PERFORMANCE COMPARISON OF FRACTIONAL ORDER PID CONTROLLER AND LINEAR QUADRATIC REGULATOR FOR LEVEL CONTROL IN COUPLED TANK SYSTEMS

M.SC. THESIS

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HAWASSA UNIVERSITY, HAWASSA, ETHIOPIA

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EXAMINERS’ APPROVAL SHEET

We, the undersigned, members of the Board of Examiners the final open defense by **Mr. Aredo Haile**Id.No.**PGCon/009/09**, have read and evaluated his thesis entitled **“Performance Comparison of Fractional Order PID Controller and Linear Quadratic Regulator for Level Control in Coupled Tank Systems”,** and examined the candidate. This is, therefore to certify that the thesis has been accepted in partial fulfillment of the requirements for the degree.

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DECLARATION

I hereby declare that this thesis work is my original work, has not been presented for a degree in this or any other university, and all sources of materials used for the thesis work have been fully acknowledged.

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This thesis has been submitted for examination with my approval as a university advisor.

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LIST OF SYMBOLS

λ…………………………………Fractional order lamda

μ…………………………………Fractional order miu

p…………………………………Proportional

I…………………………………Integral

D…………………………………Derivative

k_p………………………………...Proportional gain

k_I………………………………....Integral gain

k_d…………………………..........Derivative gain

k…………………………….…...A gain of second-order plant

e(t)……………………………..Error signal

r(t)………………………………Input

G(s)…………………………....Controller transfer function(s-domain)

$G_P(s)$……………………………The transfer function of the plant

S…………………………………Shaped or oscillatory open-loop dynamics

m…………………………………Relative dominance

$G_a(j\omega)$…………………………...The transfer function of the frequency domain

T_P………………………………...peak time

T_r………………………………...Rise time

T_s………………………………...Settling time

ξ…………………………………..damping ratio of the second-order plant

ω_n………………………………The natural frequency of the second-order plant

$Q_{in}(t)$……………………………The flow of liquid into tanks

$Q_{out}(t)$…………………………..Flow of liquid

Im……………………………..Imaginary

Re……………………………..Real

Q………………………………...Weighting matrix

R…………………………………Scaler quantity

K………………………………...State feedback gain

A………………………………...(nxn) square state matrix

B………………………………...(nxr) input matrix

C………………………………...(mxr) direct feedback matrix

P……………………………………Riccati matrix

h(t)………………………………..Height of the tank

A……………………………………Area of tank

$\frac{dx}{dt}$...The derivative of x concerningthe time

R……………………………………Resistance flow

Cm………………………………..Cent meter

ω_b ………………………………...Low frequency

ω_h ………………………………...High transition frequency

LIST OF ABBREVIATION

Cl……………………………………………..Closed-loop

LTI……………………………………………Linear time-invariant

SISO…………………………………………Single input single output

MIMO………………………………………..Mult input multi-output

TF……………………………………………Transfer function

PI……………………………………………..Proportional plus integral

PD………………………………………......Proportional plus derivative

PID……………………………………………Proportional plus integral plus derivative

FOC…………………………………………...Fractional order controller

FOPID………………………………………...Fractional order proportional plus integral plus derivative

TID……………………………………………Tilt-Integral Derivative

IOPID……………………………………….....Integer order proportional integral derivative

FOPDT…………………………………...……First-order plus dead time

LQR…………………………………..……......Linear quadratic regulator

ISE……………………………………………..Integral square error

ZN……………………………………..….........Zeigler Nichols

CV…………………………………….........Control variable

SP……………………………………………....Setpoint

PV……………………………………………..Process variable

MV……………………………………………Manipulated variable

BIBO…………………………………………Bounded input bounded output

FOTF………………………………….............Fractional-order transfer function

OL……………………………………………..Open-loop

TF2SS…………………………………………Transfer function to system space

LQ……………………………………….Linear quadratic

ARE……………………………………...Algebraic Riccati equation

ODE……………………………………...Ordinary differential equation

MATLAB………………………………...Matrix laboratory

FOMCON……………………………….Fractional-order modeling control

ABSTRACT

Process control is essential in the industrial process because it guarantees the safety and optimization in a process. Additionally, process control is a useful tool to satisfy the environmental procedure and product quality necessities. In industries, one of the controlling process variables are level process, the liquid level controllers are asignificantconcern popular process &collectiveillustrativealsoreal-world in engineeringmethods, liquid level coupled tank system, and can be arranged fashionable form of interacting and non- interacting, in this thesis work only focus on interacting coupled tank control systems, industries there many problems which affected the liquid level, so to overcome this problem and obtain constant stable output and fast response various controllers are required.

In the automated function of the interacting container, the scheme is commonly recycled exclusively in the level process. A mechanism to regulate the process height in container and movement among tanks are problematic in the process automation. Industrial engineering requires fluid moved and reserved in a container, and then stirred to another to the container. This thesis work presents the improvement concerning sectional order (FOPID) and Determinate the Optimal controllerfor governinga desired liquid level of the tank system. Various fractional-order (FOPID, Crone, TID, and FOLL) controllerand control techniques will be tested to get a great performance. the output response is conducted within MATLAB®/Simulink®situation to verify the performances of the system in terms of rising Time (Tr), Settling Time (Ts), and Overshoot (OS).from the controller included in this thesis work the Linear quadratic regulator with integral action (LQI) controller have better performance compare to the other, the overshot 7.690% and rise time$218.48 msec$..

Keywords:-Process control, Level process, coupled tank, proportional integral derivative (PID), Fractional-order-controllerProportional-Integral-Derivative(FOPID),Linear Quadratic-Regulator (LQR).

CHAPTER ONE

INTRODUCTION

1.1.BACKGROUND

Many industrial and scientific processing needs anunderstanding of the amount of the contents in the tank and other containers. In many cases, it is not conceivable or not real-world to directly view the inner. The more understandable industrial application includes tank level gauging of milk, beer, wine in the industry,level gauging of acid, oil, water treatment, and solvent vessels in chemical plants; level checking of liquid in reservoirs. Done the last several decades, computer control of the manufacturing system has been the focus ofthe generalinvestigation.

As a significant considerationof fashionablemethodregulator, impressionfluid heightproceedingmaking cannot remainoverlooked, becausesafeness, actual and appropriateregulatory process heights areessential. Controlling of height processregulator is themainproblem in a process variable along with a mutualdemonstrative&concrete in engineeringpractices. the precedingmanufacturing, human-contained the fluid heightterminated a permanentfluid heightadjustmentwhile a liquefiedcorrespondingwith a convincedlevel; now this thesis work paper, the success of both fractional-order and linear-quadratic a controller to control the process level concerning the interactingcontainer is analyzed. The multitudeof industriesprocessessuch as petrochemical, water treatment, and beer iswith a container to governor the height. A fluidheightnecessarily governed withinappropriate controllers. a key purpose of a controller in the systemskeeps a height at the chosen set point and adaptable tothe new set-point[1].

The integer-order PID controlsarenormallyrecycled in controlling a height the containerin industries since governing the liquid in many tanks and undertaking between the tanks is a straight forward complication in the industries. Proportional controllers are appropriate to lead industrial controllers and for that reason areissues of steady exertion for the progress of their performance and robustness. One of the prospects to increase proportional integral derivative controls are to practice $PI^{\lambda}D^{\mu}$amongfractional-order integer portions. It permits us to adjust derivativeMiu(μ) and integral lambda (λ) order in addition to the $K_p,\ k_i\ and\ k_d$ constants where the values of μ and λinvention between zero and one[1]. The process industries needed liquid to stand pumped and stored fashionable the tanks and formerly pump it to another tank. Repeatedly the level of the tank must be controlled. Infurthermost industries, chemical processes present many stimulatingoutstanding to their non-linear dynamics behavior.

Numerousschemesare nonlinear in type as well as such type of plantcontrol is very tedious.In such systems, a proper controller can be designed through linearization techniques, and then the linearized system is easily controlled by two controllers such as fractionalorder controller and the LQRcontrollers [2].Inearlier industry, people controlled the liquid level done a fixed liquid height switch. When the liquid is up to a convinced level, the switch is mechanically closed or open to control the level. With the continuousprogress of mechanization, continuous control of the liquid level, that is, perceiving the fluid level at all a time is required in the industry. From the time whenthe non-linearity, the controller develops a challenging task to attainreasonable performance using different shapes of tanks. The non-linearity risesoutstanding to its shape by way ofnext to the end, it is broad and at the lower end, it becomes thin.

The primary task of the controls toretains a scheme at a setpointalternatively, from a FOC, FOPID controller which is widely applied in feed-back control of the industrial system. These controllers are defined with their simple construction and method. FOPID controllers are also enabled to provide expectedperformance for systems[3]. This thesis work is intendedto analyze the achievement of FOC like the FOPID, Crone, TID, and FOLL, controller on process level control. In this study, I chose the LQ Regulator and a FOPID controller for level process control in coupled tanks then correlate a performance response of two investigators, after that will select which controllers have a better performance for process-level control in coupled tank systems.

1.2.STATEMENT OF THE PROBLEM

The statement of the problem is toinvestigate the performance comparison of FOPIDcontroller and LQRfor process-level control in a combined tank in an instance oftheinteracting method & also control the external disturbance of the system, there iscomplicationallowanalogous a heightappropriate to fluidpopular the reservoir& move among tanks within a selected controller those are fractional order controller those are (fractional order proportional controller(FOPID) and optimal control techniques.

A particularexplanation to succeed in the achievement of the systems is to applyingthe fractional-order and LQR controller. Now there two-controllerthat have been applied to control the system, In this project, the Fractional-orderandLQ-Regulator monitorwill be selected by way of the feed-back detective for the process. Finally, examinethe performance of the systems.

1.3.MOTIVATION

The improvement of the control process for coupled tank systems is complex and more stimulating because of dynamics or non-linear by nature which showsmaximumpoint behavior. The interacting containers are a challenge as a result ofthe following issues.

- Non-linear & maximumstage system.
- Generally, multi-variable descriptioneffectscollaboration within a coupled tanks extremely liquid could move either two direction
- A level characterized by the coupled tanks has to be conserved at the chosensetpoint.

Performance analysis of the process has a substantialproblem in control as well as increasingly accepted as an area of importance in many processes industry. This problem has individual importance among the engineering community because it offers a solution to control the processes that cannot be designated by a SISO model. This thesis aims to comparethe assessment of the FOPIDand LQRcontrollers in order to getthe process-level to regulate when the system is interacting.

1.4.OBJECTIVE

1.4.1. GENERAL OBJECTIVE

The main purpose of this study is to evaluatetheperformance of the FOPIDcontrollers and LQRfor process-level controlsin the face based oninteracting systems.

1.4.2. SPECIFIC OBJECTIVES

The specific objectivesin order to address the general objective of the study are:

- To develop a mathematical model of the interacting tank and control the system.
- To develop the numerical modeling of both controllers.
- To designa Fractional-ordercontroller as well asLQRfor level process control systems in case of the coupled interacting system.

- To simulate the overall system using MATLAB/SIMULINK software for evaluation of the system performance.

1.5.RELEVANCE OF THE STUDY

In the modern industrial process controlling system is the concentration on its performance. Due to the problematic of numerous factories that interrupt the system, so to reduce the disruption and increase the quality of the product and reduced operating cost it is beneficial to analyze the attainment ofa system by using Fractional-orderand linear - Quadratic controller for level process control in case of coupled interacting tank system.

In this thesis, an optimal controller called the linear- quadratic and fractional-order controllers are proposed.A technique using a linear- quadratic regulator and fractional-order proportional -integral -derivativecontroller andalso theevaluation of both the controller performance isinvestigated with MATLAB®/SIMULINK®simulation.

1.6.SCOPE OF THE STUDY

The main opportunities of this study are to analyses and comparethe performance of the scheme using fractional-order PID controlsand (LQ) regulators, all of the models areimplemented through simulation using MATLAB®/SIMULINK®software.The effectiveness of both controllers is studied popular detail of the trainset response within achosen value of input value.

1.7.METHODOLOGY

The following methodologies have been used for thecompletion of this thesis work:

- Variousworks which are related to this thesis are studied.
- A numerical modelinga plant is establishedand a Fractional-order PID controller and linear-Quadratic regulator designed for process-level control in the case of the coupled-interacting tank system.
- The proposed system model is developed and analyzedthrough simulation of MATLAB®/SIMULINK® software.
- A responseof both controllers fromimitation results investigated and the conclusion is done.

1.8.ORGANIZATION OF THESIS

This thesisis prearrangedinto sixchapters:-

Chapter onepresents the overall background of the study. The main parts of this section areproblem statement, objective, significances, scope, and methodology of the thesis work.

Chapter twocovers the literature review of previous study and related overall information about Level process control,linear Quadratic regulator,and Fractional-order controller.

Chapter three presents the mathematical modeling of level process control ofthe coupled interacting tank.

Chapter four deals withthe designing of the Fractional-order PID and linear Quadratic-regulator controller in order to analyzetheir performancefor level process control in coupled tanks.

Chapter five presents the outcome of the coupled-tank system simulation results obtained with and without the Fractional-orderPID controller,linear-Quadratic regulator,andalso analysis performance of the system finally evaluated.

Chapter sixconcluded the thesis work and recommendation for future work also presented.

CHAPTER TWO

LITERATURE REVIEW

2.1.RELATED WORK

Someimportant concepts and previously accomplished works related to this thesisare reviewed asfollows.

K.Sundaravadivu et al (2011) proposed the schematic of Fractional -Order Controller for height regulation of Spherical Container, which is formed as a First Order PlusDead Time method near a setpoint. A reply to the considered Fractional-order-proportional-integral-derivative controller has correlated withthe classical (IOPID) controllersarranged the investigationalcircumstance. $API^{\lambda}D^{\mu}$controllers arecalculatedconsuming the decreasing of the Integral Square Error (ISE) method. This mechanism action a real-world & analytical technique of the controller's design for the considered class a the controller's plant [4]

Mostafa.A. Fellani et al (2015) studied the industrial application of the Coupled Tank System is widely used especially in chemical process industries. The control of liquid level in tanks and flow between tanks is a problem in the process technologies. The process technologies requireliquids to be pumped, stored in tanks, and then pumped to another tank systematically and development ofthe Proportional-Integral-Derivative (PID) controller for controlling the desired liquid level of the CTS Various conventionaltechniques of PID tuning method will be tested to obtain the PID controller parameters.

Dharam et al (2016) carried out that control height a tank is an ultimate concern in the mechanism of control. Theimperative a force up liquid & next toa certaindominance them indoors containers &afterwarda particularconversationto another tank.Generally, the moltenwouldremainsetthroughbiochemicalresponses or blending actioninside affecting a container, now somewhat situations, regularly analtitude of fluidmust be governed. Accordingly, fluidregulators are a dynamicmovement in systems. Conical reservoirs acquiresolicitations in several processes. Theshape permits animprovedrun of fluids. Extremely, the heightgovernor tanks are anexactproblematictopicas nonlinear & continuouslyillustrations anadjustmenthappeningthe area [5].

S.Nagammai et al (2016) proposed the scheme of optimal control for a three-tank process a typical to control the system is very difficult in controller design because of assumed non-linear flow and interaction between tanks. The state feedback controller (SFB) with a controller yields better achievement related to another controller. The transient response specifications and performance indices are compared and which indicates the fact of the FSFB over LQR[6].

G.Ganesh Naidu et al (2016) studied the advantage of a crone controller for level processor level process system first and second-generation crone controlling approaches used for scheming fractional-order controllers. The crone-controller is designed using the open-loop constraint which means it is based on the posterior synthesis method.The first and second-generation crone controller applied to the level process system and compared it with the integer-order PID controller. By comparing the step, frequency response, we can say the crone-controller is giving better performance compared with the conventional proportional integral derivative controller[7].

AmrutaS.jondhale et al (2015) carried out the level control of a tank system using the PID controller, in this paper to perform and control the system by using tuning the parameter of the PID controller gain, open-loop tuning method, closed-loop tuning method, and cohen-coon method[8].

Sasidharan et al (2017) performed the control of the fluid level in the tank and movement in between the tanks is a fundamental issue in industries. The liquid process requires be pumping, reserving in the tank. The liquids will be handled many times in the industries but continuously the level of fluids in the container must be adjusted. It is essential to understand how the tank is controlled and how the level control problems are solved [8].

P. Siva Sankar et al(2017) studied the performance analysis of the FOPID controlsconsidering amulti-tank, The sectional-order-PID regulator is an expansion to the integer-order PID controller which consists of lambda, Miu certain forward within Kp, Ki, Kd. In many cases, the sectional-order PID has verified additional efficiently the traditional PID controller. A fractional-order-PID supervisoraffordsoutstanding startup reply& a choseneffective feedback[9].

Scholar (2018) et alstudied Level control processes that are usually controlled based on the error signal. They have an upstream or downstream control valve and are drained by free fall or using a pump. After the inlet flow is used for controlling the level inside the tank, the output flow represents the main disturbance in the control system for such a case to design a rectilinear quadratic regulator and FOC and analyze the enactment of two controllers [10].

Kheleswarada(2017) et al performed the analytical modeling of the two-tank system, and also analyze the performance of a system without a controller with trial and error method by changing the position of the resistance(valve)[11].

Generally, I review thepaper related to my title, their similarity and also the difference when revising the above paper, the similarly the level a tank are modeling analytical, some of the paper designs a controller for the control the system, know when analysis the gap between the revising paper and performance comparison of fractional order Proportional Integral Derivative and LQ regulator considering the heightof control in the coupled tank system, the controller gain is investigated by trial and error method(tuning) and some paper have lack of controller design and also correlate with other controller but when control the level the coupled tank system using fractional order controller like(FOPID, TID fractional lead-lag compensator) and linear quadratic regulator, I have designed a controller with a specific value, performance assessment, and compare the performance of controller used in terms of transient response, eventually selected which controller has a better performance.

2.2.PROCESS CONTROL

Process control is essential in theindustrial process because it guarantees the safety and optimization in a process. Additionally, process control is a useful tool to satisfy the environmental procedure and product quality necessities. For chemical engineers, process control is widely appropriate in the industrial process. In many processes, such as petrochemical, paper, and watertreatment engineering is using the container togovern the height of the fluid.Anequitable of the controller in the level control is to maintain at a given level of the set-point and to receive a recent set-point. The proceeding industrynecessitates liquid prospectinflated, deposited containers, then inflate to an alternativereservoir, the fluidshandledviafraternizationbehaviorfashionable acontainer, neverthelesscontinually the height of fluidareservoiressentialorganized, besides the movementafluidcommitmentremain regulated. Height& outflowgovernor chamber isby the side of acoreaindustries' environments. Interacting tank(coupled tank) regulation of system manufacturing is aninspiringassignment for frequentsourcesoutstanding to non-linearity. Controls the fluid height in the coupled container arethemost importantconcern in the system. The height of the fluid is too high for variousdisappointedresponsestability, cause destruction. When the liquid height islow, it mightrequireunstablesignificances for thesystem. The industrialprocess presents many challenging control problems due to their non-linear dynamics behavior. Nonlinear models are used where accuracy over a wider range of operations is required where they can be directly incorporated into the controller algorithm. Because of the characteristic nonlinearity, most of the chemical process industries require control techniques, those to control such systems used are fractional order controller and linear quadratic regulator. The nonlinear system taken up for the study is the coupled tanks [11], [12].The engineeringapplication of liquid level control is tremendous, specifically in chemical process industries.Generally, the liquid level control happens in some of the control loops of a process control system. Much other industrial application is concerned with level control, may it be a single loop level control means single-input-single-output (SISO) or sometimes multi-input multi-output (MIMO). In some cases, level processcontrols that are available in the industries are for interacting and non-interacting tanks.But now for the two systems, I studied interacting coupled tank system to control the liquid level in tanks and flow between tanks is a problem in the process industries. Most of the industry uses different shapes of tank systems those arethe cylindrical tank, conical tank, and spherical tank to control the flow of rate and liquid level.in our design process, I have taken a cylindrical tank because it is inexpensive and no product loss at the end-point and thus most efficient.Here all the tanks under a study have their type of manipulated variables and thus we can regulator amovementspeed of the fluid ininteracting coupled system through regulating a variable for our desired output. Often the tanks are so coupled together that the levels also interact this must also be controlled.so it is necessary to analyze the response in the interacting coupled mode of various tanks with applying step point. Two design controls system tomanage the height of fluid on container[13].Analyticalforms areanexplanation of a systemin terms of equation [14]. A control system consists of sub-systems and processes accumulated to obtain the desired output with the desired input, given a specified input.Thesystemillustration isactuallynecessary for numerousexplanations such as to:-

- Controller designing for a specific system,
- Examine the effects of the process
- Estimate the development of the system
- Categorizeissues that prompting a system

- Develop a mathematical model of a process
- Classify the correlationamonginteracting tanks.

