MATHEMATICAL MODELING AND ANALYSIS OF CRIME WITH MEDIA IMPACT

MSC.THESIS

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Approval sheet-I

This is to officially state that the study entitled "The Mathematical Modeling and Analysis of Crime With Media Impact" submitted to department of mathematics in partial fulfillment of the requirement for the degree of master of science in applied mathematics from Hawassa university, and record of original work carried out by Abduro Mamu Dekema (Id.No.PGmathk/001/11) under my guidance and supervision. Daily acknowledgements are done during his course of investigation. Therefore, I recommend that it would be accepted as fulfilling the thesis requirements.

________________________________ __________ __________________

Advisor Signature Date

Approval sheet – II

We undersigned member of the board of examiners read this thesis entitled "The Mathematical Modeling and Analysis of Crime With Media Impact " and evaluated the final defense made by Abduro Mamu Dekema ID.NO. PGmathk/001/11. Accordingly, we examined the candidate and therefore we certify that it has been accepted as partial fulfillment for the degree of masters of Science in applied mathematics.

Advisor	Signature	Date
External examiner	Signature	Date
Internal examiner I	Signature	Date
Internal examiner II	Signature	Date
PG coordinator	Signature	Date
Department Head	Signature	Date
SGS	Signature	Date

DECLARATION

I declare that this thesis is my original work and that all source materials used for this thesis have been properly acknowledged. This thesis has been submitted partial fulfillment of the requirements for masters of Science in applied mathematics at Hawassa University. I earnestly declare that this thesis is not submitted to any other institutions anywhere for the award of any academic degree, diploma or certificate.

________________________ ____________ ______________

Name **Signature** **Date**

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Acronyms

MatLab:matrix laboratory

CFE: crime free equilibrium

CPE:crime persistent equilibrium

ODE: ordinary differential equation

Abstract

In this thesis, we proposed and analyzed a nonlinear mathematical model that explains the dynamics of crime that includes media coverage. Some fundamental properties of the model including existence and positivity as well as boundedness of the solutions are investigated. Algebraic expression for the basic reproduction number is computed using the next generation matrix method. The model exhibits two equilibria: the crime-free equilibrium and the persistent equilibrium. The analysis shows that the crime-free equilibrium point is locally asymptotically stable if the basic reproduction number is less than unity ($R0 < 1$) and the persistent equilibrium point is locally asymptotically stable when the basic reproduction number is greater than one ($R0 > 1$). It is pointed out that the crime dies out if there is effective media coverage and awareness about crime, otherwise, the crime spreads rapidly in the society and it becomes an epidemic. The Numerical simulation is carried out using Ode45 of Matlab, sensitivity analysis of the basic reproduction number is also constructed. Analytical and numerical result shows that effective use of media coverage is the strategy to reduce the dynamics of crime transmission.

Key Words: Mathematical model, Crime, Basic reproduction number, Stability, Sensitivity Analysis, Numerical Simulation, Media coverage.

CHAPTER ONE

1. INTRODUCTION

1.1 Background of Study

The term "**crime"** is one of the critical and common problems to all societies.Crime can simply be defined as a deviant behavior that violates some prescribed norms and values [8]. Crime can be defined as, an unlawful act that is accounted by state or authority [2]. Crime is an act of harmful not only to some individual but also to a community, society or the state. That is, crimes usually are defined as acts or omissions forbidden by law that can be punished by imprisonment and/or fine [12]. Murder, robbery, burglary, rape, drunken driving, and failure to pay taxes all are common examples. Criminal is a person who has committed a crime. Anyone who breaks the law is a criminal. All criminal behaviors involve the use of force, fraud, or stealth to obtain material or symbolic resources. Mass media have been defined as the publishers, editors, journalists and others who constitute the communications industry and profession, and who disseminate information; largely through newspapers, magazines, television, radio and the internet [39]. There are different types of media such as television, internet, newspaper, radio, etc.

