA WAKENA 13.8 GR2	13.8	13.8	100	16.00 816	0	14	31.962 39	34.89 405	22	Ok
MELKA WAKENA 13.8 GR3	13.8	13.11	95	0	0	81.47 16	25.810 64	85.46 237	22	Ok
MELKA WAKENA 13.8 GR4	13.8	13.8	100	16.00 816	0	14	31.962 39	34.89 405	22	Ok
DILLA 132KV	132	117.0 832	88.699 36	6.045 478	0	0	0	0	22	Limit reached
HAGEREMARIAM1 32KV	132	117.0 834	88.699 52	5.593 031	0	0	0	0	22	Limit reached
Gilgel Gibe II UI	15	15	100	31.33 738	0	50	24.883 68	55.84 978	22	Ok
Gilgel Gibe II U2	15	15	100	31.33 738	0	50	24.883 68	55.84 978	22	Ok
Gilgel Gibe U3	15	15	100	31.33 738	0	50	24.883 68	55.84 978	22	Ok
Gilgel Gibe U4	15	15	100	31.33 738	0	50	24.883 68	55.84 978	22	Ok
Gilgel Gibe 400KV	400	391.5 043	97.876 08	28.97 464	0	0	0	0	22	Ok
N1853	400	388.4 183	97.104 57	28.75 307	0	- 0.087 99	- 42.431 7	42.43 184	22	Ok
N1921	132	120.7 073	91.444 9	23.58 219	0	0	0	0	22	Limit reached

TABLE E.1
LINE DATA OF IEEE 14 BUS SYSTEM

Line No2.	From Bus	To Bus	Resistance	Reactance	Susceptance
1	1	2	0.01938	0.05917	0.026
2	1	5	0.05403	0.22304	0.025
3	2	3	0.04699	0.19797	0.022
4	2	4	0.05811	0.17632	0.019
5	2	5	0.05695	0.17388	0.017
6	3	4	0.06701	0.17103	0.017
7	4	5	0.01335	0.04211	0.006
8	4	7	0	0.20912	0
9	4	9	0	0.55618	0
10	5	6	0	0.25202	0
11	6	11	0.09498	0.1989	0
12	6	12	0.12291	0.25581	0
13	6	13	0.06615	0.13027	0
14	7	8	0	0.17615	0
15	7	9	0	0.11001	0
16	9	10	0.03181	0.0845	0
17	9	14	0.12711	0.27038	0
18	10	11	0.08205	0.19207	0
19	12	13	0.22092	0.19988	0
20	13	14	0.17093	0.34802	0

TABLE E.2
BUS DATA OF IEEE 14 BUS SYSTEM (IN MW)

Bus No	Bus Type	Voltage	Angle (rad)	P _L	Q _L	P _G	Q _G
1	1	1.06	0	0	0	0	0
2	2	1.045	0	0	0	40	0
3	0	1.01	0	94.2	19	0	0
4	0	1.019	0	47.8	-3.9	0	0
5	0	1.02	0	7.6	1.6	0	0
6	0	1.07	0	11.2	7.5	0	0
7	0	1.062	0	0	0	0	0
8	0	1.09	0	0	0	0	0
9	0	1.056	0	29.5	16.6	0	0
10	0	1.051	0	9	5.8	0	0
11	0	1.057	0	3.5	1.8	0	0
12	0	1.055	0	6.1	1.6	0	0
13	0	1.05	0	13.5	5.8	0	0
14	0	1.036	0	14.9	5	0	0

APPENDIX B

Injection and flow measurement of southern region 21 –bus system

A. Estimation southern region before PMU\estimation\zdatas.m

```

%      |Msnt |Type | Value | From | To | Rii |
zdata21 = [ %---- Voltage Magnitude -----%
            1      1      1      1      0      2.86483e-09;
            %-----%
            %---- Real Power Injection -----%
            2      2      2.57      1      0      1e-4 ;
            3      2      0.75      2      0      1e-4 ;
            4      2      0.4      3      0      1e-4 ;
            % pseudo measurments /zero injection
            5      2      0      12      0      1e-4 ;
            6      2      0      13      0      1e-4 ;
            7      2      0      14      0      1e-4 ;
            8      2      0      17      0      1e-4 ;
            9      2      0      19      0      1e-4;
            10     2      0      20      0      1e-4 ;

            %-----%
            %---- Reactive Power Injection -----%
            11     3      -0.455      1      0      1e-4 ;
            12     3      0.829      2      0      1e-4 ;
            13     3      0.012      3      0      1e-4 ;
            %pseudo measurments /zero injection
            14     3      0      12      0      1e-4;
            15     3      0      13      0      1e-4;
            16     3      0      14      0      1e-4;
            17     3      0      17      0      1e-4;
            18     3      0      19      0      1e-4;
            19     3      0      20      0      1e-4;

            %-----%
            %----- Real Power Flow ----- %
            20     4      0.006      1      4      64e-6;
            21     4      0.010      1      5      64e-6;
            22     4      0.750      2      16     64e-6;
            23     4      0.400      3      18     64e-6;
            24     4      -0.005      4      1      64e-6;
            25     4      -0.100      6      1      64e-6;