The presenteddemonstratingmethodscould be categorized into three main. The process couldrequirenumerous inputs otherwise excitation of the system a purpose a time. The output of the variable must be calculated. There is a correlation between input & output[15]. The process control is an important part of almost every process operation. The structure of a process is frequently measured by process variables such aswe manipulated a variable, measured variable, and controlled variable. The control of process variables is attained by the controller. Process control is repeatedly held responsible for dissimilar processes taking place in industries. Our objectives are to analyze the performance of the Fractional-order controller and linear quadratic regulator for processcontrol in case of level control. Applying an in the effect of a controller construction to control a process provides us numerous benefits: - better control system yield, better performance, and improved the response of the system. In-process control loop a controller job is to influence the control system via the control controlled variable so that the value of the controlled variable (system) equals the value of the reference model. The controlleristhe “Brain “of the process control and also generates analarmtowards the ending(controlled variable)dependent upon adeviation between the desired point and the measured value of the controlled variable.

2.3.LEVEL PROCESS CONTROLS

Control the height of a process in a coupledcontainerand flow among tanks is a basic problem in the process industries. The process industries want the liquid to be forced, stored in tanks, and then forced to another tank. Numerous times the determination of the liquid be treated by chemical or mixing treatment in the tanks, nevertheless continuously the level of liquid in the tanks requirement must be contained and outflowamong containers needs controlling.The flow &level regulate a containeris the brain of process industries. The liquid level tank can be arranged in the form of the interacting and non-interacting system can be controlled in different ways.

CHAPTER THREE

SYSTEM MODELING

3.1.ANALYTICALMODELING OF SINGLE AND COUPLED TANK SYSTEM

Mathematical modeling of level processes is characterized by large time constants and/or large time constants and time delays (dead time).In the instance of constant flow, in the stationary regime, the input and output quantities are equals.

Mathematical models are needed for these processes, to: determine the mechanism of the process, simulate the behavior, establish the control structures, and establish the control strategies[16].

3.1.1. MATHEMATICAL MODELING OF SINGLE TANK SYSTEM

Modern control assumption has been established on effectiverepresentations that are characterized as differential equations (DE) in other word transfer functions (TF). This single tank scheme depending onthe relationship amongst the movement of fluidwithin the container, $q_{in}(t)$& height of the liquid. Against the model, to design a feedback controls the output of the system at the desired point.

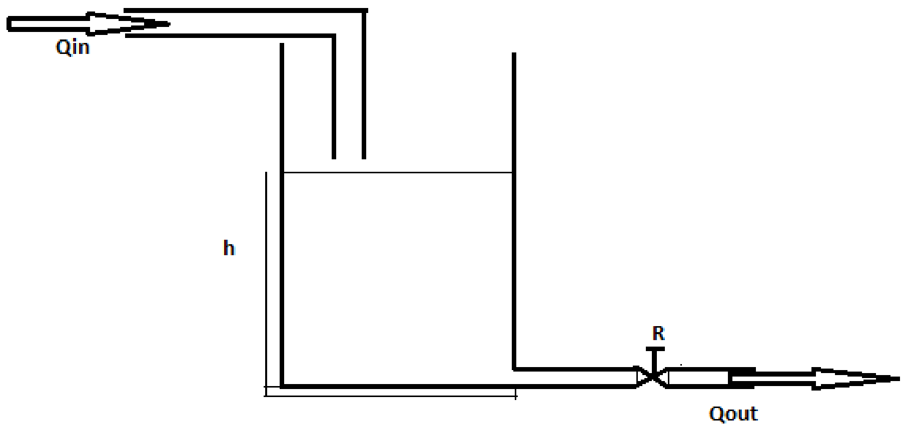


Figure 3.1:- Schematic diagram of the single tank

The liquid outflow of the tanks isequivalent to the height of fluid$h$ if the flow rateis laminar

$$q_{out} = kh(t) \tag{3.1}$$

Where$k$ *is* a constant parameter to control the flow of liquid

A mass balance across the tank gives:

$$A\frac{dh}{dt} = q_{in}(t) - q_{out}(t) \tag{3.2}$$

Where q_{in} −the flow rate of the liquid in

q_{out} –the flow rate of liquid out

A − Cross-sectional area of the tank

When we re-arrange the above equation

$$\frac{dh}{dt} + \frac{1}{A}q_{out}(t) = \frac{1}{A}q_{in}(t) \tag{3.3}$$

Equation (3.3) is re-considered and a new variable, x_1 is currentlydefined

$x_1(t) = h(t)$

Eqn (3.3) inscribed as:

$$\frac{dx_1}{dt} = -\frac{k}{A}x_1 + \frac{1}{A}q_{in}(t) \tag{3.4}$$

Generally when expressed the above equation in the state-space form:

$$\frac{dx}{dt} = \dot{X} = Ax(t) + Bu(t) \tag{3.5}$$

The previous equations are in general, linear equation when x is state variablesof a system. A liquid ina tank, there is a variable$h(t)$and a particular input $q_{in}(t)$ variable.so the matrices**A**and B are scalars. So the nonlinear system can be written in the form of:

$$\frac{dx}{dt} = y(x, u, t) \tag{3.6}$$

Where $y$is a non-linear, function.

3.1.2. ARTIMETICAL MODELING OF TWO INTERACTING TANK SYSTEMS

Plant modeling[17][18]

In this case, two tanks connected to form a coupled tank system; here the level of the first tank depends on the level of the second tank. The straightforward principle of control interacting two containersis continued the height of the tank steady after the flow of fluid into the tank and flow out of the liquid. The parameter used to control the coupled-tank system in process industries height, and cross-sectional area. To retain&governor the level liquid at anindicatedvalue, theflow input rate is regulated[19].The interacting (coupled) tank in figure 3.2.

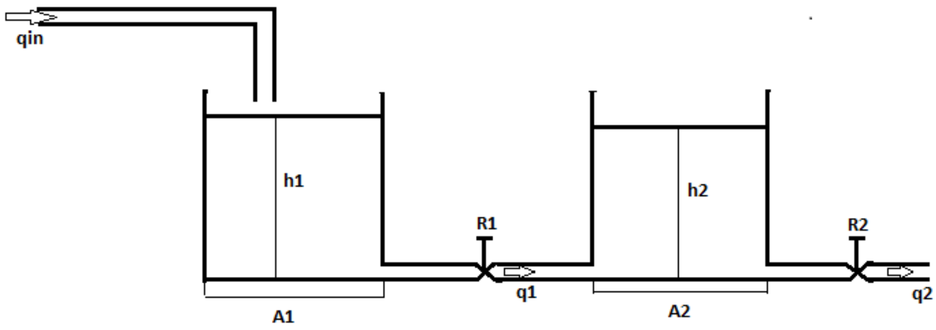


Figure 3.2:- Block diagram of interacting (coupled) tank [20]

Plant transfer function Resistance modeling,[18]

Consider the flow of the liquid through a pipe from the pump to the process tank and another pipe from the process tank to another process tank.

The resistance R for liquid-flow in such a container is defined as the change in the level difference to unit change in flow rate, that is

$$R = \frac{\text{change in level difference, cm}}{\text{change in flow rate, cm3 / sec}} \tag{3.7}$$

Where,

q_{in} =Volumetric flow rate input ($cm^3/_{sec}$)

q_1=Volumetric flow rate from tank 1 to tank 2 ($cm^3/_{sec}$)

q_2= Volumetric flow rate from tank output ($cm^3/_{sec}$)

h_1 =Height of the liquid level input (cm)

h_2 =Height of the liquid level in tank 2 (cm)

A_1= Cross-sectional area of tank 1 (cm^2)

A_2= Cross-sectional area of tank 2 (cm^2)

R_1=Linear resistance of flow from tank 1 through valve 1 ($cm^{-2}sec$)

R_2=Linear resistance of flow from tank 2 through valve 2 ($cm^{-2}sec$)

For tank 1:-

Mass balance equation can be written as

$$A_1\frac{dh_1}{dt} = q_{in} - q_1 \quad (3.8)$$

From the definition of resistance, the relationship between q_1 ,h_1 and h_2 is given by

$$q_1 = \frac{h_1 - h_2}{R_1} \quad (3.9)$$

$$A_2\frac{dh_2}{dt} = q_1 - q_2,\ q_2 = \frac{h_2}{R_2} \quad (3.10)$$

$$A_1\frac{dh_1}{dt} = q_{in} - \frac{h_1 - h_2}{R_1} \quad (3.11)$$

$$R_1A_1\frac{dh_1}{dt} = R_1q_{in} - h_1 + h_2 \quad (3.12)$$

By taking Laplace transform

Note that R_1A_1 is the time constant of the system, Taking the LT of both sides of equation (3.12), we obtain

$$R_1A_1sh_s(s) = R_1q_{in}(s) - h_1(s) + h_2(s) \quad (3.13)$$

$$h_1(s) = \frac{R_1q_{in}(s) + h_2(s)}{1 + R_1A_1s}, \quad \text{when } \tau_1 = R_1A_1$$

$$h_1(s) = \frac{R_1q_{in}(s) + h_2(s)}{1 + \tau_1 s} \quad (3.14)$$

For tank2, the mass-balance equation is

$$\left.\begin{aligned} A_2\frac{dh_2}{dt} &= q_1 - q_0 \\ A_2\frac{dh_2}{dt} &= \frac{h_1 - h_2}{R_1} - \frac{h_2}{R_2} \end{aligned}\right\} \quad (3.15)$$

$$R_2R_1A_2\frac{dh_2}{dt} = R_1h_1 - R_2h_2 - h_2R_1 \quad (3.16)$$

On dividing by R_1 and taking Laplace transform

$$A_2R_2h_2(s) + \frac{R_2}{R_1}h_2(s) + h_2(s) = \frac{R_2}{R_1}h_1(s) \quad (3.17)$$

$$h_2(s)\left(\tau_2 s + \frac{R_2}{R_1} + 1\right) = \frac{R_2}{R_1}h_1(s) \quad (3.18)$$

Where τ_2=R_2A_2

$$h_2(s)\left(\tau_2 s + \frac{R_2}{R_1} + 1\right) = \frac{R_2}{R_1}\frac{R_1 q_{in}(s) + h_2(s)}{1 + \tau_1 s} \quad (3.19)$$

$$h_2(s)\left(\tau_2 s + \frac{R_2}{R_1} + 1\right)(R_1 + R_1\tau_1 s) = R_2R_1q_{in} + R_2h_2(s) \quad (3.20)$$

$$h_2(s)\left(\tau_2 s + \frac{R_2}{R_1} + 1\right)(R_1 + R_1\tau_1 s) - R_2h_2(s) = R_2R_1q_{in} \quad (3.21)$$

Therefore the transfer function of the two tanks interacting system is expressed as

[21] , [22].

$$\frac{h_2(s)}{q_{in}(s)} = \frac{R_1R_2}{R_1\tau_2\tau_1S^2 + (R_1\tau_1 + R_1\tau_2 + R_2\tau_1)S + R_1} \quad (3.22)$$

$$= \frac{R_2}{\tau_2\tau_1S^2 + (\tau_1 + \tau_2 + R_2A_1)S + 1} \quad (3.23)$$

Accordingly, effecting state-space Eqn. of a plant can beimitated from the above Eqn(3.23)

$$\left.\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}\right\} \quad (3.24)$$

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ \frac{-1}{\tau_1\tau_2} & -(\frac{\tau_1 + \tau_2 + R_2A_1)}{\tau_1\tau_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{R_2}{\tau_1\tau_2} \end{bmatrix} [q_{in}] \tag{3.25}$$

$$y = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{3.26}$$

$$A = \begin{bmatrix} 0 & 1 \\ \frac{-1}{\tau_1\tau_2} & -(\frac{\tau_1 + \tau_2 + R_2A_1)}{\tau_1\tau_2} \end{bmatrix} \tag{3.27}$$

$$B = \begin{bmatrix} 0 \\ \frac{R_2}{\tau_1\tau_2} \end{bmatrix}, C = [1 \quad 0], D = 0 \tag{3.28}$$

When the interacting coupled tanks with disturbance system

Disturbance Analysis for an interactingcontainer of fluid height control systems can be as follows:-

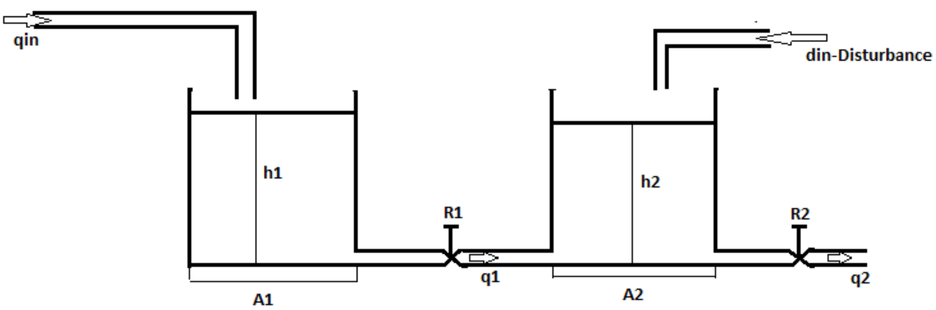


Figure 3.3:-Block diagram of the interacting two with a disturbance

The disturbance transfer function analysis as below

$$A_2 \frac{dh_2}{dt} = d_{in} - q_2 \tag{3.29}$$

Since from equation (3.7), $q_2 = \frac{h_2}{R_2}$

$$A_2 \frac{dh_2}{dt} = d_{in} - \frac{h_2}{R_2} \tag{3.30}$$

The LT of equation (3.30)

$$\frac{H_2}{D_{in}} = \frac{R_2}{R_2 A_2 s + 1} \tag{3.31}$$

CHAPTER FOUR

CONTROL METHODS AND DESIGN

4.1.INTRODUCTION

A control structure is a system or set of devices, that manages the signal or regulates they presence of additional processes or systems to accomplishthe chosen set-point.in other words, the definition of the control method can be simplified as a system, which controls other systems, twoforms of controlmechanism are: -Open and closed-loop controlmethod.Open-loop control methods arewherein a control accomplishment is self-regulating the response the processes but a (CLS) in which the response affects the input insure that the input will regulate itself based on the response generated[23]. The closed-loop summarized below in the tabular form.

Controller	Advantage
Closed Loop	Disturbance rejection
	Guaranteed attainmentuniform withinprototypical uncertainties, while the structure does not much faultlessly the physical process and model specification are not exact
	The unbalanced process can be steadied
	Compactfeeling to stricturedifferences
	Improved performance using reference tracking

	Increased productivity
	Energy and Material Conservation
	Safety

Table 4.1:- summarization of closed-loop controller

If the process does not happen the desired performance specification, Controllers are used&A also connected either in series with or parallel to the plant depending upon the specification.A show figure 4.1 feedback control system with a controller.

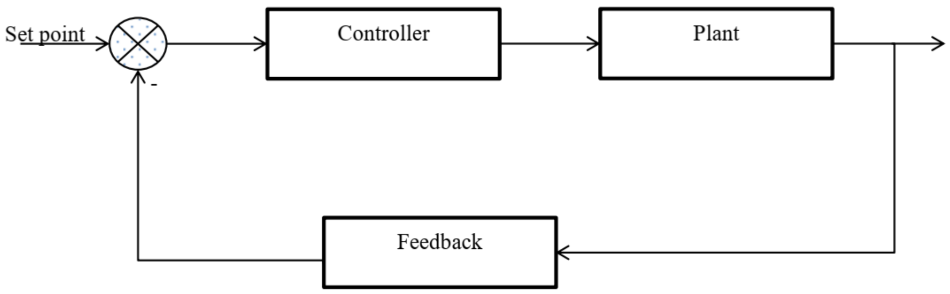


Figure 4.1:- closedControl system

As shown above,figurecontrolsignalsgenerated which are the changeamongthe set-point signal and the feedback signal.The control signal resolves the scalethroughwhatever an output responseturnsagainstthe desired point.The controller is staying to keeping the controlled variable such as liquid level, motor speed;robot joint angle at a set-pointin addition tothe input signal is applied to plant which will give good results. Feedback control is an action in which a measured variable is compared to its desired value to produce an error signal which is acted upon in such as to decrease the amplitude of the error. For a plant with MIMO, the situationdesiresnumerousregulators. For a SISO, need a single controller for managing.

4.2.TYPE OF CONTROLLERS

There is various type of controller used for performance analysis of the system within a given specification. Controllers can be categorized into, feed-back, &feed forwardregulator.A contributionnearthe feed-back controls identical as anythingfrustratingtowardgovernoroutputregulates are feed-back interested in the supervisor. The feed-back method frequentlyaffects in-betweensomeplacewell-ordered variables aren’tpreferredsetpoint. The feedforwardgovernorput upescape slow-ness a feed-back regulatethrough feedforward controller.

$$\left.\begin{array}{l}\text{proportional} - \text{integral} - \text{derivative controller}\\ \text{pole} - \text{placement controller}\\ \text{optimal controller}\end{array}\right\}\text{are feedback controller}$$

A first two-controller is the feedback controller & A third is anFSBcontrol. Pole-placement regulators are a feed-back controller that used for placing closed-loop-poles to the setpointposition in the plane. Pole-placement used only for (SISO) system. For the(MIMO)multiple-input multiple-output system, the complication of over-affluence of the designed controller is challenged, for such onesystem, we didn’trecognizein what wayto define all the construct elements, sincesimply a restrictedamount of system can be initiateon or after the closed loop-pole position.A pole-placement-technique some trial& error technique with pole-position was a prerequisitefor we don't recognizepreviouslywhatever pole location will contribute betterachievement. The controller is equipment introduced to monitor the process and adjust some variables to maintain the system at or near desired conditions. Controller design has a significant place in the region of industrial engineering. The type of controller to be selected is important for achieving the desired design criteria. In practice, simple and low-degree controller structures are preferred most of the time. Among these controller structures, fractional-order controller those are fractional-order proportional-integral controller, crone controller, TID controller,fractional-order lead-lag controller,and LQ regulator isdesired the most. There are two varietiesof controllers are selected to evaluate the performance of process-level control in form of the interacting in this thesis work .those are fractional order controller (FOC) and linear quadratic controller (LQR). Then the application of those two different controllers in industrial applications is compared [14],[23]

The overall importance feature of feedback control analyzed as follows:

Controller	Importance feature of feedback control
Feedback control	To stabilize the system
	To reduce system output sensitivity to system parameter variation due to the established of the system components.
	To reduce the impact of external a system effect on the system response

	To improve the system output transient
	To improve the system output steady-state response
	To reduce system sensitivity inaccuracies in the system measurements

Table 4.2:- importance features of the feedback controller

4.2.1. INTRODUCTION PID CONTROLLER

PID (proportional–integral–derivative) controller used as a linear controller for the control process with a good response. A PID controller is commonly used in the process and applying accurately the grouping of three types of corrective actions to the error signal, which characterizes how far or near is the desired set point from the actual output. As widely known, these three control actions are proportional, integral and derivativeThe key feature when tuning PID controllers is in determining how to best syndicate those three terms to accomplish the most well-organized instruction of the process variable for the well-thought-out problem. As well known, the most understandable way is to use a simple weighted sum where each term is multiplied by a tuning constant or gain[24] PID's working principle is that it calculates an error value from the processed measured value and the desired reference point. The work of the PID-controller is to reduce the error by changing the inputs of the system. Proportional-integral derivative control is the basic control scheme of the classical control system. [25]. The assessment of aprocess could beimproved[26] using the proper value of gain$k_p, k_d,$and k_i.an analytical equation for (IOPID) control of a system within a value of input $u(t)$, value of output $y(t)$ and error signal $e(t)$ is communicated as (4.19) & (4.20) where $k_p -$ is the proportional gain, $k_i -$ integral gain, and $k_d -$derivative gain.