Media coverage of crime and criminal justice helps to correct these misconceptions and shed light on important realities of crime and criminal justice. Incorporating media is a direct influence upon perceptions of a crime is critically examined. The notion that acts such as murder, rape, and robbery are to be prohibited exists worldwide [11]. What precisely is a criminal felon is defined by criminal law of each country. Due to this, the public and policy makers have always been concerned about preventing the criminals.

Mathematical modeling is an important tool used in analyzing the dynamics of prevalence of crime. Several models have been formulated and analyzed to explain the dynamics of crime transmission. The intervention which is always associated with crime is punishment. The purpose of punishment is to incapacitate criminals from society and have a prevention effect on other possible offenders. Punishment is supposed to make criminal behavior less attractive and more threatening. Also, the punishments like capital punishment or incarceration prevent the criminals to commit crime[20, 21]. Hence any attempt to reduce crime needs to place a high priority on addressing juvenile delinquent. Recommendations for doing so emphasize a preventative approach of early interventions, alternative sanctions such as community service and strategies which encourage rehabilitation of lawbreakers while minimizing the use of institutionalization for all but the most serious offenders [35].

1.2 Causes of crime

There are different causes of crime. Crime is a highly complex phenomenon that changes across cultures and across time. Legal activities in one country (e.g. alcohol consumption in the UK) are sometimes illegal in others (e.g. strict Muslim countries). As cultures change overtime, behaviors that once were not criminalized may become criminalized (and then decriminalized again-e.g. alcohol prohibition in the USA). As a result there is no simple answer to the question 'what is crime?' and therefore no single answer to 'what causes crime?' Different types of crime often have their own various causes [31].

It seems straightforward to ask what causes crime but the question is not always as simple as it appears. Depending on the context, it might mean

- What stimulate individuals to get involved in crime?
- Why do certain individuals offend more frequently than others?
- Why do some individuals remain in crime longer than others?

Different sets of factors may be related in the answers to each of these questions. Family may be the main reason individuals get involved in crime but drug dependence may be the main specific factor between those who offend frequently and those who offend rarely. Individuals may commit theft to raise cash to buy illegal drugs but theft rates may be more prevalent in some areas because of the greater supply of attractive commercial targets(e.g. banks). In asking what causes crime we need to be clear about the precise question we aretrying to response[3, 16]. Crime is primarily the outcome of multiple adverse social, economic, cultural and family circumstances. To present crime it is important to have an understanding of its backgrounds. These are complex and interconnected, but can be condensed in three main groups.

- Economic Factors/Poverty.
- Social Environment
- Family Structures.

Economic Factors/Poverty: Economic factors are the main causes of crime which includes: deficiency of financial resources, lack of meaningful employment options, poor housing, lack of hope and the prejudice against persons living in poverty.

Social Environment: Social root causes of crime are: inequality, not sharing power, lack of leadership in communities, low value placed on children and individual well-being. Family Structures: Families are uniquely placed in contributing to raising healthy responsible members of society. Dysfunctional family conditions contribute to future delinquency. These conditions include:

- Parental inequality, parental conflict, lack of communication.
- Lack of respect and responsibility, family violence.

1.3 Mathematical Model

Different individuals or scientists give different meanings to mathematical model. Here, we discuss the quite commonly used definitions. Mathematical model is an idealization of a real-world phenomenon. According to Rutherford Aris, a mathematical model is a set of mathematical equations that provide an adequate description of a physical system. A "physical system" can be broadly interpreted as any real-world problem-natural or man-made, discrete or continuous, and can be deterministic, or random in behavior. A mathematical modeling involves the use of mathematics to describe, explain, or predict behavior or phenomena in the real world. That is, modeling is one of the ways by which formulae are deduced to solve a real life situation [33]. It can be particularly useful in investigating questions or testing ideas within complex systems. The model is then analyzed, solved, or simulated on a computer. The results can be interpreted in physical terms to aid understanding of the underlying system or to point to parts of the system that might be targeted for change [28].