```

```

26      4      -2.374      7      1      64e-6;
27      4       0.790      7      8      64e-6;
28      4      -0.655      8     16      64e-6;
29      4      -1.390      9      7      64e-6;
30      4       1.06       9     10      64e-6;
31      4       0.140     10     11      64e-6;
32      4       0.030     11     15      64e-6;
33      4       0.154     18     20      64e-6;
34      4      -0.015     21     20      64e-6;

%-----%
%----- Reactive Power Flow ----- %
35      5       0.181      1      4      64e-6;
36      5       0.004      1      5      64e-6;
37      5       0.829      2     16      64e-6;
38      5       0.012      3     18      64e-6;
39      5      -0.2        4      1      64e-6;
40      5      -0.00       6      1      64e-6;
41      5       0.812      7      1      64e-6;
42      5      -0.804      7      8      64e-6;
43      5      -0.804      8     16      64e-6;
44      5       0.027      9      7      64e-6;
45      5      -0.027      9     10      64e-6;
46      5      -0.022     10     11      64e-6;
47      5      -0.010     11     15      64e-6;
48      5       0.015     18     20      64e-6;
49      5      -0.02      21     20      64e-6;];

```

B. Estimation southern region after PMU\estimation\zdatas.m

```

%      |Msnt |Type | Value | From | To | Rii |
zdata21 = [ %---- Voltage Magnitude -----%
            1      1      0.942116      13      0      2.86483e-09 ;
            2      1      0.954311      19      0      2.86483e-09 ;
            %-----%
            %---- Real Power Injection -----%
            3      2       2.57       1      0      1e-4;
            4      2       0.75       2      0      1e-4 ;
            5      2       0.4        3      0      1e-4 ;

```

```

%-----%
%---- Reactive Power Injection -----%
6      3      -0.455      1      0      1e-4 ;
7      3      0.829      2      0      1e-4 ;
8      3      0.012      3      0      1e-4 ;

```

```

%-----%
%----- Real Power Flow ----- %
9      4      0.006      1      4      64e-6;
10     4      0.010      1      5      64e-6;
11     4      0.750      2      16     64e-6;
12     4      0.400      3      18     64e-6;
13     4      -0.005     4      1      64e-6;
15     4      -0.100     6      1      64e-6;
16     4      -2.374     7      1      64e-6;
17     4      0.790      7      8      64e-6;
19     4      -0.655     8      16     64e-6;
20     4      -1.390     9      7      64e-6;
21     4      1.06       9      10     64e-6;
22     4      0.140     10     11     64e-6;
23     4      0.030     11     15     64e-6;

```

```

%.....
% measurment from PMU at bus 13 and 19
%.....
24     4      0.035     13     14     2.86483e-09;
25     4     -0.135     13     12     2.86483e-09;
26     4      0.215     10     12     2.86483e-09;
27     4      0.017     19     17     2.86483e-09;
28     4     -0.109     19     20     2.86483e-09;
29     4      0.736      8      17     2.86483e-09;
%.....
30     4      0.154     18     20     64e-6;
31     4     -0.015     21     20     64e-6;

```

```

%-----%
%----- Reactiv Power Flow ----- %
32     5      0.181      1      4      64e-6;
33     5      0.004      1      5      64e-6;
34     5      0.829      2      16     64e-6;
35     5      0.012      3      18     64e-6;
36     5     -0.2        4      1      64e-6;
37     5     -0.00       6      1      64e-6;
38     5      0.812      7      1      64e-6;

```


APPENDIX C

Power System State Estimation using Weighted Least Square Method..