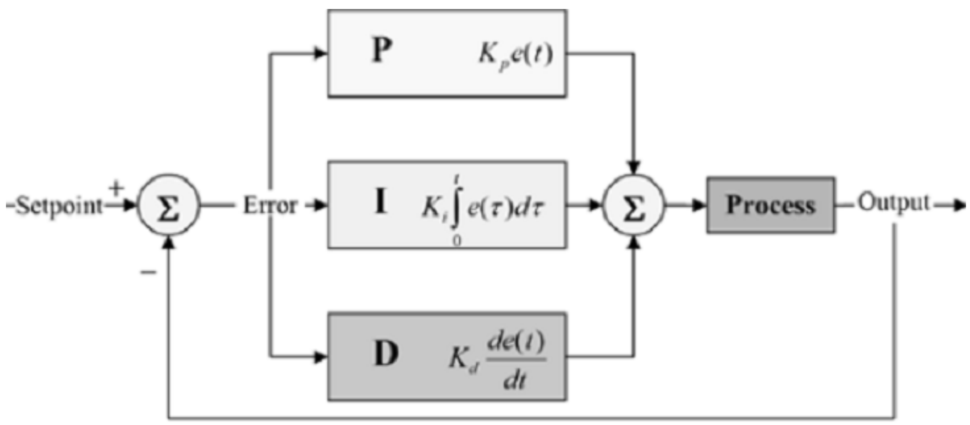


Figure 4.2:- PID Controller [27]

Proportional gain k_p −improve the structure to create it protected a capacitydisorder. $\mathrm{k_i}$ −Supportsto decrease errors of steady-state.k_d −Supports to improve the establishment of a closed loop [26].IOPID controller is the type of feed-back whose output a control variable (CV), arebuilt on the erroramongasetpoint (sp) and process variable.

$\mathrm{k_p}$ −is responsible for system stablemoreover low and the process variable can implication;moreover high and a process variable can oscillate.

$\mathrm{k_i}$ −Accountable for afault to zero, but towardestablished $\mathrm{k_i}$ toogreat is to requestvibration.

$\mathrm{k_d}$ −Isaccountable for a plant output to great&the process variable will vibrate. Proportional-integral-derivative-controller challenges to accurate the inaccuracyamong an (MV) &the setpoint value by computing. Error is the reference to regulate some input to the system to awell-defined set-point. The output of the controller can be defined in terms of the receptiveness of the controller to an errorsignal, the amount to which the controller overshoots the set-points and the amount of system oscillation [26].

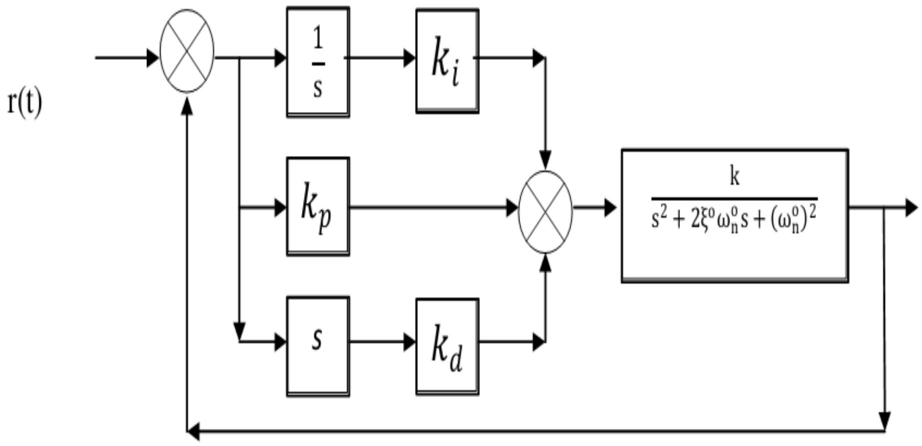


Figure 4.3:- PID controller design for a system

The input of PID controller r (t) is equivalent to the sum of the three signals, the signals obtained by multiplying the error signal by a constant proportional gain plus the signal obtained by differentiating and multiplying the error signal by constant integral gain [28]. The output of the PID controller is given Laplace to transform and itis given as

$$r(t) = K_P\ e(t) + K_d \frac{de(t)}{dt} + K_i \int e(t)\, dt \qquad (4.1)$$

The three terms of integer order (IOPID) controls have three modes of a control character. Those three basic modesare -proportional, integral, and derivative terms. IOPID-the regulators are used in monitoring the closed-loop practice open-loop process. Controller parameterized mathematically as:-

Taking Laplace transform

$$G_C(s) = \left[k_p + \frac{K_i}{s} + \quad k_d s\right] \qquad (4.2)$$

Where

K_P − Proportional gain

K_i − Integral gain

K_d − Derivative gain

$$G(s) = K_p(e(t) + \frac{1}{T_i} e(t) + T_d \frac{de(t)}{dt} \qquad (4.3)$$

Where T_i − is the integral is time andT_D − is the derivative time,TheP-parts entertainmentarrangedcurrent value of the error signal, the integral -represents a normal of aforementionedmiscalculations and animitative can assumed a forecast of forthcoming faultconstructed on linear-extrapolation well-meaning of note that the control-unit$u(t)$ arefashionedexclusivelystarting the inaccuracy $e(t)$. Generally, in a process, the PID controllers are termsdonates and accomplishing transient response&stability [28], [29].

		Effect of Performance			
Term	Response	Settling time	Rise time	Overshoot	Steady-state error
Proportional	K_p	Small change	decrease	Increase	Decrease
Integral	$K_i = K_p(\frac{1}{T_i})$	Increase	decrease	Increase	Eliminate

Derivative	$k_d = K_p T_d$	Decrease	Small change	Decrease	Small change

Table 4.3:- Effect of the parameter in a closed-loop system

Where $k_p$denotes the proportional gain, $k_i$denotes the integral gain, and $k_d$denotes the derivative gain correspondingly. The proportional term determines the change of the yield that is comparative to the presentinaccuracy. The comparative gainduration is disturbed within the obtainable state of the variable. The important gain term is comparative for mutually the greatness andinterval of anerror. The comparative term increasesthe speed of the process towards the set-point&reduces the remainingsteady-state error that couldhappen within a proportionate only regulator. Theaggregate of a difference of the progression error is intended by responsible the varianceincline of the derivative concerning interval. Ratio of a transformation of the error is reproducedviathe $k_d -$ gainclassical PID regulators have been extensively used in industry, due to their effectiveness for linear systems, ease to understand, and simple to implement. As shown in a figure,the actual controller of the process can be investigated by the transient response.

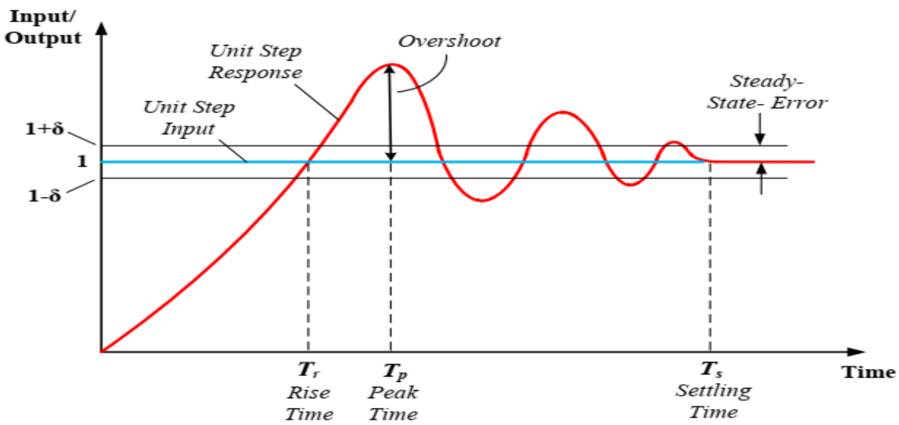


Figure 4.4:- parameters of transient response analysis of a controller

4.2.1.1.COMPARATIVE-TERM

A comparativeterm is formedthrough a yieldassessmentto the stimulating error significance. The comparativeresponse is attuned by enlarging the error by a relentlessimprovement called the comparative gain constant.

The proportional term is given by:-

$$\left.\begin{array}{c} u(t) = k_p e(t) \\ \text{or} \\ \frac{(S)}{E(S)} = K_P \end{array}\right\} \qquad (4.4)$$

Anextraordinary $K_p$resultshappening a large differenceoutput for theparticularmodification in the inaccuracy. When the gain of proportionalityis high, the processunsteady, whereas aninsignificantyieldreply to anenormous input error& a less reaction to anoutsizedcontributionissmall, the control accomplishmentmightstandbesideunimportantas soon asreplyingtoward the processinstabilities. In the Manufacturingpreparation,the comparativeterm indicates that tounpackaged of the output adjustment.

4.2.1.2.INTEGRAL TERM

An effect of integral gains arecomparable to together the amount of the error & the interval of error. APID controller, the essentialgain are sums of immediate inaccuracy over interval&contributes the collected off-set that wouldneed been modifiedformerly. Gatheredinaccuracy is then increased by the fundamentalimprovement (K_i) and further to the regulator output.

Basic is expressed as follow:

$$u(t) = k_p e(t) + k_i \int_0^t e(\tau) d\tau \qquad (4.5)$$

Or

$$\frac{u(s)}{e(s)} = k_p \left(1 + \frac{1}{\tau_i s}\right) \qquad (4.6)$$

4.2.1.3.DERIVATIVE TERM

Imitative of a systeminaccuracy is premeditated by responsible the gradient of the inaccuracy over interval&aggregatepercentage of deviation by the $k_d$relentless.

A Derivative term is set by

$$u(t) = k_p e(t) + k_d \frac{de(t)}{dt} \quad (4.7)$$

Or

$$\frac{u(s)}{e(s)} = k_p(1 + \tau_d s) \quad (4.8)$$

The function of the derivative term slows the rate of variation of the controllerresponse to control the system plant. The derived controllers are used to decrease the level of the overshoot designed by the integral element and increase the combined controller system stable. Conversely, the derivative term reduces the momentary response of the regulator, and similarly, differentiation of a signal increasessound and thus this term in the controller is highly sensitive to noise in the error term and can source a process to become unstable if noise and the derivative improvement are sufficiently large.

In general, The PID controller has the following form in the time-domain,The Overall Characteristics of Proportional, Integral, Derivative Controller gain in Tabular Form.

Proportional control	$u(t) = K_P e(t)$
Integral control	$u(t) = K_i \int e(t)\,dt$
Derivative control	$u(t) = K_d \frac{de(t)}{dt}$

Table 4.4:- Features of the PID controller

Steps for the scheme a Proportional-integral-derivative controller are:-

Step1: determine each value of PID controller gain within the desired system

Step2:- decide what features of the processessentialdeveloped.

Step 3:-usedcomparative gain to reduce a rise-time

Step4:- use derivative gain to decline the overshoot and settling time

Step5:-use integral gainto remove the steady-state-error.

Proportional-integral-derivative (PID) controllers are the feedback loop controlling mechanism. It corrects the error between a measured variable value and the desired set-point by calculating and then a corrective action adjust the process (plant) as per the requirement. The biased addition of these three arrangements is used to adjust the process (plant) via a control element. By calculating the value of three constants in the PID controller, it can offer a control achievement designed for specific activities.

The overall structure of the system control is shown in Figure 4.5.

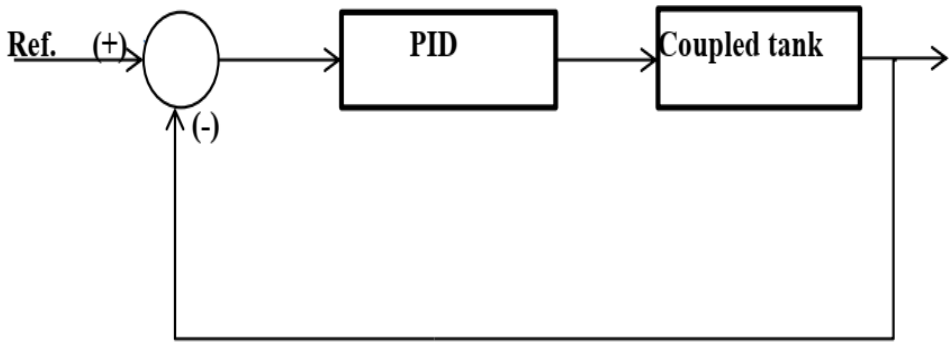


Figure 4.5:- closed-loop control systems

Now to design the proportional-integral-derivative controller a certain dominant pole placement method is used. The transfer-function of PID controller need to Control 2nd -order plant ($G_p(s)$) (with slow "S" designed or oscillatory open-loop changing aspects).

$$G_C(s) = \left[k_p + \frac{K_i}{s} + \quad k_d s\right]$$

$$= \frac{k_d s^2 + k_p s + k_i}{s} \qquad (4.9)$$

Where,

$$\left.\begin{array}{l} K_p - \text{proportional gain} \\ K_i - \text{integral gain} \\ K_d - \text{derivative gain} \end{array}\right\} \text{of the PID controller}$$

In general, a second-order plant is categorized by an open loop transfer-function,

$$G_p(s) = \frac{K}{S^2 + 2\xi\omega_n s + \omega_n^2} \qquad (4.10)$$

Where,

the ξ-damping-ratio of a second-order plant

ω_n-The natural- frequency of a second-order plant

K-Constant of a2nd -order plant

Then, the CL transfer function becomes,

$$G_{c1}(s) = \frac{G_c(s)G_p(s)}{1 + G_c(s)G_p(s)} \qquad (4.11)$$

Substituting for $G_C(s)$ and $G_p(s)$ from Equation (4.8) and (4.10) respectively into Equation (4.11) gives,

$$G_{C1}(s) = \frac{k(k_d s^2 + k_p s + k_i)}{S^3 + (2\xi\omega_n + kk_d)S^2 + (\omega_n^2 + kk_p)s + kk_i} \qquad (4.12)$$

From the equation (4.10) perfectly, the open-loop plant has two poles at$\left[\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}\right]$and from (4.11) are also obvious the PID-controllers have onesextreme at derivation& also two-zeros at$\left[\frac{K_p}{2K_d} \pm j\sqrt{\frac{K_i}{K_d} - \frac{K_p^2}{4K_d^2}}\right]$.Dignified that together the poles of the system &regulator zero are complex conjugates in the complex s-plane. FromEqn (4.12), assumed that the CL classification has two zeros & three poles in the complex s-plane and position of aCLzeros continuemodest as in (4.11) although the location of CL poles differencesdependent on the PID controller gains. For definitefull state feedback with PID- controllers, the CL system (4.12) would require one real-pole which necessity be extreme away since the realpart of the additional two complexes (conjugate)CL poles as considered[30]. Currently, if the chosenCL calculations of a 2nd -order scheme can be specified as the qualifications on ξ^{C1}(Damping ratio of the CL scheme) andω_n^{c1} (Natural frequency of the CL system) as in one can simplysubstitute the location of the real zero (α) by$(-\alpha\xi^{C1}\omega_n^{c1})$, providing that $s = -\alpha\xi^{C1}\omega_n^{c1}$ is chosen to be hugesufficientregarding $(-\alpha\xi^{C1}\omega_n^{c1})$.Consequently, selectinganappropriate ofcomparativegovernance (α), the third-order CL system (3rd)[31]determinationaccomplish like second-order classificationconsuming the user-specified CL damping-ratio ξ^{C1}(percentage of maximum overshoot) & closed-loop natural frequency ω_n^{c1} (Rise- time). In this situation, the distinguishing polynomial is inscribedas:

$$(S + \alpha\xi^{C1}\omega_n^{c1})(S^2 + 2\xi^{c1}\omega_n^{c1} + (\omega_n^{c1})^2 = 0 \qquad (4.13)$$

After multiplication Equation (4.13) is written as,

$$s^3 + (2+\alpha)\xi^{C1}\omega_n^{c1}s^2 + (1+2(\xi^{C1})^2(\omega_n^{c1})^2)s + \alpha\xi^{C1}(\omega_n^{c1})^3 = 0 \quad (4.14)$$

Relating the coefficients of Equation (4.13) with the denominator of Equation (4.12), mathematical modeling of PID controller gain can be calculated as follow.

$$\omega_n^2 + kk_p = 1 + 2\,\alpha(\xi^{C1})^2(\omega_n^{c1})^2 \quad (4.15)$$

$$k_p = \frac{(1 + 2\,\alpha(\xi^{C1})^2(\omega_n^{c1})^2) - \omega_n^2}{K} \quad (4.16)$$

$$kk_i = m\xi^{C1}(\omega_n^{c1})^3 \quad (4.17)$$

$$k_i = \frac{\alpha\xi^{C1}(\omega_n^{c1})^3}{K} \quad (4.18)$$

$$2\xi\omega_n + kk_d = (2+\alpha)\xi^{C1}\omega_n^{c1} \quad (4.19)$$

$$kk_d = (2+\alpha)\xi^{C1}\omega_n^{c1} - 2\xi\omega_n \quad (4.20)$$

$$k_d = \frac{(2+\alpha)\xi^{C1}\omega_n^{c1} - 2\xi\omega_n}{K} \quad (4.21)$$

The overall PID gains are:-

$$\left.\begin{aligned} K_p &= \frac{(1 + 2\,\alpha(\xi^{C1})^2)(\omega_n^{c1})^2 - \omega_n^2}{K} \\ K_i &= \frac{\alpha\xi^{C1}(\omega_n^{c1})^3}{K} \\ K_d &= \frac{(2+\alpha)\xi^{C1}\omega_n^{c1} - 2\xi\omega_n}{K} \end{aligned}\right\} \quad (4.22)$$

The Four major characteristics that affect systems dynamically can be summarized in the table below

No	Transient response	Characteristics of the response
1	Rise-time	period it profitsaimed at process response to risingaway after90% of aset-pointsignificance forthe first time
2	Over-shoot	In what wayconsiderably the ultimateassessment is high than the steady-state regularizedbeside the steady-state
3	Settling-time	The historical itsreceipts for the system to touch to its steady-state
4	Steady-state-error	The modificationamong a steady-state& the preferredreply

Table 4.5:- Transient response for Performance analysis

4.3.BASIC CONCEPT OF FRACTIONAL ORDER CALCULUS

FactionalCalculus is an old accurate topic since the 17th century. The sectional-order calculus is a zone of mathematics that expressesthrough derivatives and integrals since non-integer guidelines. In additionalverses, it is anoversimplification of the old-fashionedcalculus that indicatesequivalentconceptions&apparatuses but throughconsiderablylargerappropriates. Happening the last two periods, fractional calculus has remainedrecollected by scientists and engineers and applied in an accumulative number of grounds, specifically in the area of regulatorconcept.Fractional calculus benefitsto estimate$\frac{d^{\alpha}}{dt^{\alpha}}, \alpha -$fold integrals where α is fractional, irrational, or complex. Fractional-order systems,α is measured to be fractional. These mathematical occurrencespermittelling anactualpurposeadditionalcorrectly than conventionalinteger order approaches. The foremostpurposeofexpending the integers order simulationsremained the interval ofanexplanationaimed at sectional differential equivalences.The success of fractional-order controllers is absolute with a lot of success due to the development of effective methods in differentiation and integration of non-integer order equations.There are dissimilarexplanations of Fractional-Order-differentiations and integrations. Some of the definitions range directly from integer-order calculus. The well-established definitions include the Grunwald-Letnikov definition, the Cauchy integral formula, the Caputo definition, and the Riemann-Liouville. Classificationdeterminationstandsbrief as follows &formerly their belongingsdeterminationremainsparticular.

4.3.1. FRACTIONAL ORDER CALCULUS

Cauchy'sdescribesectional Order integration.characterization is anall-purpose the per-request of the integer-order Cauchy formulation

$$D^{\gamma} f(t) = \frac{\Gamma(\gamma+1)}{2\pi j} \int_{C} \frac{g(\tau)}{(\tau - t)^{\gamma+1}} d\tau \tag{4.23}$$

Wherever C is the smooth curve neighboring the single-valued purposeg (t).

Grunwald–Letnikov'sexplanation is perhaps the best known due to its most suitability for the realization of discrete control algorithms.The Grunwald-Letnikov definition is expresses as[32]

$$_{a}D_{t}^{\alpha} f(t) = \lim_{h\to 0} \frac{1}{h^{\alpha}} \sum_{j=0}^{[\frac{t-a}{h}]} (-1)^{j} \binom{\alpha}{j} f(t - jh) \tag{4.24}$$

Where $w_j^{\alpha} = (-1)^j \binom{\alpha}{j}$denotes the factors of the polynomial$(1 - z)^{\alpha}$the factors can likewiseremaindeveloped recursively beginning

$$w_0^{\alpha} = 1, w_j^{\alpha} = (1 - \frac{\alpha+1}{j} w_{j-1}^{\alpha}, \text{when } j = 1,2,3 \ldots\ldots\ldots\ldots\ldots$$

Riemann–Liouville definesFractional Order differentiation the Fractional Order integration is defined as [33]

$$_{a}D_{t}^{-\beta} f(t) = \frac{1}{\Gamma(\beta)} \int_{b}^{t} (t - \tau)^{\beta-1} f(\tau) d\tau \tag{4.25}$$

Where 0 $<\beta<$ 1and b- theleadingintervaldesign, repeatedlysupposedtowardsremainnothing, i.e., b= 0.