Why do we deal with a model instead of a real world problem? Mathematical model used for a number of different reasons, how well any particular objective is achieved depends on both the state of knowledge about a system and how well the modeling is done [9]. It is an important tool in analyzing the spread and control of infectious diseases. The model formulation process clarifies assumptions, variables, and parameters. Moreover, models provide conceptual results such as the basic reproduction numbers.

The main motivations for working with models include:

- ✓ To avoid the need of costly, undesirable, or impossible experiments with the real-world.
- ✓ Prepares the way for better design or control of a system.
- ✓ To explore what is inaccessible in the real world. Identify all the variables that affect the performance of a system.

1.4 Statement of the problem

Almost everywhere on the world, the incidence of crimes against persons or property is on the rise and needs serious control. Challenges of insecurity that face in the society is a serious problem. Crime has become a major issue for public and political discourses but we are not see that good casing of media coverage about depressing of crime minimize the challenges. Many studies emphasis the dynamics of the crime by system of differential equations without considering the effect of media coverage and consider all the individual leads to be criminal (are prone to commit crime) but we consider the susceptible individuals are prone to commit crime or not prone to commit crime, non-criminal individuals and media coverage.

Media coverage is widely acknowledged as a key tool for influencing people behavior towards the crime to devise proper policies for controlling. We analyze a mathematical model for crime transmission incorporating media coverage to assess the effect of media on the prevailing crime in the society, applied a mathematical model$\boldsymbol{SACQR}$ to this problem.

In this study we are going to examine the following questions:

- ➢ How could a mathematical model be formulated to describe the dynamics of crime with media impact?
- ➢ How can we show the stability analysis of equilibrium points?
- ➢ What are the condition that influence the prevalence of crime
- ➢ What are the most sensitive parameters to spread the crime dynamics?

1.5 Objective of the study

Objective of the study includes general objective and specific objective.

1.5.1 General objective

The general objective of this study is to analyze the dynamics of crime with media impact.

1.5.2 Specific objectives

The specific objectives of this study are to:

- Formulate a modified mathematical model for crime with media impact.
- Compute the equilibrium points.
- Determine the basic Reproduction Number and investigate stability analysis of the equilibrium points.
- To investigate the most sensitive parameters of the model.

1.5.3 Significance of the study

This study could contribute for the community from different aspect. The primary benefits addressed to people who are suffered by different crimes. It can insight media leaders to focus on the dynamics of crime transmission and make awareness for the society due to media coverage. The analysis of dynamics of crime prevalence could be used to predict crime outbreak. The study findings are helpful for decision makers and governments in policy formulation, planning and making decision in order to control the crime prevalence. The study also helps researchers to get knowledge on the application of mathematical models in the field of crime prevalence, which is the concept of equivalence epidemiology. Finally, researchers may use the study findings as a standing point and can conduct a research in a more detail manner.

CHAPTER TWO

2. LITERATURE REVIEW

In different time many researchers are done about the spread of crime with media impact. Now we will see some of literatures that are done before this study and its guide on how to do our work.

Several authors have presented empirical as well as practical study on the generation and prevention of crime. The possible contributions of mathematical modeling of crime have been nicely reviewed in [17]. Iglesias et al., [38] presented an economic analysis of crime based on the idea that a crime results from a tradeoff between exported benefit and the risk of punishment. In particular, the tradeoff between crime and punishment has been studied. Zhao et al.,[19] proposed a mathematical model to study the interplay between criminality and poverty and explored the possibility of crime control via government interventions. Vito GF et al.,[36] proposed the prevention of juvenile delinquency is an essential part of any crime prevention program in society. Many criminals who begin their criminal careers as youths and teenagers continue this behavior into adulthood. Hence any attempt to reduce crime needs to place a high priority on addressing juvenile delinquency.