Matlab code

```
num = 21; % 21 bus system.....
ybus = ybusppg(num); % Get YBus..
zdata = zdatas(num); % Get Measurement data..
bpq = bbusppg(num); % Get B data..
nbus = max(max(zdata(:,4)),max(zdata(:,5))); % Get number
of buses..
type = zdata(:,2); % Type of measurement, Vi - 1, Pi - 2,
Qi - 3, Pij - 4, Qij - 5, Iij - 6..
z = zdata(:,3); % Measurement values..
fbus = zdata(:,4); % From bus..
tbus = zdata(:,5); % To bus..
Ri = diag(zdata(:,6)); % Measurement Error..
V = ones(nbus,1); % Initialize the bus voltages..
del = zeros(nbus,1); % Initialize the bus angles..
E = [del(2:end); V]; % State Vector..
G = real(ybus);
B = imag(ybus);

vi = find(type == 1); % Index of voltage magnitude
measurements..
ppi = find(type == 2); % Index of real power injection
measurements..
qi = find(type == 3); % Index of reactive power injection
measurements..
pf = find(type == 4); % Index of real powerflow
measurements..
qf = find(type == 5); % Index of reactive powerflow
measurements..

nvi = length(vi); % Number of Voltage measurements..
npi = length(ppi); % Number of Real Power Injection
measurements..
nqi = length(qi); % Number of Reactive Power Injection
measurements..
npf = length(pf); % Number of Real Power Flow
measurements..
nqf = length(qf); % Number of Reactive Power Flow
measurements..
```

```

iter = 1;
tol = 5;

while(tol > 1e-4)

    %Measurement Function, h
    h1 = V(fbus(vi),1);
    h2 = zeros(npi,1);
    h3 = zeros(nqi,1);
    h4 = zeros(npf,1);
    h5 = zeros(nqf,1);

    for i = 1:npi
        m = fbus(ppi(i));
        for k = 1:nbus
            h2(i) = h2(i) + V(m)*V(k)*(G(m,k)*cos(del(m)-
del(k)) + B(m,k)*sin(del(m)-del(k)));
        end
    end

    for i = 1:nqi
        m = fbus(qi(i));
        for k = 1:nbus
            h3(i) = h3(i) + V(m)*V(k)*(G(m,k)*sin(del(m)-
del(k)) - B(m,k)*cos(del(m)-del(k)));
        end
    end

    for i = 1:npf
        m = fbus(pf(i));
        n = tbus(pf(i));
        h4(i) = -V(m)^2*G(m,n) - V(m)*V(n)*(-
G(m,n)*cos(del(m)-del(n)) - B(m,n)*sin(del(m)-del(n)));
    end

    for i = 1:nqf
        m = fbus(qf(i));
        n = tbus(qf(i));
        h5(i) = -V(m)^2*(-B(m,n)+bpq(m,n)) - V(m)*V(n)*(-
G(m,n)*sin(del(m)-del(n)) + B(m,n)*cos(del(m)-del(n)));
    end

    h = [h1; h2; h3; h4; h5];

```

```

% Residue..
r = z - h;

% Jacobian..
% H11 - Derivative of V with respect to angles.. All
Zeros
H11 = zeros(nvi,nbus-1);

% H12 - Derivative of V with respect to V..
H12 = zeros(nvi,nbus);
for k = 1:nvi
    for n = 1:nbus
        if n == k
            H12(k,n) = 1;
        end
    end
end

% H21 - Derivative of Real Power Injections with
Angles..
H21 = zeros(npi,nbus-1);
for i = 1:npi
    m = fbus(ppi(i));
    for k = 1:(nbus-1)
        if k+1 == m
            for n = 1:nbus
                H21(i,k) = H21(i,k) + V(m)*V(n)*(-
G(m,n)*sin(del(m)-del(n)) + B(m,n)*cos(del(m)-del(n)));
            end
            H21(i,k) = H21(i,k) - V(m)^2*B(m,m);
        else
            H21(i,k) = V(m)*
V(k+1)*(G(m,k+1)*sin(del(m)-del(k+1)) -
B(m,k+1)*cos(del(m)-del(k+1)));
        end
    end
end

% H22 - Derivative of Real Power Injections with V..
H22 = zeros(npi,nbus);
for i = 1:npi
    m = fbus(ppi(i));
    for k = 1:(nbus)
        if k == m

```

```

        for n = 1:nbus
            H22(i,k) = H22(i,k) +
V(n) * (G(m,n) * cos(del(m) - del(n)) + B(m,n) * sin(del(m) -
del(n)));
        end
        H22(i,k) = H22(i,k) + V(m) * G(m,m);
    else
        H22(i,k) = V(m) * (G(m,k) * cos(del(m) - del(k))
+ B(m,k) * sin(del(m) - del(k)));
    end
end
end

% H31 - Derivative of Reactive Power Injections with
Angles..
H31 = zeros(nqi,nbus-1);
for i = 1:nqi
    m = fbus(qi(i));
    for k = 1:(nbus-1)
        if k+1 == m
            for n = 1:nbus
                H31(i,k) = H31(i,k) + V(m) *
V(n) * (G(m,n) * cos(del(m) - del(n)) + B(m,n) * sin(del(m) -
del(n)));
            end
            H31(i,k) = H31(i,k) - V(m)^2 * G(m,m);
        else
            H31(i,k) = V(m) * V(k+1) * (-
G(m,k+1) * cos(del(m) - del(k+1)) - B(m,k+1) * sin(del(m) -
del(k+1)));
        end
    end
end

% H32 - Derivative of Reactive Power Injections with
V..
H32 = zeros(nqi,nbus);
for i = 1:nqi
    m = fbus(qi(i));
    for k = 1:(nbus)
        if k == m
            for n = 1:nbus