The separation is formallydesignated as $_{a}D_{t}^{-\beta} f(t)$

The explanationexceedingformula is the nethermostrepeatedlyrecycledclassification in fractional- order calculus. The indexes on collectededges of D characterize the lower and upper restrictions in the combination.

Caputo'sdescribe sectional Order separation). Caputo's classification is specified by

$$_{0}D_{t}^{\gamma} y(t) = \frac{1}{\Gamma(1-\gamma)} \int_{0}^{t} \frac{y^{(m+1)}(\tau)}{(t - \tau)^{\gamma}} d\tau \tag{4.26}$$

Where α=m+ γ, m is an integer, and $0 < \gamma \leq 1$. Similarly,Caputo's sectional Orderaddition is well-defined as

$$ {}_0D_t^{\gamma} y(t) = \frac{1}{\Gamma(-\gamma)} \int_0^t \frac{y(\tau)}{(t-\tau)^{\gamma+1}} d\tau, \gamma < 0 \quad (4.27) $$

4.3.2. FRACTIONAL ORDER DIFFERENTIATIONPROPERTIES

The FOD issucceedingbelongings[34]

The FOD ${}_0D_t^{\alpha} f(t)$concerningt of an investigativepurposef (t) istoosystematic

Sectional Order differentiation is correctlymatchingthrough integer order one while α=n is a whole number. Also ${}_0D_t^{\alpha} f(t) = f(t)$

The FOD is rectilinear; i.e., aimed ateach constant aand b takes

$$ {}_0D_t^{\alpha} [af(t) + bg(t)] = a\, {}_0D_t^{\alpha} f(t) + b\, {}_0D_t^{\alpha} g(t) $$

FODinfluencesaccomplish the commutative-law and likewise,please

$$ {}_0D_t^{\alpha} \left[{}_0D_t^{\beta} f(t) \right] = {}_0D_t^{\beta} \left[{}_0D_t^{\alpha} f(t) \right] = {}_0D_t^{\alpha+\beta} f(t) $$

The sectional-Order-differentiation Laplace transforms is well-defined as

$$ \mathcal{L}\left[{}_0D_t^{\alpha} f(t) \right] = s^{\alpha} \mathcal{L}[f(t)] - \sum_{k=1}^{n-1} s^k \left[{}_0D_t^{\alpha-k-1} f(t) \right]_{t=0} $$

In specific, condition the byproducts of f (t) is allidentical to 0 at t = 0, the singleconsumes

$$ \mathcal{L}[{}_0D_t^{\alpha} f(t)] = s^{\alpha} \mathcal{L}[f(t)] $$

Presentations of sectional -calculus can remaininventingin the region of regulatorclassifications. Sectional order calculus decides the derivatives and integrals are of any physicalamount. The FOD can be characterized by anall-purposeimportantoperator ${}_aD_t^{\alpha}$ as anoversimplification of the differential and integral operators, which is defined as follows

$$a^{D_t^\alpha} = \begin{cases} \frac{d^\alpha}{dt^\alpha} & R(\alpha) > 0 \\ 1 & R(\alpha) = 0 \\ \int_a^t (dt)^{-\alpha} & R(\alpha) < 0 \end{cases} \quad (4.28)$$

Where

a–Lower limit of integration

t–Upper limit of integration

α–Order of fractional differentiation or integration Negative α - shows integration and positive α -shows differentiation[35],[36].

- Frequency and time-Domain Investigation of Fractional Order Systems

FOSarestraightsaddition ofconventional integer-order schemes. The sectional Order system is accepted up-on the FODE, and the FOTF of a particularadjustablescheme can be definite as

$$G(s) = \frac{b_1 s^{\gamma 1} + b_2 s^{\gamma 2} + b_3 s^{\gamma 3} \ldots \ldots \ldots b_m s^{\gamma m}}{a_1 s^{\eta 1} + a_2 s^{\eta 1} + a_3 s^{\eta 1} \ldots \ldots . \ldots a_n s^{\eta n}} \quad (4.29)$$

Anywhere $(a_n, b_m) \epsilon R^2 \& (\eta_n, \gamma_m) \in R^2$

- Frequency Domain Investigation of Fractional Order Systems
  We conclude fractional-order basic terms are in three categories: - fractional-order pure differential term, fractional-order derivative term, and fractional-order integral termIt can be understood that, when jω is used to adding for the variable *S* in the FOTFmodel the above, the frequency domain response $G_a(j\omega)$ can be easily calculated. Thus, Fractional Order Bode diagrams and Nyquist plots can be simply evaluated.
- Investigation Of Fractional-OrderTime Domain(IFOTD)Systems

Assessment ofFractional Ordertime-domain response is additionallydifficult. As shownin the succeedinganextraordinarymethod of a FOD equation.

$$A1^{D_t^{\eta 1}} y(t) + a2^{D_t^{\eta 2}} y(t) + \cdots + an - 1^{D_t^{\eta 1n-1}} y(t) + an^{D_t^{\eta 1n}} y(t) = u(t) \quad (4.30)$$

When $u\ (t)$characterized by aninfluencedpurpose& its sectional Order derivative. Receive that the y (t) has zerosprimarysituations. Beginning the Laplace, transformfunction determine the transfer function

$$G(s) = \frac{1}{a_1 s^{\eta 1} + a_2 s^{\eta 1} + a_3 s^{\eta 1} \dots \dots . \dots a_n s^{\eta n}} \tag{4.31}$$

The fractional-order linear time-invariant (LTI) system can also be represented by the following state-space form

$$0^{D_t^q} x(t) = Ax(t) + Bu(t) \tag{4.32}$$

$$y(t) = Cx(t)$$

Where $x \in R^n$, $u \in R^r$ and $y \in R^p$ are the state-input and output vectors and

$A \in R^{nxn}$, B∈ R^{nxr}, $C \in R^{pxn}$ and q= $[q_{1,} q_2, q_3 \dots \dots \dots\ q_n]^T$is the fractional-order system.

4.3.3. STABILITY ANALYSIS OF FRACTIONAL-ORDER SYSTEM

The stability is an exceptionallysignificant property of the non-linear systemthat can be examined in several domains'normalconception of BIBO. A fundamentalLTIsystem using impulseresponse $h\ (t)$towardbounded input bounded output stable if the essential and appropriate condition is fulfilled:-

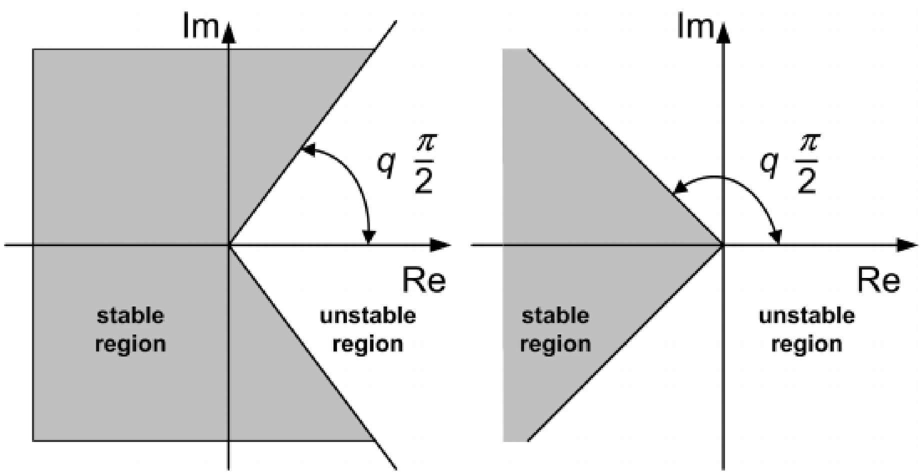
$$\int_0^\infty \| h(\tau) \|\ d\tau < \infty \tag{4.33}$$

Where the output of the system is defined by convolution

$$f(t) = h(t) * u(t) = \int_0^\infty h(\tau)\, u(t - \tau) d\tau \tag{4.34}$$

The additional very important domain is the frequency-domain. In terms offrequency domain for estimating the stability, transform the *s*-coordinate intothe complex plane Go(jω) and the transformation is understoodallowingthe transfer-function of the open-loop system Go(jω). Throughout the conversion, all roots of the characteristic equation of a polynomial are a map from *s*-coordinate into the critical-point (-1, j0) in the plane Go(jω).However, we cannot directly use an algebraic tool, for example,Routh-Hurwitz criteria for the fractional-order system, since we don’t have a distinctive equation[37]. An interesting point is stable of FOSmight root in the right- half of complex ω- plane, as presented in fig.4.6 Since the principal sheet of the Riemann surface, is defined –π

<arg(s) < π, by using the mapping $w = s^q$, the corresponding w domain is defined by –qπ < arg (w) < qπ, and the w plane region corresponding to the right half-plane of this sheet is defined by –qπ/2 <arg (w) < qπ/2



(a) 0<q<1 (b) 1<q<2

Figure 4.6:- stability region of the fractional-order system

The FOis stable or not if satisfied the following condition

The fractional transfer function of equation (4.29), $G(s) = \frac{Z(s)}{P(s)}$is stable if and only if the following condition is satisfied in w-plane.

$$|\arg(w)| > q\pi/2, \forall w \in C, P(w) = 0 \quad (4.35)$$

Where $0 < q < 2$and $w = s^q$. When $w = 0$ is a particular root of $p(s)$, for $q = 1$, this is the traditional theorem of pole-place in the complex-plane, no-pole is in the right-plane. The fractional-order linear-time-invariant system state-space model (4.32) is stable if

$$|\arg(\text{eig}(A))| > \frac{q\pi}{2} \quad (4.36)$$

Where $0 < q < 1$ and eig(A) represents the eigenvalues of matrix A.

4.4.TYPES OF FRACTIONAL ORDER CONTROLLER

AFOPIDregulator is the extension of the classical PID-controller based on fractional-calculus. For several years, engineeringPID controllers have been presenting very commonly used in process industries. The significance of proportional-integral-derivative controls, incessant efforts isexistenceended to developrobustness. An automatic regulator method, FOC is oversimplification integer-order controllers would regulate the robust control achievement and also more accurate.

The FOC which stands the overview of conventional IOC would main to additional detailed & robust governor performance. However, it is practical; the fractional-order simulations involve the FOCtoward accomplishing the betterassessments. TheFOCis functional to rectilinear or non-rectilineardynamic forces to improved performances of a scheme. Generally, there are four maincategories of FOC[38].

- Fractional order PID-controller
- Tilted proportional and integer (TID) controller
- Crone controller
- Fractional lead-lag compensator

4.4.1. $PI^{\lambda}D^{\mu}$ CONTROLLER

The FOPID controller has two more adaptable parameters than the PID controller, and the order of the controller can be chosen randomly, so the FOPID controller owns more flexibility. In addition to that, the FOPID controller has great adaptability to the parameter variation of the control system. When the constraint of the control system change contained by a certain range, the system characteristics remain unchanged, so the FOPID controller has the characteristic of strong robustness.Control techniques including feedback control, optimal control, predictive control, neural network control, fuzzy logic control, and so on, have been establishedmeaningfully. However, the PID control technique widely used in several industrial applications such as process control, motor drives, flight control, etc. More than 90% of the control loops in the industry are PID based control loop This is mostly because of PID controller possesses robust performance to fulfill the internationaltransformation of industry process, simple construction to be easily understood by engineers, and open-mindedness to design and implement. Recently, there are increasing the performance of the proportional-integral-derivative controller by using the knowledge of fractional calculus. Proportional integral derivative controllers are linear and symmetric and they have complications in the occurrence of non-linearity. This problem can be solved by using a fractional-order PID (FOPID) controller. A$PI^{\lambda}D^{\mu}$regulatorthrough integrator of actual order λ& differentiator actual order $\mu$is a more common cause of the classical PID-controller which delivers more flexibility and robustness in tuning and control by an additionaltwo Degree of Freedom in the system.

These degrees of freedom are related to the orders of the integral and derivative parts which are extended to non-integer (fractional) values.The FOPIDcontrolsuggested by Podlubny is a simplification of atraditionalproportional-integral-derivative regulatorexpending the fractional multiplication. The TF is well-defined by Equation (4.38).

AnF-OPIDcontrol is characterized by five constraints, i.e. the Proportional gain (Kp) the integral gain (Ki), the $d$erivative gain (Kd), integral order (λ), and derivative order (μ)[39].The FOPID controller is the increase of the classical conventional PID controller based on fractional mathematics. Due to the dominant importance of PID controllersdevelop their features and robustness[40]:

Fractional-order controllers have received significant responsiveness in the last years both from the academic and industrial point of view. Podlubny suggested anoverall form of the IOPIDcontrol, is called $PI^{\lambda}D^{\mu}$control, where the values of λ and μ lie between 0&1. Related with the PID controller, the FOPID controller has more two adjustable parameters, which makes the parameter tuning more flexible, it is very essential significative for successful the control accuracy, therefore, the $FOPI^{\lambda}D^{\mu}$ a controller applied for managing aLevel process in interacting two-tank [41],[42].

The differential equation of fractional order $PI^{\lambda}D^{\mu}$ the controller is shown as equation (3.37)

$$u(t) = k_p e(t) + k_i Dt^{-\lambda} e(t) + K_d Dt^{\mu} e(t) \tag{4.37}$$

Relating Laplace transforms to this equation with zero initial conditions, the TF of the controller can be expressed by:

$$G_{fc}(s) = \frac{U(s)}{E(s)} = K_p + K_i s^{-\lambda} + K_d s^{\mu} \;\; ,(\lambda > 0, \mu > 0) \tag{4.38}$$

Taking λ= 1 and μ = 1 it is the conventional PID controller, if μ = 0 and λ = 1 it is the conventional PI controller, if λ= 0 and μ = 1 it is the unadventurous PD controller and if both μ&λ = 0, it $PI^{\lambda}D^{\mu}$can be seen that the adjustable range of the fractional $PI^{\lambda}D^{\mu}$ the controller is wider than the conventional PID controller, therefore, the guideline performance of the FOC is greater than the IOPID controls.

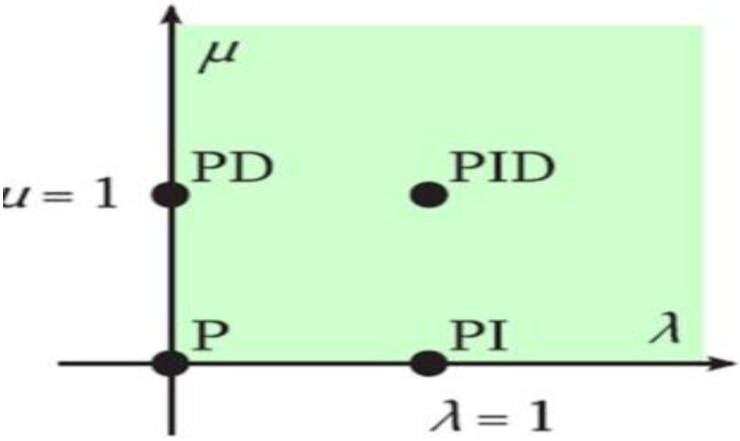


Figure 4.7:- Graphs of PID controller

4.4.2. TILTED PROPORTIONAL AND INTEGER (TID) CONTROLLER

Tilted-Proportional and Integral (TID) Controller is to provideanenhanced feed-back loop controller gains of the classical Proportional integral controller. In TID structure the proportional compensating unit is exchanged with a compensator having a transfer function characterized by ${}^{1}/_{s^{1/n}}$or${}^{1}/_{s^{-1/n}}$. This compensator is herein referred to by way of a “Tilt” compensator, as it delivers a feedback gain as a function of frequency which is tilted or formed for the gain/frequency of a conservative or positional recompense unit. The perfect controller is herein referred to as a Tilt-Integral Derivative (TID) controller. It contains three components tunable feedback loop control system which contains a proportional integral derivative controller. The individualtransformation from the conventional controller is that the proportional recompensing part of the system is replaced with a more suitable compensator which is having a transfer function. The term ‘Tilt’ suggests that it can provide a feed-back improvement as a frequency purposethat is shaped or tilted to improvement frequency of predictablereparationentity. A transfer function of TID can be written:-

$$G(s) = K_t(1/s)^{\frac{1}{n}} + \frac{k_i}{s} + sk_d \tag{4.39}$$

Where n is a non-zero real number, the above transfer function is shown in the figure below as:

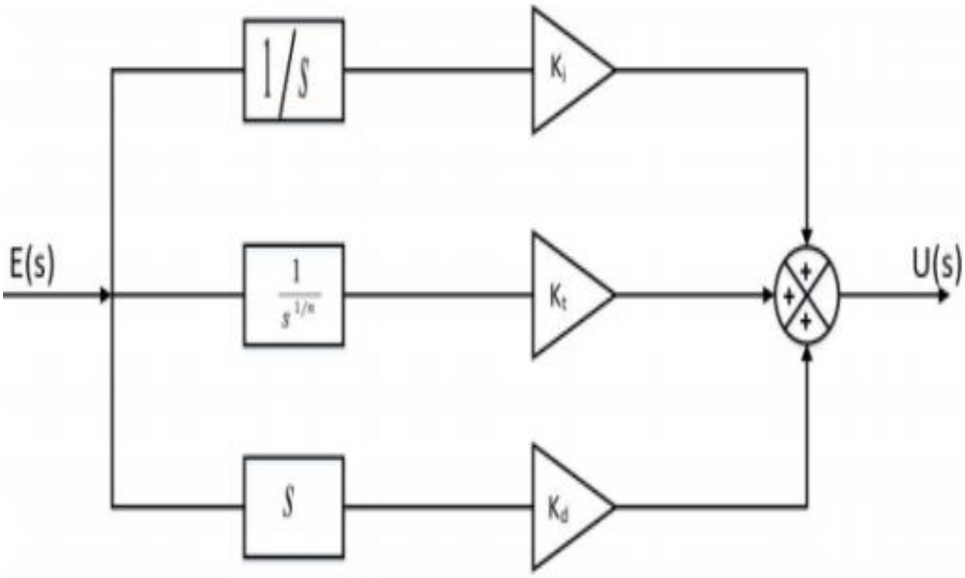


Figure 4.8:- Tilt-integral derivative controller

The effects of TID controller can be summarized as:

- ✓ Simple Tuning
- ✓ Feedback control is improved
- ✓ Disturbance rejection ratio is improved
- ✓ system parameter variation has less influence on closed-loop response

The main parameter for tuning in the TID controller is the tunable coefficient “n” and the other parameters (Ki and Kd) are the same as we used in the conventional PID controller. Thus, the most optimum value of “n” is obtained by this technique for the lower percent of under/overshoot, settling time, and steady-state error[43].

4.4.3. CRONE CONTROLLER

Crone Controller is a French acronym for fractional-order robust control. By the use of these controllers, it is conceivable to confirm almost constant closed-loop features and ensure a small variation of the CL system stability degree in the meanness of the plant perturbation and improbability in model parameters.A crone controller exists in three generations. Once the phase and the margin are constant, the first generation is used. Though, when the phase is only constant nearby the unity gain frequency, the second generation is used. The structure’sinstruction for the first two generations is non-integer changing between 0 and 1 however, for the third generation, the complex order originates. It has a frequency domain design methodology engaging fractional differentiation. It is possible to control plants, unstable, time-varying, and non-linear plants with crone controller.one of the applications of crone controllers has been in the domain of flexible transmission, car suspension control, and hydraulic actuator.

The Crone controllers are designed using this method.To move from the rational or fractional representation of the rational form transfer function,Oustaloup method is used.

Fractional form: $$C(s) = \frac{K_0}{S}\left(\frac{1+\frac{1}{\omega_n}}{1+\frac{1}{\omega_b}}\right)^n \quad (4.40)$$

Rational form:

$$I_N^n(s) = \frac{K_0}{S}\prod_{i=1}^{N}\left[\frac{1+\frac{s}{\omega_i}}{1+\frac{s}{\omega_u}}\right] \quad (4.41)$$

Where$N$characterizes the estimate of cells (commonlyessentials one cell which is collected of a pole and zero with a minimum of four cells to reconstruct the fractional system), k_0 is the gain responsible to set the unit gain frequencyω_u, ω_i and $\omega_u$being respectively the poles and the zeros of the rational transfer function. Regarding the combination of the crone controller, the identical method is used for the three generations. The posteriormethod is organized where the TF of the controller is deduced after defining the behavior of the open-loop system. Hence, the transfer function of the open-loop is the following:

$$\beta(s) = \beta_0\left(\frac{1+s/\omega_b}{s/\omega_b}\right)^{n_b}\left(\frac{1+s/\omega_h}{1+s/\omega_b}\right)^{n}\left(1+\frac{s}{\omega_h}\right)^{-n_h} \quad (4.42)$$

- The first step contains defining the necessary specifications for the synthesis of the nominal plant transfer function. The frequency method is also used in this part as the standardization of the sensitivity functions is easier.
- The second step consists ofreselecting the frequency closed-loop conditions into open-loop frequency specifications for the nominal plant. These new conditions take into explanation the plant behavior at:

  - ✓ Low-frequencies requirerespectable accuracy in the steady-state;
  - ✓ intermediate frequencies, specifically around the frequency ω_u, to get the stability degree robustness;
  - ✓ High frequencies to have good input plant sensitivity. Hence, ω_b and $\omega_h$characterize the low as well as high intermediate frequencies, n is the

fractional-order fluctuatingamong 1 and 2, n_b andn_h is asymptotic order performances for small and high frequency and β_0 is a constant that guarantees a unit gain at the frequency ω_u[44].

4.4.4. FRACTIONAL LEAD-LAG COMPENSATOR

Fractional Lead-Lag compensators are commonly used to stabilize slightly stable systems. Lag compensators are mostly used to decrease the amplitude of the high-frequency loop gain of a system. The use of fractional order elements in this lag-lead compensator gives greater flexibility to fashionable, to form the loop frequency responses since the order of the clean can take any real value as a replacement for of only integer values the TF of a generic fractional-order lead-lag compensator[45].

The fractional lead-lag controllers are often desired in the strategy of control systems. Although the fractional-order lead-lag controllers are comparable to other terms of the control structure, they are different in terms of placement of the zerosand poles.

$$C(s) = K_c\left(\frac{s+\frac{1}{\lambda}}{s+\frac{1}{x\lambda}}\right)^{\alpha} = K_c x^{\alpha}\left(\frac{\lambda s+1}{x\lambda s+1}\right)^{\alpha}, 0 < x < 1 \tag{4.43}$$

Where$0 \leq \alpha \leq 2$, the transmittance corresponds to the frequency bounded fractional derivative/integrator which is at the very origin of the crone control.

4.5.MATLAB TOOLBOXES FOR ANALYZING FRACTIONAL ORDER SYSTEMS

In recent years, as fractional calculus develops more and more broadly used across different academic disciplines, there are growing difficulties for the numerical tools for the calculation of fractional integration/differentiation or the simulation of fractional-order systems.

A) @fotf

@fotf (fractional-order transfer- function) is a control tool-box for fractional-order systems developed by **Xue**. Most of theFOTFexclusive are extended from the MATLABbuiltin-functions. @fotf tool-box uses the overload programming method to enable the related methods of the Matlab built-in functions to covenantthrough fractional-order models. TheFOTF objects created from it can be collaboratingutilizing generated from the MATLAB transfer-function course. Conversely, the over-loading of related functions such as impulse (), step (), etc. @fotf toolbox supports the time delay in the transfer function. It doesn'tstraight support the TFmatrix; hence, multi-input multi-output systems can't be simulated. Nevertheless, meanwhile, it deliversSimulink-block -encapsulation of the complicated function @fotf (), MIMOcorrelationrecognizedviaphysically adding roundinterfacesSimulink – block – diagrams. A draw-back of the fraction order transfer function is that the sample time has a moderately big influence on accurateness[46].

B) Ninteger

Ninteger, a non-integer MATLAB control toolbox, is a toolbox proposed to help with emergingFOC and evaluating their performance. Fractional order Simulink block diagram are involving the following function,such as nid,nipid,fotf, etc. [47]

C) CRONE

The crone-Tool-box, established meanwhile19th by a*crone*group, is a Matlab and Simulink Toolboxdevoted to presentations of non-integer derivatives in industrialskill[48]. It developedbeginning the inventive script version of theremaining programming. A respectablearticle of the crone toolbox arecertain of the approaches is appreciated for multi-input multi-outputfractional transfer functions. Numerousextratoolboxesare encouraged by crone, e.gninteger and FOMCON.thedraw-back of the crone toolboxare intervalsuspension can't be combinedinterested in the engenderedFOTF. Cronea MATLABtool-box considerably more dominant than just simulating FOS.

D) FOMCON

The FOMCON (Fractional- Order-Modeling, and Control) *toolbox* is established by AlekseiTepljakov [16]. Its kernel workssystems in @fotf, ninteger, and Crone.Summarizesthe particularmain function of individuals three–fractional-order (FOMCON) toolboxes, therelativeFOMCONwithin three-toolboxes is obtainable in fig4.9particulardistinguisheddeviations of the unique@fotf are

- Newfotf() practices string parser to allowoperators to input-transfer- function as a string

- tf2ss() are loaded and foss() are additional, whichvarieties the adaptation between fotf(0entity and foss()article. The croneMATLAB-toolbox is consistentlyintelligent to do atask;contrariwise, an index is encoded in Matlab P-code format. In this project,the FOMCON Matlab toolbox is used because it contains the main functionalities of fotf, ninteger, and Crone toolboxes[49].

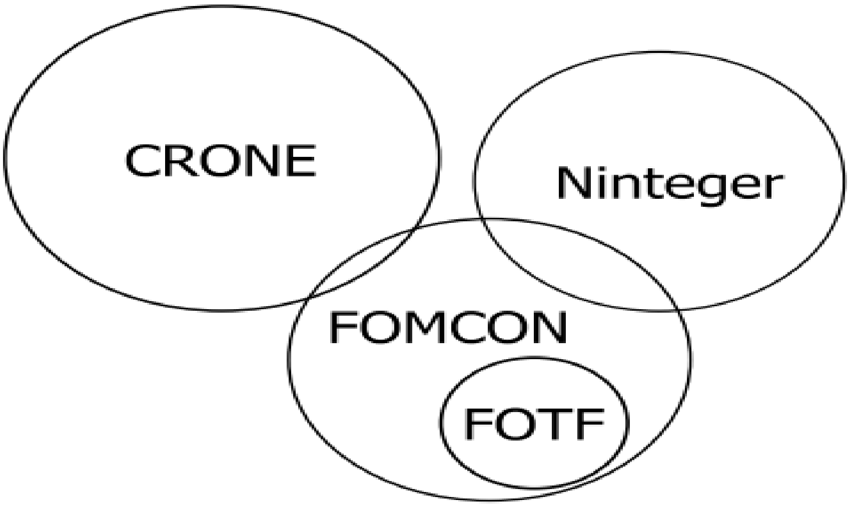


Figure 4.9:- FOMCON relation to other fractional-order MATLAB toolboxes

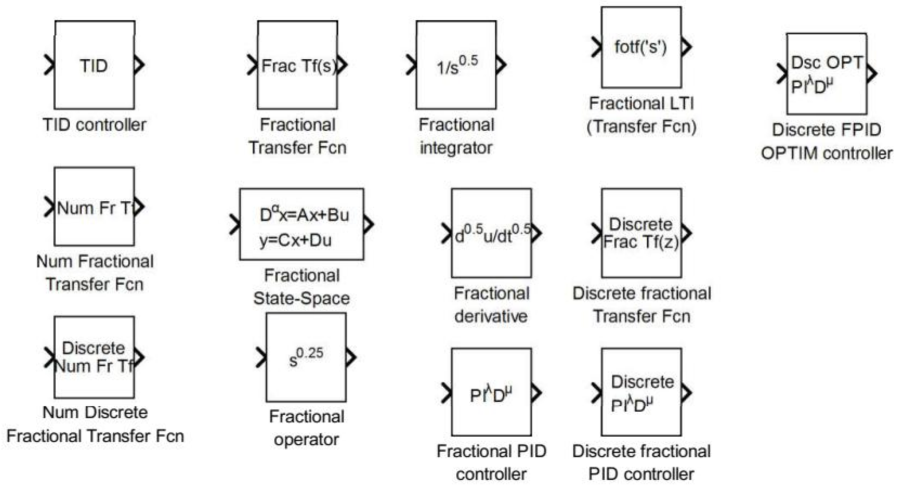


Figure 4.10:- FOMCON Simulink library

4.6.INTEGER ORDER VS FRACTIONAL ORDER PID CONTROLLER

Even though the integer-order proportional integral derivative controlmechanism well concerning the consummation of the design description. Nevertheless,through the modification of constraintassessment& aging, the yieldis likely to denigrate and change. A conventional proportional-integral-derivative Controller doesn'trequire the competencetowardaccomplishing within a changingconstraint. Bythis yield inclinestoward

*d*iffering after its insignificant importance FOPID theconstraint of a traditionalPID regulator can be overawed by the impression of a additional modification constraint in order ofintegrator &differentiator .

Surely, moreover situation proportional gain Kp, integral gain Ki, and derivative, gainK_d have two additionalconstraintsλand μ, whateverdevelops theopportunity of control scheme design. The FOCoversimplifies theIOPIDdesign and risesfrom point to surface. Thedevelopmentimprovesadditionalsuppleness to CSDand we can control our real-world method more effortlessly. Within the benefit of FOPID controls, it is conceivable to design a controller to guarantee thatthe CLscheme is robust to improvementvariance& step replyshows astage bode scheme of the structure is horizontalachievement crossover-frequency as a consequencetechniquedevelopsadded robustness to improvementmodification. $PI^{\lambda}D^{\mu}$Controllers are a smaller amount of sensitivity to constraint variation. Allfurthermost of the practicalprogressions can be competentlydemonstrated within the help of FOdifferenceequivalencesmoderately than integer ones[40].Consequently,FOC is appropriatelyaimed at these sectional-order models to represent better control.

4.6.1. ADVANTAGE OF FRACTIONAL AND INTEGER ORDERPID PID CONTROLLER

Throughrelatedaninteger-order controller, &fractional-order is theoretical to suggest the followingrecompenses[50]

- ✓ If the constraint of measured variabledeviations, sectional-order PID controller are less sensitive rather than traditional PID controller
- ✓ FOPID controller hasestablishedsignificantconsideration in the preceding years togetherafterthe theoretical&manufacturingthemes of view concerning the ordinary PID controllers as five parameters to choose.

Generally,the use of fractional order models and controllers is expected to lead a momentous overall improvement of industrial control loop quality thus providing an increase in control system precision, performance, and energy efficiency. When FOPID controllers implementation for real-time used estimate method because two additionalparameters of the FOPID controller to improve the flexibility of the system.

The extension of derivation order from integer to fractional numbers provides a more flexible tuning strategy and an easier achieving of control requirements concerning classical controllers.The potential use of FOPID controllers lies in the industrial domain, where the majority of control loops are based on traditional PID controllers, including those dedicated to liquid level control. The application of FOPID controllers to the problem of liquid level control is justified since such controllers offer more tuning flexibilityparameter and allow taking into account more robustness criteria and also unmolded dynamics of real industrial systems, small advantages in performance arising from using fractional-ordercontrollers (FOPID)in place of integer order of Proportional Integral Derivative(IOPID) will lead to an overall enhancement[51].The most common design method for fractional-ordercontrollers is based on frequency domain analysis[52].

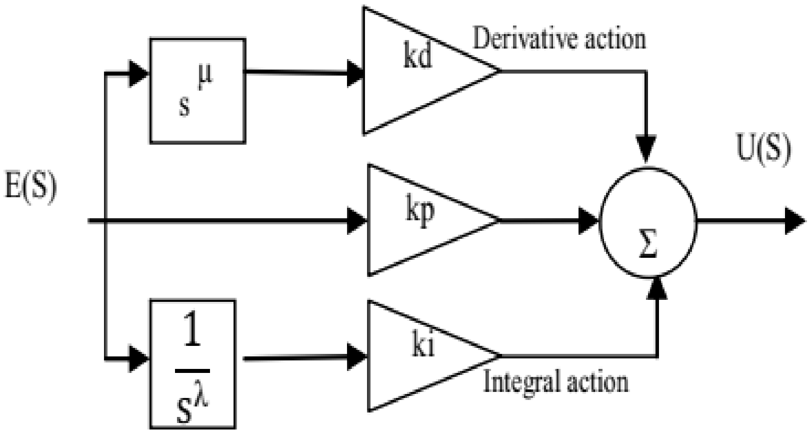


Figure 4.11:- Block diagram of FOPID controller

It can be predicted that the $PI^{\lambda}D^{\mu}$the controller may upgrade the efficiency of the system. Additional feature lies in the fact that $PI^{\lambda}D^{\mu}$controllers are low sensitive to the constraintsvariations of the controlled system also FOPID providesfurther flexibility in the controller compared with theconventional PID controller[53].The feedback control loop of a fractional-order system with a fractional controller is similar to the integer-order the feedback control loop is shown in fig 4.11

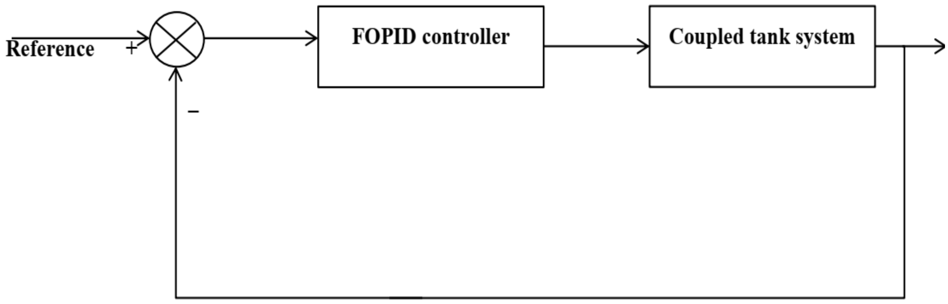


Figure 4.12:-Block diagram of the coupled tank system with FOPID controller

4.7. INTRODUCTION TO LINEAR QUADRATIC REGULATOR

A principle of optimal techniques is workingdynamics scheme at minimum cost functionof a methodwhose subtleties arespecifiedthrough the convenience of rectilineardifferenceequivalences&cost functionthroughthe rectilinear quadratic problematic. The situation of a regulator that manageswhichever anapparatus or method is found by anexactprocess that reduces a cost function containsconsideringfeatures. A truthfulprocedure is an impartialpurpose that essentialto be reduced in the strategypractice. Fashionable, two methods controls are planned&clarified in detail which is theFOPID&LQRcontroller. Furthermore, the succeedingdesigncondition has completedevaluating the performance of LQR control schemes those performances are evaluated by transient response. To stunnedselected problems faced by the PID regulator, and develop another controller method are linear-quadratic-regulator. The performance measure of the linear-quadratic-function collected of the state space form & control signal.

Linear quadratic regulator [54],[55]strategysystem is well recognized in modern optimal control theory and has been commonly used in many applications. Thus, the LQR theory has establishedsignificant attention since the 1950s. The LQRmethod seeks to find the optimal controller that reduces a given cost-function. Two parameters of cost function matrices are Q and R, whichconsider the state-vector and the system-input respectively. These allowance matrices regulate the consequences on the excursion of state-variables and control signals. One practical method is to Q and R to be a diagonal matrix. The value of the parameter in Q and R is related to thecontrolled plant. The LQR is a control design that delivers the best conceivableassessmentsregardingparticularportion of enactment. Linear quadratic regulator isovercomethestrategy of state − feedback controller K such that the opportunity of cost-function J is reduced. In this thesis work, thefunction of state feedback control is used to control strength, increase the amplitude of the response with a desired point and gives a better response of system.The commonly LQR used for analysis the state feedback gain from closed loop system. Linear Quadratic Regulator is, in fact, a feedback controller for control the level of the process that provides the solution in the cases wherever the system dynamics are described by a set of linear-differential-equations and the cost is designated by a linear-quadratic quadratic function. The LQR procedure is an automated way of outcome a raised feedback controller.

Control engineers can also prefer alternative ways for designing theappropriate controller like a full-state feedback controller or known as pole-placement,LQRhas proposed to regulator the Level in a coupled-tank arranged in the form of the interacting system. Where the parameters of controller governing the system are establishedviaexpendinganalytical modeling to reduce the cost function within weighting structurescarried by the engineer in Layman's terms.The LQR modeling is the core, just a mechanizedtechnique of finding a suitable state feed-back controller description of the LQR system [56].

4.7.1. LQR CONTROLLER DESIGN

The LQR is an optimal technique in modern control theory that uses a state-space method to investigate a system. Using state-space approaches it is comparatively simple to effortutilizing a multi-output system [57]. The LQ controller designed is categorized as optimal control techniques. The purpose of the LQR is to realize a system with practical components that will provide the desired set point. The preferred point can be readily stated in terms of time-domain performance indices. For example, the maximum overshoot and rise time for a step input are respected time-domain indices, In the case of steady-state and transient performance, the performance indices are normally specified in the time -domain[58], [59],[60]. The schematic of this type of control systems are existing in Fig. 4.13.

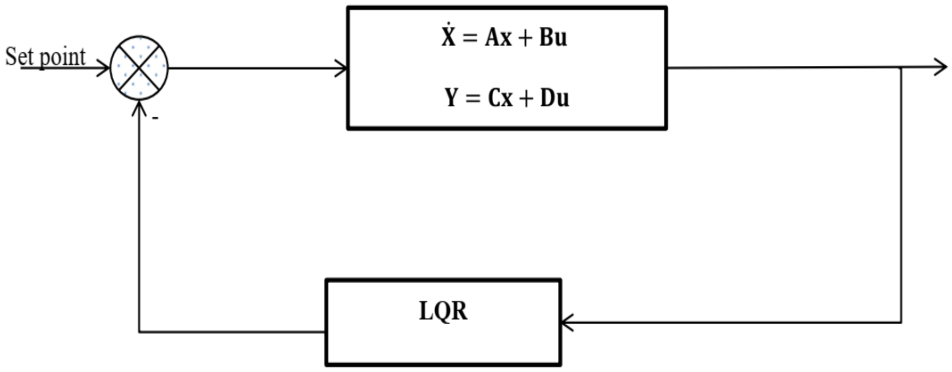


Figure 4.13:- Block diagram of the coupled − tank using LQR optimal controller

4.7.2. PRINCIPLES OFLQPERFORMANCE ANALYSIS

Optimal Techniques say (FSB) achievement matrix "K" is considered&also invented that completely n state variable are computable ones and obtainable for the regulator[61], [62] .the state and output equivalence can be specifiedthe fig. (4.12)

$$\dot{x} = Ax + BU, y = Cx + DU \tag{4.44}$$

Where x variable is a column vector of length n, u is the control input − vector of length r, A is an $(n \times n)$ square state matrix; B is an $(n \times r)$ input matrix;y is a column output vector, C is an $(m \times n)$ the output matrix and lastly, D is an $(m \times r)$ throughresponse matrix.

The LQR is used to control complex-systems that requirea high-performance .a system described by linear differential equations can be shown in the steady-state form given in equation (4.1).

For a continuous − time described by

$$\left.\begin{array}{l} \dot{x} = Ax + BU \\ y = Cx + DU \end{array}\right\} \tag{4.45}$$

By a cost function well-defined as

$$J = \int_0^{\infty} (X^T Q x + U^T R U) dt \tag{4.46}$$

Where Q and R are the weight matrices, $Q -$ is required to be a positive-definite or positive-semi-definite symmetry matrix; $R -$ is required to be a positive definite symmetry matrix. One practical method is to Q and R to be a diagonal matrix. The value matrix Q and R are related to its involvement in the cost function J. The feedback control law that reduces the value of the cost. Designing a linear quadratic regulator controller consist of the following steps

Step 1: Q and R matrix, minimizing Jmust be chosen.

Step 3: Optimum feedback gain matrix K iscalculated using (4.48).