```

```

        H32(i,k) = H32(i,k) +
V(n)*(G(m,n)*sin(del(m)-del(n)) - B(m,n)*cos(del(m)-
del(n)));
        end
        H32(i,k) = H32(i,k) - V(m)*B(m,m);
    else
        H32(i,k) = V(m)*(G(m,k)*sin(del(m)-del(k))
- B(m,k)*cos(del(m)-del(k)));
    end
end
end

% H41 - Derivative of Real Power Flows with Angles..
H41 = zeros(npf,nbus-1);
for i = 1:npf
    m = fbus(pf(i));
    n = tbus(pf(i));
    for k = 1:(nbus-1)
        if k+1 == m
            H41(i,k) = V(m)* V(n)*(-G(m,n)*sin(del(m)-
del(n)) + B(m,n)*cos(del(m)-del(n)));
        else if k+1 == n
            H41(i,k) = -V(m)* V(n)*(-G(m,n)*sin(del(m)-
del(n)) + B(m,n)*cos(del(m)-del(n)));
        else
            H41(i,k) = 0;
        end
    end
end
end

% H42 - Derivative of Real Power Flows with V..
H42 = zeros(npf,nbus);
for i = 1:npf
    m = fbus(pf(i));
    n = tbus(pf(i));
    for k = 1:nbus
        if k == m
            H42(i,k) = -V(n)*(-G(m,n)*cos(del(m)-
del(n)) - B(m,n)*sin(del(m)-del(n))) - 2*G(m,n)*V(m);
        else if k == n
            H42(i,k) = -V(m)*(-G(m,n)*cos(del(m)-
del(n)) - B(m,n)*sin(del(m)-del(n)));
        else

```

```

                                H42(i,k) = 0;
                                end
                            end
                        end
                    end
                end

% H51 - Derivative of Reactive Power Flows with
Angles..
H51 = zeros(nqf,nbus-1);
for i = 1:nqf
    m = fbus(qf(i));
    n = tbus(qf(i));
    for k = 1:(nbus-1)
        if k+1 == m
            H51(i,k) = -V(m) * V(n) * (-G(m,n) * cos(del(m) -
del(n)) - B(m,n) * sin(del(m) - del(n)));
        else if k+1 == n
            H51(i,k) = V(m) * V(n) * (-G(m,n) * cos(del(m) -
del(n)) - B(m,n) * sin(del(m) - del(n)));
        else
            H51(i,k) = 0;
        end
    end
end
end

% H52 - Derivative of Reactive Power Flows with V..
H52 = zeros(nqf,nbus);
for i = 1:nqf
    m = fbus(qf(i));
    n = tbus(qf(i));
    for k = 1:nbus
        if k == m
            H52(i,k) = -V(n) * (-G(m,n) * sin(del(m) -
del(n)) + B(m,n) * cos(del(m) - del(n))) - 2*V(m) * (-B(m,n) +
bpq(m,n));
        else if k == n
            H52(i,k) = -V(m) * (-G(m,n) * sin(del(m) -
del(n)) + B(m,n) * cos(del(m) - del(n)));
        else
            H52(i,k) = 0;
        end
    end
end
end

```

```

end

% Measurement Jacobian, H..
H = [H11 H12; H21 H22; H31 H32; H41 H42; H51 H52];

% Gain Matrix, Gm..
Gm = H'*inv(Ri)*H;

%Objective Function..
J = sum(inv(Ri)*r.^2);

% State Vector..
dE = inv(Gm)*(H'*inv(Ri)*r);
E = E + dE;
del(2:end) = E(1:nbus-1);
V = E(nbus:end);
iter = iter + 1;
tol = max(abs(dE));
end

CvE = diag(inv(H'*inv(Ri)*H)); % Covariance matrix..

Del = 180/pi*del;
E2 = [V Del]; % Bus Voltages and angles..
disp('----- State Estimation -----');
disp('-----');
disp('| Bus |      V      | Angle | ');
disp('| No  |      pu      | Degree | ');
disp('-----');
for m = 1:n
    fprintf('%4g', m); fprintf(' %8.4f', V(m)); fprintf('
%8.4f', Del(m)); fprintf('\n');
end
disp('-----');

```