Step 4: System response is checked. If the system response is not met with the required specifications, repeat all steps[63].

$$U = -Kx \tag{4.47}$$

K − is given by

$$K = R^{-1}B^{T}P \tag{4.48}$$

P can be initiatethroughresolving the nonstop time(ARE)[64]

$$A^{T}P + PA - PBR^{-1}B^{T}P + Q = 0 \tag{4.49}$$

4.7.3. ALGEBRAIC APPROACH FOR A SECOND-ORDER SYSTEM

Contemplate a second-order LTI schemecharacterized in well-regulated canonical form,

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ A_{21} & A_{22}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ B_{21}\end{bmatrix}u \tag{4.50}$$

Scheming (FSB)regulatorthrough linear quadratic regulator (LQR) needs the minimization of a cost function specified in eqn(4.45) which seats the weight notsingle on mechanism input but likewise on the state of the structure. Hence the (SFB)law is specified by equation (4.46).One of the importantstructures LQR- are the Q −should be asymmetric positive semi-definite matrix and Rmust be a positive-definite-matrix. So,Q and R are weighting matrices,are chosen as

$$Q = \begin{bmatrix}q_{11} & 0\\ 0 & q_{22}\end{bmatrix}, R = r, p = \begin{bmatrix}p_{11} & p_{12}\\ p_{21} & p_{22}\end{bmatrix} \tag{4.51}$$

In an optimal method, the (SFG) matrix can be calculated

$$K = R^{-1}B^{T}P = \frac{1}{r}\begin{bmatrix}0 & B_{21}\end{bmatrix}\begin{bmatrix}p_{11} & p_{12}\\ p_{21} & p_{22}\end{bmatrix} \tag{4.52}$$

$$K = \frac{B_{21}}{r}\begin{bmatrix}p_{21} & p_{22}\end{bmatrix} \tag{4.53}$$

The element of p matrix analysis through (ARE) (4.28).

$$\begin{bmatrix} 0 & A_{21} \\ 1 & A_{22} \end{bmatrix}\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} + \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}\begin{bmatrix} 0 & A_{21} \\ 1 & A_{22} \end{bmatrix} - \frac{1}{r}\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}\begin{bmatrix} 0 \\ B_{21} \end{bmatrix}\begin{bmatrix} 0 & B_{21} \end{bmatrix}\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} + \begin{bmatrix} q_{11} & 0 \\ 0 & q_{22} \end{bmatrix} = 0 \quad (4.54)$$

$$\begin{bmatrix} P_{21}A_{21} & P_{22}A_{21} \\ P_{11} + P_{21}A_{22} & P_{12} + P_{22}A_{22} \end{bmatrix} + \begin{bmatrix} P_{21}A_{21} & P_{11} + P_{21}A_{22} \\ P_{22}A_{21} & P_{21} + P_{22}A_{22} \end{bmatrix} - \frac{1}{r}\begin{bmatrix} P_{12}P_{21}B_{21}^2 & P_{12}P_{22}B_{21}^2 \\ P_{22}P_{21}B_{21}^2 & P_{22}^2B_{21}^2 \end{bmatrix} + \begin{bmatrix} q_{11} & 0 \\ 0 & q_{22} \end{bmatrix} = 0 \quad (4.55)$$

$$\begin{bmatrix} 2P_{21}A_{21} - \frac{P_{12}P_{21}B_{21}^2}{r} + q_{11} & P_{22}A_{21} + P_{11} + P_{21}A_{22} - \frac{P_{12}P_{22}B_{21}^2}{r} \\ P_{11} + P_{21}A_{22} + P_{22}A_{21} - \frac{P_{22}P_{21}B_{21}^2}{r} & 2(P_{12} + P_{22}A_{22}) - \frac{P_{22}^2B_{21}^2}{r} + q_{22} \end{bmatrix} = 0 \quad (4.56)$$

4.7.4. THE CLOSED LOOP SYSTEM

The CL state equation of the system toinscribed as

$$\dot{x} = (A - BK)x(t) = (A - BR^{-1}B^{T}P)x(t) \quad (4.57)$$

Eigen-values of a CL methodessentialhave negatives real-parts of the closed-loop method stable. According to a direct-substitution technique fordesign pole-placement control.

$$|SI - A + BK| = 0 \quad (4.58)$$

Replacement ofequivalent method matrix A&Binput the (SFC) gain matrix-K in a characteristic

$$S^2 + \left(\frac{P_{22}B_{21}^2}{r} - A_{22}\right)S + \left(\frac{P_{12}B_{21}^2}{r} - A_{21}\right) = 0 \quad (4.59)$$

The general form of second-order characterized equation forms

$$S^2 + 2\xi\omega_n s + \omega_n^2 = 0 \quad (4.60)$$

Comparing Equations (4.59) and (4.60), the expressions for p_{21}andp_{22}can be obtained as given below

$$\left.\begin{aligned} p_{12} &= \frac{r(\omega_{n+}^2 + A_{21})}{B_{21}^2} \\ p_{22} &= \frac{r(2\xi\omega_n + A_{22})}{B_{21}^2} \end{aligned}\right\} \quad (4.61)$$

Or In general, for a second-order system, when Q is a diagonal matrix and R is a scalar quantity, the elements of the Riccati matrix P can be calculated:

4.7.5. DESIGN PROCEDURE

The numericaldesign of a plantSymbolizesorganized in a state-space differential form.

- Identify the essential damping-ratio (ξ) &accepted frequency (ω_n) a pla.
- Achieve the SF) matrix K in terms of the transformation-matrix P,
- Decide the actual characteristic equivalence of the plant.
- Additional transformation matrix (P) is(ARE) &achieve the terminologies.
- Adjust the value of R & Q matrix.
- Determinethe gain matrixof the optimal techniques.
- Findoutput the system confirms that the strategyhappens(sp) in terms of transient response.

4.7.6. DYNAMIC BEHAVIOR OF THE PLANT

To understand and control complex systems, we must obtain their quantitative mathematical models by analyzing the relationships between the variables of the system and employing Laplace transform or other mathematical tools [17]. A set of linear – differential-equation remained formulated to obtain a model transfer function that described the physical system of the plant.

4.7.7. PERFORMANCE MEASURES CRITERIA

- To test our model, we choose to use performance measures. These measures will be used in analyzing the performances of FOC and LQR.

Transient-response: - one of the furthermostessential characteristics of control is the reply of a system as a function of time .it can be defined in terms of two issues.

The speed ofthe response as represented by the rise-time (Tr) and The closeness of the response to the desired response as represented by the overshoot (Os) and settling-time (Ts)[65],[66]

4.8.LINEAR QUADRATIC REGULATOR DESIGN FOR COUPLED Tank

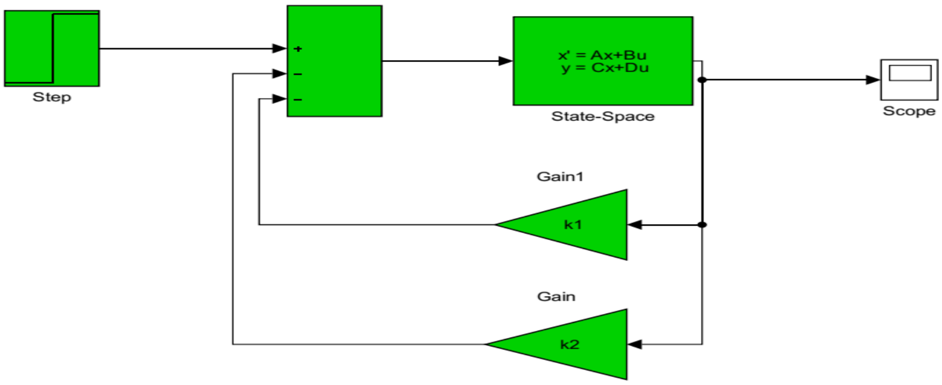


Figure 4.14:- LQR controller design for the interacting system

It is important to realize the optimal control approach to feedback design is very different from classical control.Linear quadratic regulator,it is characterized by a reference input approach to zeros.

4.9.DESIGN LINEAR QUADRATIC REGULATOR WITH INTEGRAL ACTION FOR CONTROLS COUPLED TANK SYSTEM

From a linear scheme in the form of State modeling are:-

$$\left.\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t)\\ y(t) &= Cx(t) + Du(t)\end{aligned}\right\} \quad (4.69)$$

The control strategy using state feedback has been applied to allocate the pole of the closed loop system (if the system is a completely controllable state) I any position, chosen to meet design specification. An advantage of LQR method when compared to the allocation method is that the first one provides a systematic mode of calculation for state feedback control gain matrix.

The quadratic optimal regulator aims is to find matrix k for the optimal control vector gain by so that the cost function in is minimized equation (4.47).

Determining the parameter in the above equation (4.44) must be design the LQR controllers. Analyzing a coupled tank system model, it can be seen verified that the output of a process level dynamics behavior of the coupled tank shown figure 3.2. Therefore it is necessary to place an integrator into controller in order to eliminate the static error between tracking reference and controlled variable.in other word; the LQR control must have integral action. Block diagram of the LQR with integral action shown in figure and named specifically as LQI.

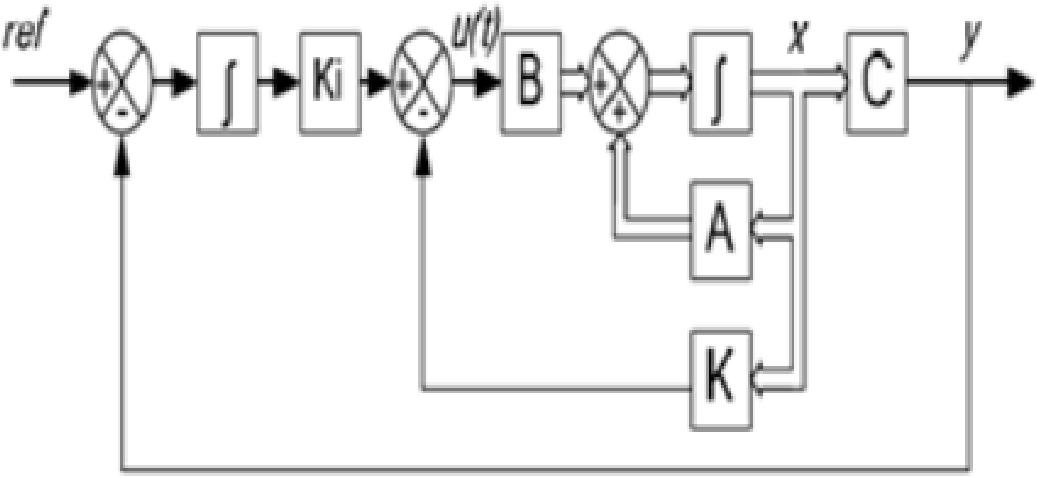


Figure 4.15:- block diagram of LQI of state feedback control systems

The LQR with integral action (LQI) assumes that the following expanded matrix arrangement equation (4.69):-

$$A_{new} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \quad (4.70)$$

$$B_{new} = \begin{bmatrix} B \\ 0 \end{bmatrix} \quad (4.71)$$

$$C_{new} = [c \quad 0] \quad (4.72)$$

$$D = 0$$

Whose gain is given by:-

$$\mathrm{K} = [\mathrm{k}_1 \mathrm{k}_2\ \mathrm{k}_3\ \ldots \ldots . . \mathrm{k}_{n+1} \quad -\mathrm{k}_i] \quad (4.73)$$

$$\mathrm{k}_{n+1} - \text{state feedback gain}$$

$$\mathrm{k}_i - \text{integral action error gain}$$

In this case, the model cost is defined by equation (4.46)

The gain k is obtained solving Riccati equation for p using equation (4.49)

After that obtaining p, k can be obtained equation (4.48). The physical implementation this control system, every state variable must be measurable and variable for feedback. The optimal gain of the state feedback vector is determined using $matlab$ command$[\mathrm{k}, \mathrm{e}, \mathrm{p}] = \mathrm{lqr}(\mathrm{A}_{\mathrm{new}}, \mathrm{B}_{\mathrm{new}}, \mathrm{q}, \mathrm{r})$. This command determine the optimal gain of feedback matrix k.

FRACTIONAL ORDER PID DESIGNED FOR COUPLED TANK SYSTEM

An open-loop transfer-function of interacting coupled tank systems as derived in equation (4.86) is given.A simulated model for the fractional-order PID control system is as shown in figure 4.15.The difference of (DSP) &the (MV) goes to aFOC. FOPID controller combines PID controllers gain and in addition to two parameters those are lambda (λ)with integral gain and Miu (μ) with derivative gain

$$G(S) = \frac{R_2}{\tau_1\tau_2 S^2+(\tau_1+\tau_2+A_1R_2)S+1} \quad (4.74)$$

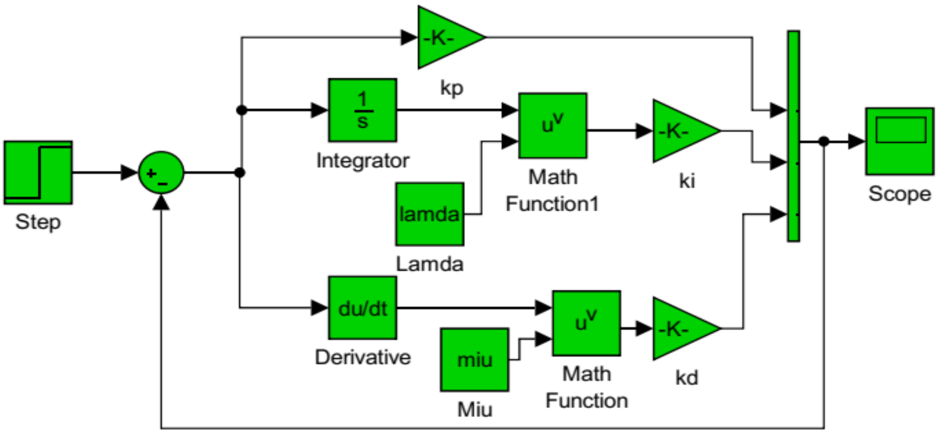


Figure 4.16: Interacting system control using FOPID controller

4.10. PARAMETERS OFLEVEL CONTROL SYSTEM

The achievement ofa process-level control system in interacting coupled tank for and compare the reply of the system with fractional order controller and linear quadratic regulator (LQR), the selected parameter of the level tank is specified in table 4.6.

No	Description	Parameter	value	Units
1	Area of tank1	A_1	250	cm^2
2	Area of tank2	A_2	250	cm^2
3	Resistance of tank1	R_1	0.01	sec./cm^2
4	Resistance of tank2	R_2	0.01	sec./cm^2
5	Height of tank1	H_1	30	Cm
6	Height of tank2	H_2	15	Cm

Table 4.6:- parameter of level control systems

Since the open-loop transfer function of the system is given above by equation (4.74) and with equation (4.10) and the unknown open-loop system parameters, ξ, and ω_n are obtained.Using the parameters given in table 4.6.

$$k = \frac{R_2}{\tau_1 * \tau_2}$$

$$\left.\begin{aligned} \omega_n &= \sqrt{\frac{1}{\tau_1 * \tau_2}} \\ \xi &= \frac{\tau_1 + \tau_2 + R_2 A_1}{2\omega_n} \end{aligned}\right\}$$

For interacting system coupled tank K, ξ *and* ω_n can be obtained by the above method. Therefore, the open-loop transfer function of the interacting coupled system without disturbance parameter can be calculated as follows.

$$G(S) = \frac{0.0016}{s^2 + 1.2s + 0.16} \tag{4.75}$$

The response of the open-loop processwithout the controller shown below

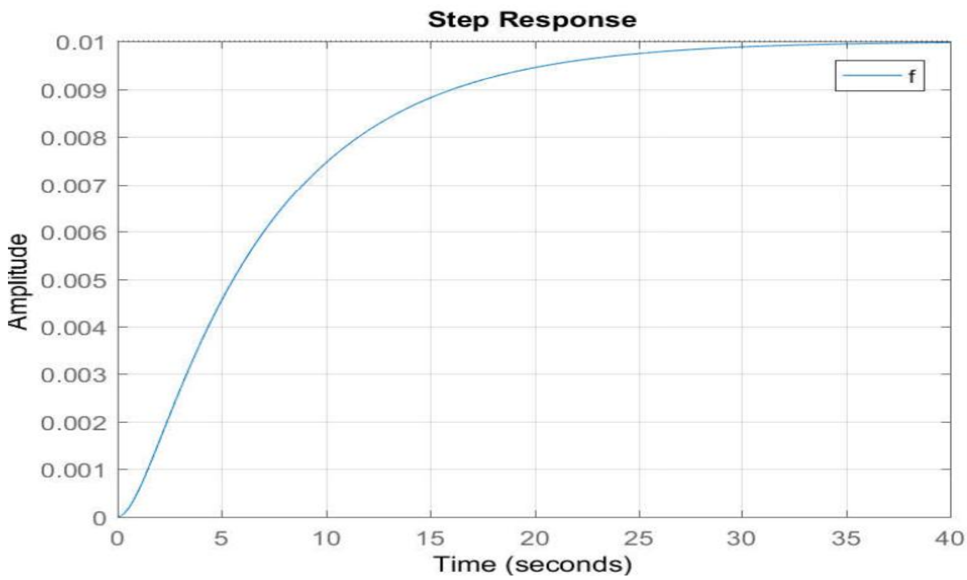


Figure 4.17:- Response of the open-loop system

When the disturbance added to the system, the disturbance transfer function can be

$$G_d(s) = \frac{H_2(s)}{D_{in}(s)} = \frac{0.01}{2.5s + 1} \tag{4.76}$$

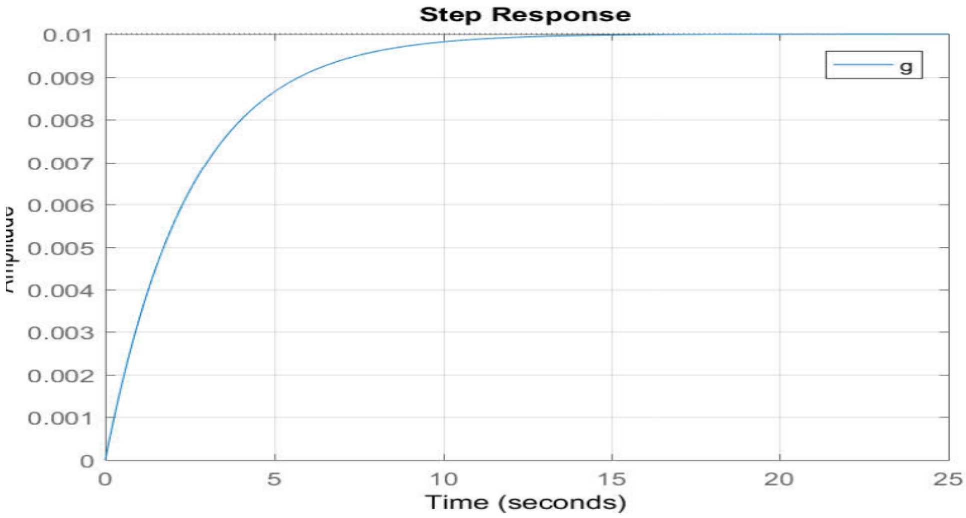


Figure 4.18:- Output response of the open-loop system when disturbance added

The selecting the desired parameters of a closed system and using the relation in equation (4.22), the gain of the classicalProportional –integral-derivative controller gain isconsidered which produce particularpole placement at specific-damping and frequency, providing relative dominance (α) in selected iteratively by examiningthe accurateness of system response. Here specificconsiderations of closedloop schemepreferred are $\xi^{cl} = 0.95$, $\omega_n^{cl} = 2\frac{rad}{sec}$ selecting the appropriate value of the relative dominance by trial and error by checking the accuracy of system response, the closed-loop transfer function of fractional order Proportional Integral Derivative (FOPID) when the value of $\mu = \lambda = 0.5$.

$$G(s)_{cl} = \frac{4.5s + 11.136s^{0.5} + 7.6}{s^{2.5} + 1.2s^{1.5} + 4.5s + 12.136s^{0.5} + 7.6} \quad (4.77)$$

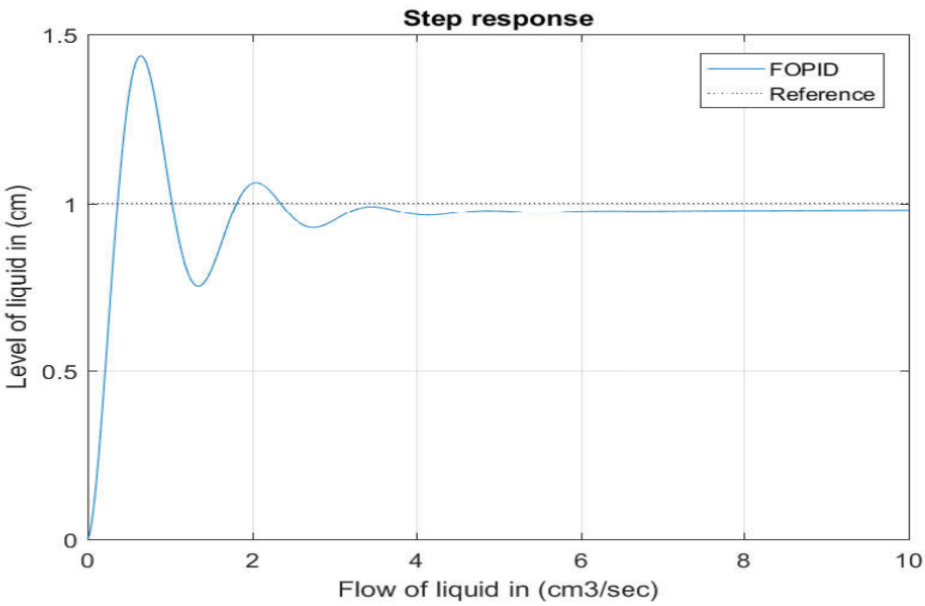


Figure 4.19:-The closed-loop of fractional-order response

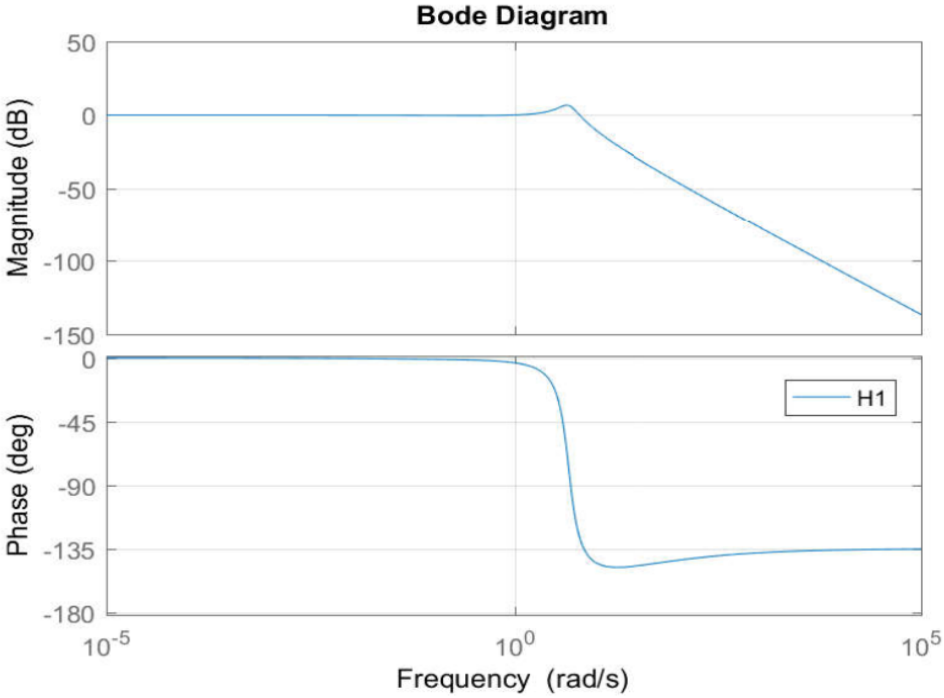


Figure 4.20:- Frequency-Response of fractional- order system G(s)

CHAPTER FIVE

SIMULATION RESULTS AND DISCUSSION

5.1.OPEN LOOP PERFORMANCE OF COUPLED TANK (INTERACTING SYSTEM)

The open-loop performance of the interacting is investigated using itsMATLAB/Simulink model,the parameters used to carry out thesimulation studies are provided in Table 4.6.

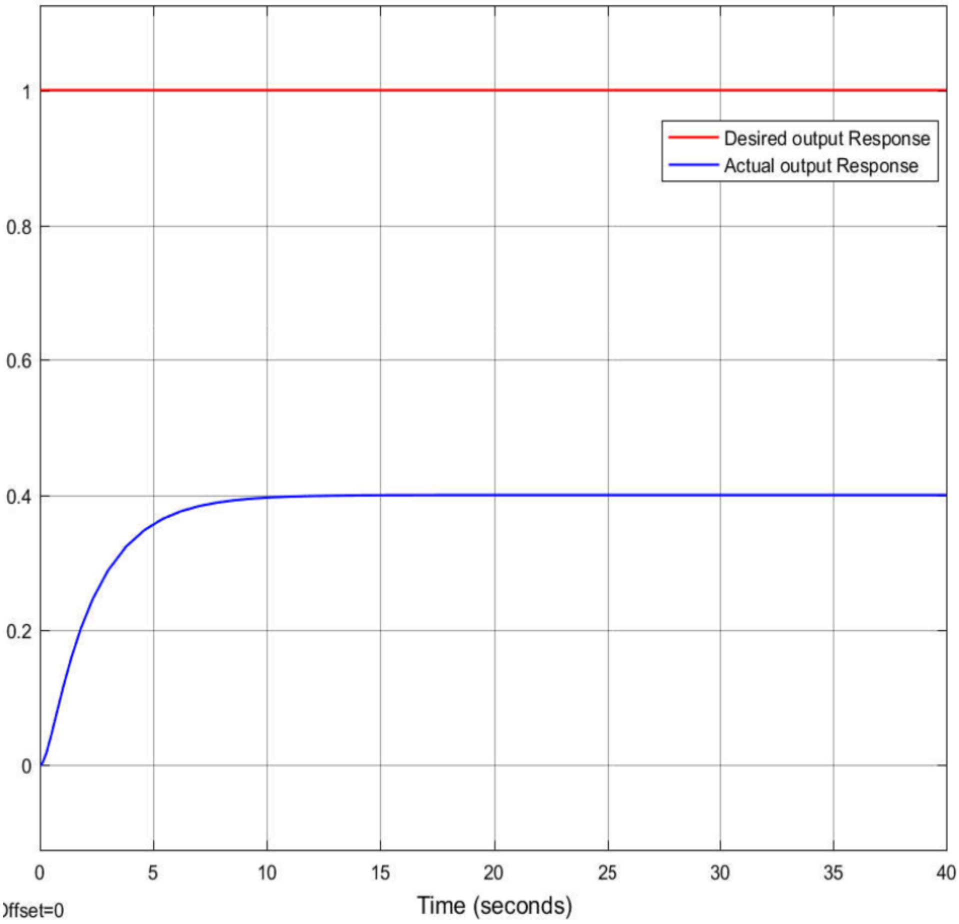


Figure 5.1:- Open-loop output response of interacting two-tank

The output response of the coupled tank (interacting system) obtained using the MATLAB/Simulink is shown in Fig. 5.1. It is observed that the rise time of output interacting system coupled tanksare 4.629seconds andthe settling time is 8.52 seconds.

5.2.PERFORMANCE OF INTERACTING COUPLED TANK WITH PID CONTROLLER

To demonstrate the performance of the interactingcoupled tank with a PID controller using in MATLAB/Simulink,the parameter of the level control is given in table 4.6. A

Simulink prototypical of the interacting coupled-tank with the PID controller is given in figure 5.2.

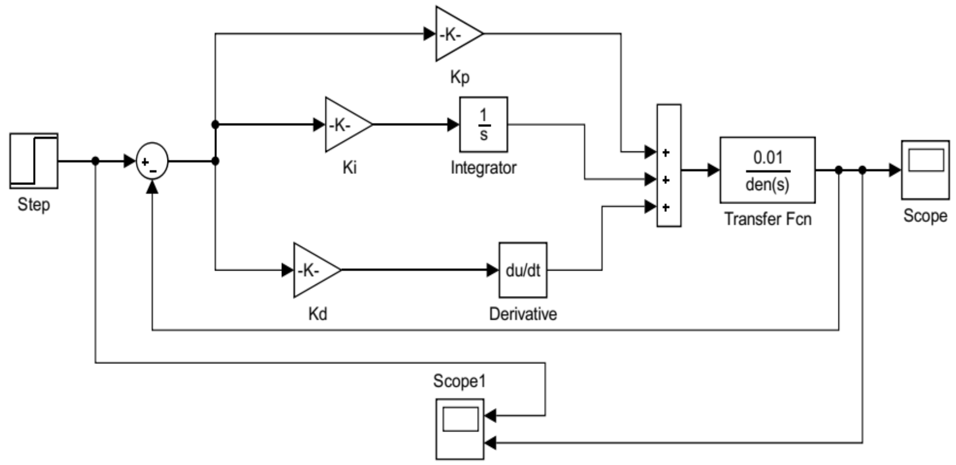


Figure 5.2:- MATLAB/Simulink model of the coupled tank within PID

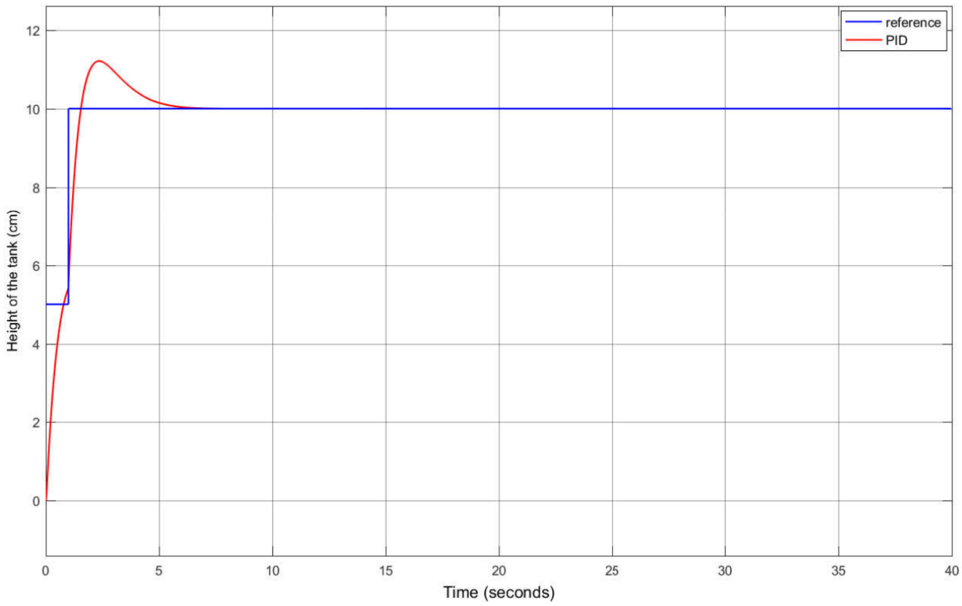


Figure 5.3:-The output response a CT with a PID

AS a result, obtained abovefigure using the PID controls for the coupled interacting tank, the overshoot is 15.117% and rise time 558.939msec.

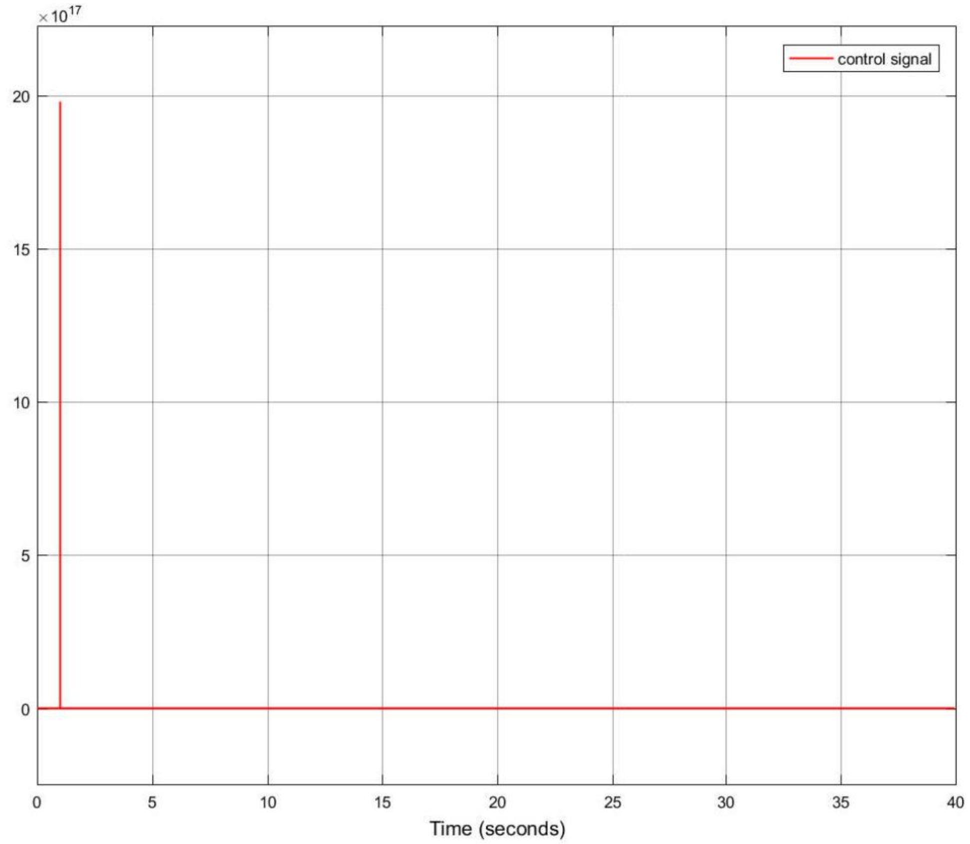


Figure 5.4:- Control signal generated by the PID controller

5.3.PERFORMANCE OF TWOCOUPLED TANK WITH FRACTIONAL ORDER PID CONTROLLER

To demonstrate the performance of coupled interacting tank with fractional order PID controller in MATLAB/Simulink,the parameters of the level control are the same as given in table 4.6.The MATLAB/Simulink models of interacting tanks withFOPID are given in below.

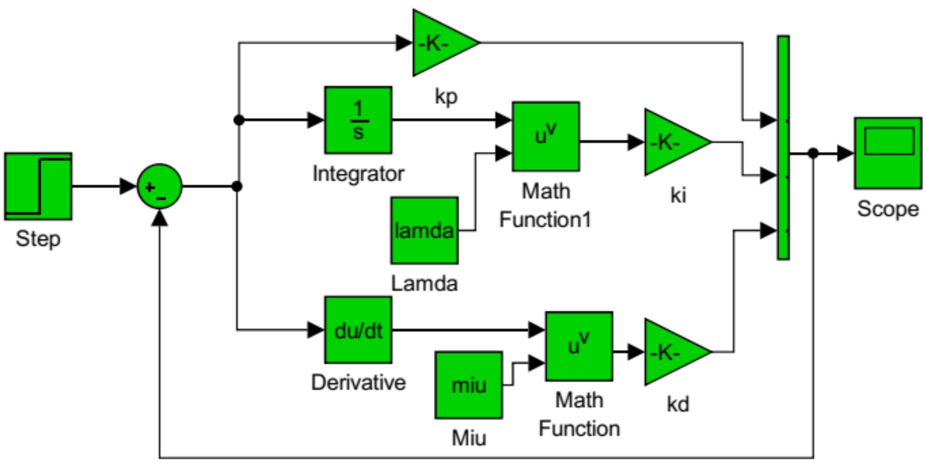


Figure 5.5:- Simulink model of the coupled tank with fractional order PID controller

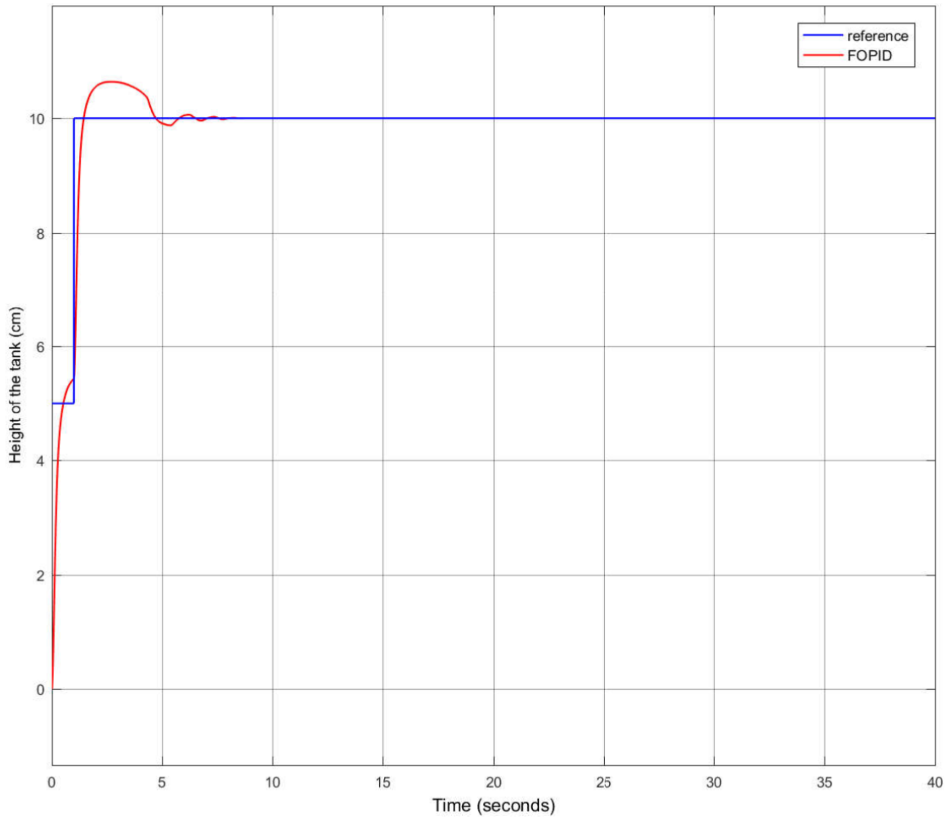


Figure 5.6:- Output of coupled tank with fractional order PID controller

As we observed fromfigure 5.5the result obtained in Figure 5.6 using fractional order PID controller forthe coupled interacting tank if the parameter value λ and μ, in between 0.35 and 0.75 the overshoot is 6.603%, The desired output of the interacting level tank is obtained Rise time 244.703msec.

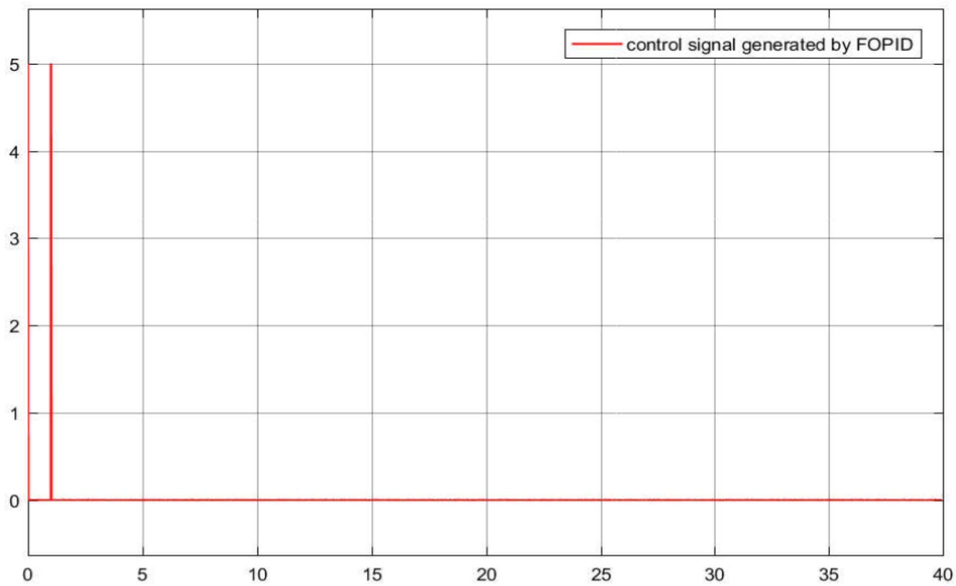


Figure 5.7:- Control signal generated by FOPID controller

5.4.PERFORMANCE OF COUPLED INTERACTING TANK WITH TID CONTROLLER

To demonstrate the performance of coupled interacting tanks with the TID controller in MATLAB/Simulink, the parameters of the level control are the same as given in table 4.6.The MATLAB/Simulink model of interacting tanks with TID control is shown in below,

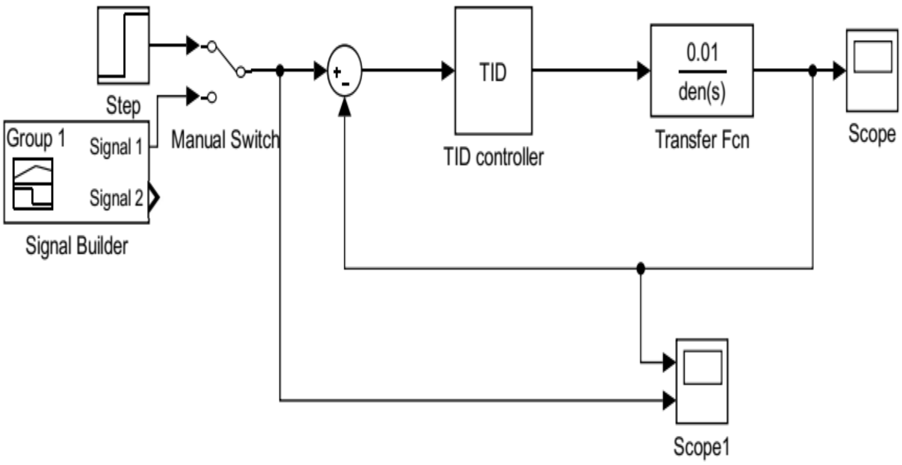


Figure 5.8:- Simulink Model of the CT with TID controller

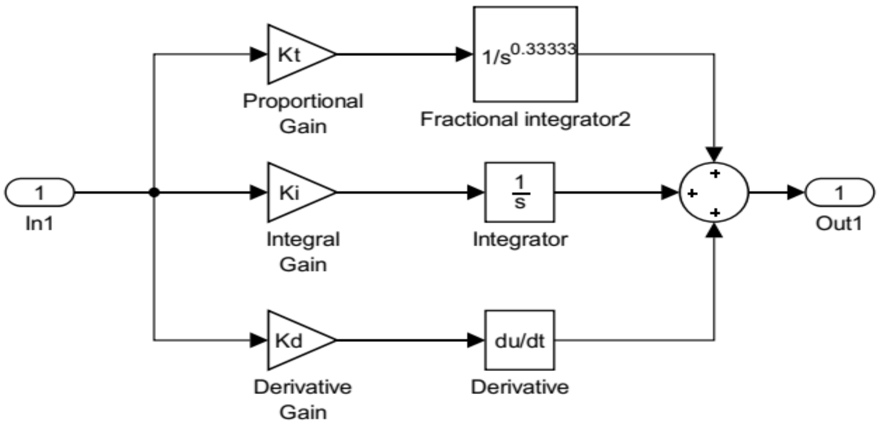


Figure 5.9:-Internal of tilted integral derivative (TID) controller

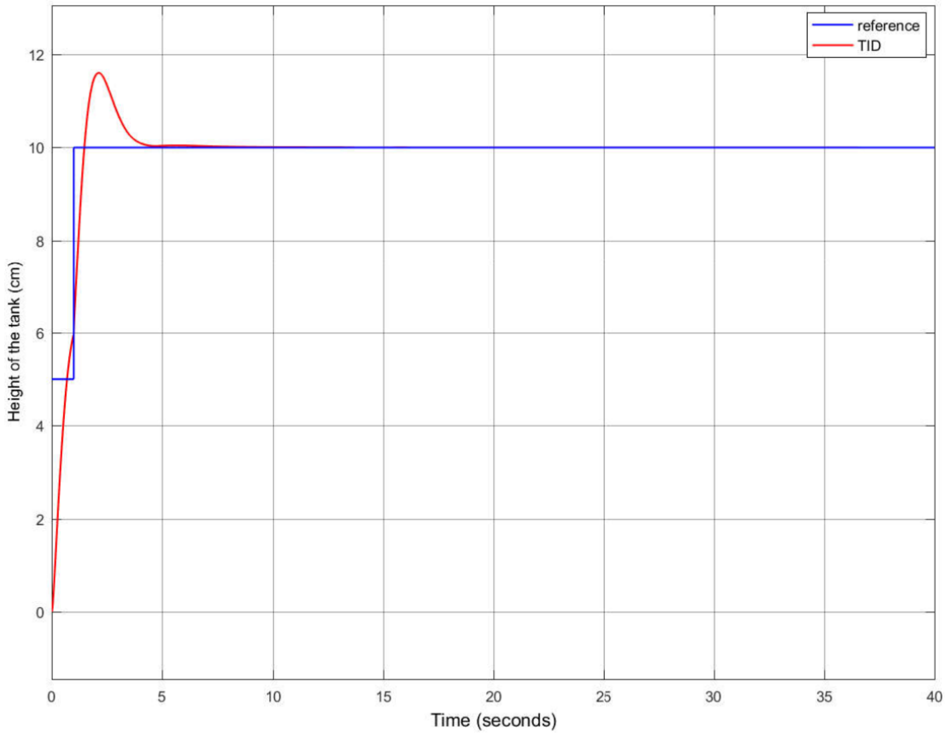


Figure 5.10:- The output response of (CT)with TID controller

From the result obtained in Figure 5.10using the TID controls for the (CT) the output response of a system below the setpoint when the value of n is an increase, but if the value of n equals to five the overshoot is 23.268% the desired output of the interacting level tank is obtained at the Rise time 504.755msec.

5.5.PERFORMANCE OF COUPLED INTERACTING TANK WITH THE FRACTIONAL LEAD-LAG COMPENSATOR

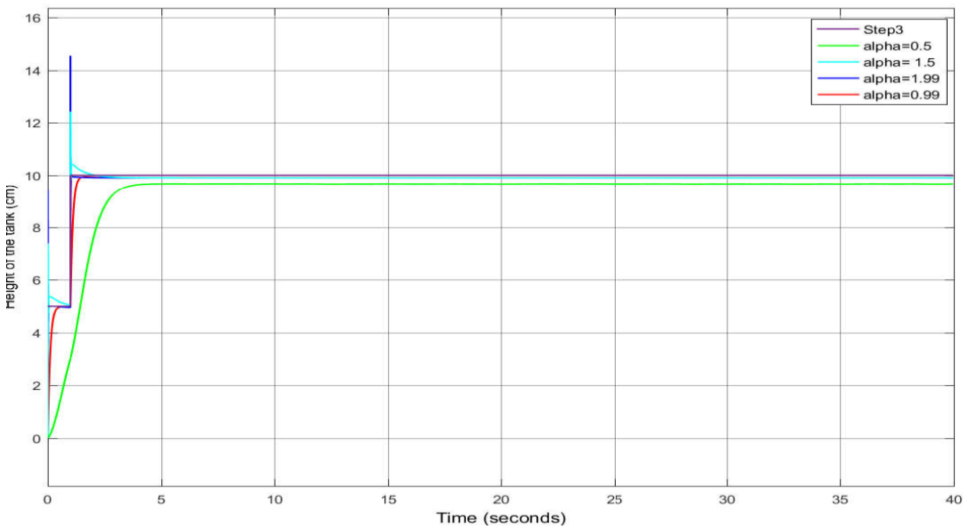


Figure 5.11:- output response of fractional order Lead-Lag controller

As we observed from figure 5.11 fractional-order lead-lag compensator when alpha in between zero and two mean that($0 \leq \alpha < 2$) if the value of alpha is an increase from 0.5 to 1.99, the system controller response fast according to shown figure 5.11 but not in line with the desired set point

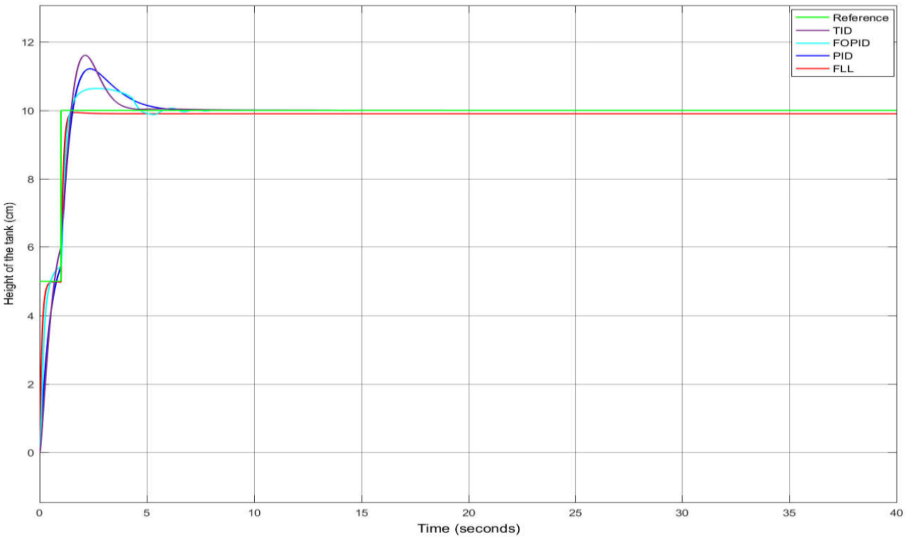


Figure 5.12:- the output response of a system using different controllers

5.6.PERFORMANCE OF COUPLED INTERACTING TANK WITH LQI FORTRACKING REFERENCE STATE FEEDBACK

To demonstrate the performance of coupled interacting tank with linear quadratic with integral action for tracking reference in MATLAB/Simulink, the parameters of the level control are the same as given in table 4.6.The MATLAB/Simulink model of interacting tanks with the LQI is shown in figure 5.13.

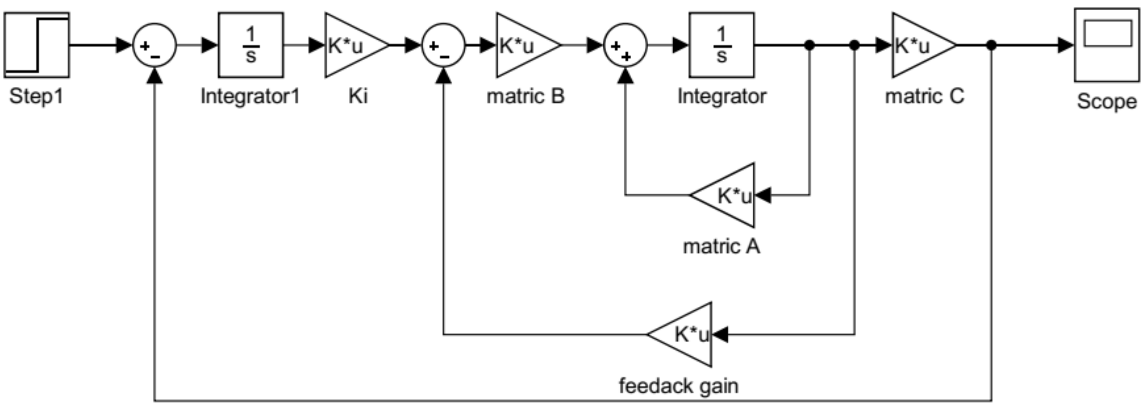


Figure 5.13:- Design coupled tank using LQIcontrollerfor tracking Reference

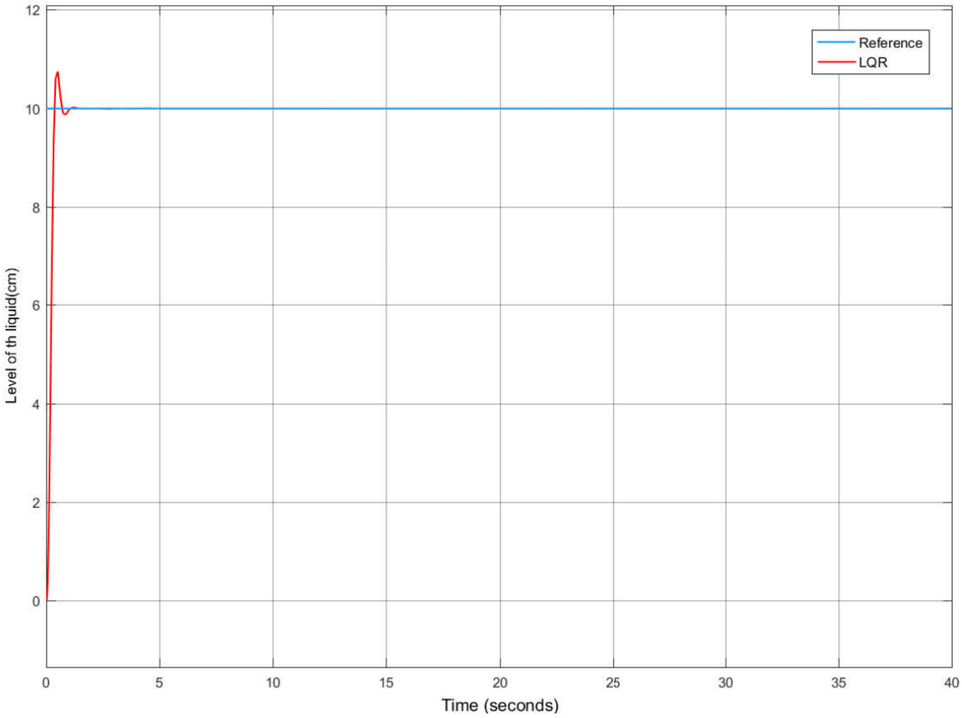


Figure 5.14:- output response of the coupled tank by LQI state feedback

From the result obtained in Figure 5.14 using a LQIfeedbackthe coupled interacting tank, the output of the system overshoot is 7.69%. The desired output of the interacting level tank is obtained at the rise time 81.013msec..

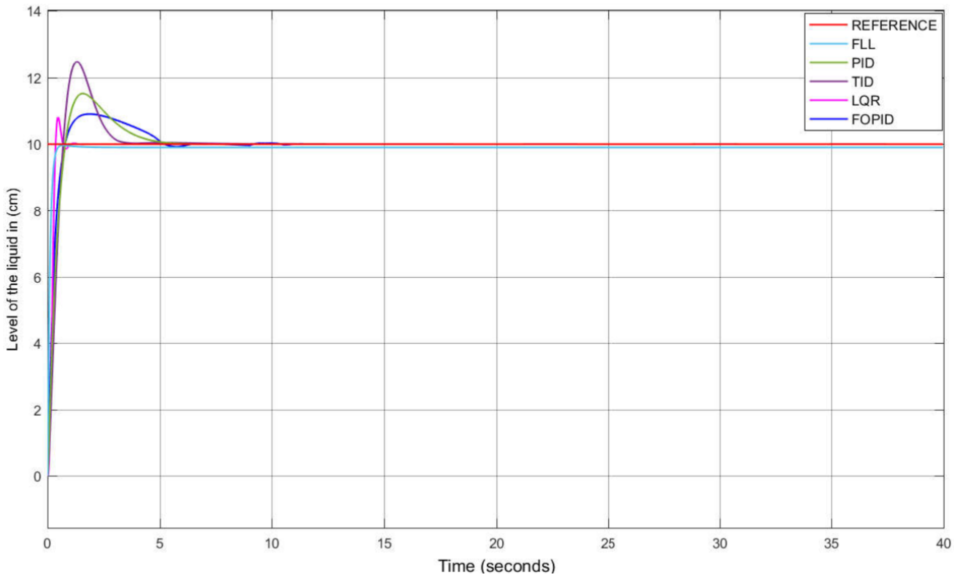


Figure 5.15:- The Over Output Response of Fractional order Controller and LQI Controllers

CHAPTER SIX

CONCLUSIONS AND RECOMMENDATION

6.1.CONCLUSIONS

Process control is essential in the industrial process because it guarantees the safety and

Optimization is a process. Additionally, process control is a useful tool to satisfy the environmental procedure and product quality necessities. For chemical engineers, process control is widely appropriate in the manufacturing system. the governing the height of the container in an industrial process i is the main issue, for that case design the appropriate controller, performance comparison of process-level control in case of the coupled tank, better performance, in this thesis work you have analysis two-controller those are fractional order controller and linear quadratic controller. The FOPID controller is the enlargement of the conventional PID controller based on fractional calculus. For numerous years, in industries, an IOPID control has been common in the function of industrial control. The greatness involves the clarity of design and its best achievement, such as a low percentage of over each and small settling time (In slow industrial processes which are essential. Fractional-order proportional integral derivative-controller have two more adaptable constraints than the PID-controller, &the order of the regulator can be selected randomly. TID Controller is to deliver a better feedback loop controller having the asset of the conventional PID controller, but if and only if a response which is nearer to the academically optimal response, fractional-order lead-lag controller commonly used to stabilize slightly stable systems and The linear-quadratic- regulator is a control scheme that provides the best conceivable performance concerning some given measure of performance. When using LQR with integral actionovercome to design a state feedback controller K and also computeintegral action tracking the reference point. In this technique, a feedback gain matrix K and LQI reference trackingare designed to use for control strength, the amplitude, and the response system.

By applying and linear quadratic regulator with integral action for state feedback control and fractional order controller, Linear quadratic regulator with integral action(LQI) feedback hasa better performance compared to the integer proportional integral derivative ad fractional order controller’s like(FOPID, TID and Fractional order Lead Lag) other than I observed from the Matlab/Simulink simulation result of table 5.1 the performance of Linear quadratic regulator with integral feedback control are better achievement compare to the other.

6.2.RECOMMENDATION

Although the intellectual of analysis of the performance of the FOPID controllerand optimal linear quadratic control for process-level control in interacting two-tank has been validated by Matlab® simulations, it would be advantageous to examine its practical implementation.

Another research issue is to test another optimal control techniques- algorithm than the used prescribed convergence algorithm. Analyze and evaluate the performance of optimal control techniques designed using different algorithms could lead to new insight.

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APPENDIX

Appendix A

% to create the transfer function of coupled tank can be used

$A_1 = 250\text{cm}^2; A_2 = 250\text{cm}^2;\ R_1 = 0.01\frac{\text{sec}}{\text{cm}^2}; R_2 = 0.01\text{sec./cm}^2$

$$G(s) = \frac{0.0016}{s^2 + 1.2s + 0.16}$$

The output response of open loop system can be

>>num=0.0016;

>>dem= [1 1.2 0.16];

G(s) =tf (num,den)

Step (G(s)

State space form of the system in controllable canonical form

$A = \begin{bmatrix} 0 & 1 & 0 \\ -0.16 & -1.2 & 0 \\ -1 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 0.0016 \end{bmatrix}, C = [1 \quad 0 \quad 0]$,D=0;

The value of relative dominance $\alpha = 1$,damping ratio of closed loop $\xi^{cl} = 0.95\ and\ natural\ frequency\ of\ closed\ loop\ \omega^{cl} = \frac{2rad}{sec}$.

And also natural frequency, damping ratio of the open loop system a respectively,$\omega = 0.4, \xi = 0.15$.

By using the formal on page 27, the gain of PID calculated

$$k_p = 6960, k_i = 4750, k_k = 2812.5$$

Linear quadratic regulator design depending on the following parameter value

$$R = 0.04; Q = \begin{bmatrix} 25 * 10\hat{}-6 & 0 & 0 \\ 0 & 75 * 10\hat{}-3 & 0 \\ 0 & 0 & 20 * 10\hat{}9 \end{bmatrix}$$

$$\text{K} = lqr(A_{new}, B_{new}, B, q, r)$$

Linear quadratic regulator with integral action for state feedback gain k.

$$\text{k} = 10\hat{}5([1.3574 \quad 0.123 \quad -7.0711])$$

For a figure page 55 of coupled tank controlled with fractional order PID controller when $\mu = \lambda = 0.5$

>>loadsets

All test sets loaded successfully.

>>kp= 6960;

>>Ki=4750;

```
>>kd=2812.5;

>> s=fotf('s')

Fractional-order transfer function:

s

>>num=0.0016;

>>den=[1 1.2 0.16];

>> G(s)=tf(num,den)

G(s) =

      0.0016

  -------------------

s^2 + 1.2 s + 0.16

Continuous-time transfer function.

>> H(s)=kp*s+ki*s^-0.5+kd*s^0.5

Fractional-order transfer function:

6960s^{1.5}+2812.5s+4750

------------------------

s^{0.5}

>> F(s)=H(s)*G(s)

Fractional-order transfer function:

   11.136s^{1.5}+4.5s+7.6

------------------------------

s^{2.5}+1.2s^{1.5}+0.16s^{0.5}

>> Y(s)=feedback(F(s),1)

Fractional-order transfer function:

        11.136s^{1.5}+4.5s+7.6

------------------------------------------
```

s^{2.5}+12.336s^{1.5}+4.5s+0.16s^{0.5}+7.6

>>step(Y(s))

For reduced the overshoot, settling time, rise time response take value of $\lambda = 0.35\ and\ \mu = 0.75$. for TID controller the value of n=5; and the value of alpha for fractional order lead lag controller used in between zero and two.