Modeling of delinquent behavior using an infectious disease approach has been done for fanatical and violent ideology [4, 7, 30] and the transmission of violent crime and burglary in the United Kingdom [5]. Becker et al.,[15], has suggested that people drift towards criminality when the benefits of crime are more than potential punishment. Hence, making additional the media coverage is obviously not the most important factor responsible for fighting the transmission of the crime, but it is a very important issue which has to be considered seriously [1]. In this study, a mathematical model is proposed to investigate the change in prevalence of a crime when an awareness program through the media is employed.

Our main aim is the study of the impact on the epidemic outbreak of the combined action of the awareness program. Hence, the epidemic modeling approach can readily be applied to study the dynamics of crime in a society. Voluminous literature is available on epidemic models comprising the transmission and control of crime [18, 29]. According to Olsson [40], crimes are associated with various kinds of losses: firstly, loss of productivity which involves time and the man power who spend in criminal activities in steady economic production; secondly, victim costs that include loss of property and other valuables, loss of capitals, injuries, and deaths; and lastly, loss of other nontangible properties, for instance, suffering and body pain, destruction of life quality, and other health and psychological problems. Arecent study of Jeke et al. [41] considered funds that are used to repair and restore the damages caused by crimes, costs of maintaining prisons, and costs of payment of prosecutors and law enforcement officers which are some of the negative impact of crime in which any governments with good governance cannot ignore.

Mathematical modeling has become one among the important tools in enhancing an understanding of the dynamics to social problems including crime, which is useful for political and economic decision-making regarding the intervention program design for crime eradication and control. The use of compartmental models in connecting mathematics in solving social challenges currently has been greatly useful, for instance, climate change, diseases, unemployment, and crime. Crime model is a new area of research connecting social studies and mathematics. Short et al. [42] developed a first statistical model for burglary crime which was motivated by criminology and social for general understanding. From that point, the series of linear and nonlinear mathematical models for crime has been developed while proposing the transmission of criminality behavior through social interaction with criminals.

Soemarsono et al. [43] develop a crime model extending mathematical model for unemployment developed by Munoli and Gani [44] by introducing a crime compartment as the effect of unemployment. Sundar et al. [45] developed a nonlinear mathematical model of crime induced by unemployment. González-Parra et al. [46] develop nonlinear mathematical model for crime and considered the transmission of criminal behavior as social epidemic while including behavior change as an important parameter and reducing the contact rate between good civilians and active criminals can help to reduce the number of criminals.

Srivastav et al. [47] investigated on crime prediction and prevention while taking into account the severity of corruption and crime in developing countries. The study considered social interaction as the main factor of changes in human behavior through imitation of what is happening in the surroundings and further emphasizes on the law enforcement for crime eradication. Some more studies that considered criminal behavior as the social epidemic which spread though social interactions can be found in [48-51].

CHAPTER THREE

3. METHODOLOGY

In this study, we used a non-linear deterministic model and the model is formulated by using system of ordinary differential equation. The stability analysis of the equilibrium points was done by classifying their steady states as crime free equilibrium point and persistent equilibrium point by using the Routh-Hurwitz stability criteria. And also we used the next generation matrix to determine the basic reproduction number. The important terms that can serve as our methods relevant to the thesis are defined and explained in the following manner.

3.1 Ordinary Differential Equation

Definition 3.1 (Ordinary differential equation)

An ordinary differential equation (ODE) is a differential equation involving one or several derivatives of a dependent variable with respect to a single independent variable. That is a differential equation of order n is said to be ordinary differential equation if it is of the

form of $f\left(y, \frac{dy}{dt}, \frac{d^2y}{dt^2}, \dots, \frac{d^ny}{d^nt}, t\right) = 0$ for certain function f.

Typical n^{th} order linear non-homogeneous ordinary differential equations given

$$\frac{d^ny}{dt^n} + a_1(t)\frac{d^{n-1}y}{dt^{n-1}} + a_2(t)\frac{d^{n-2}y}{dt^{n-2}} + \dots + a_{n-1}(t)\frac{dy}{dt} + a_n(t)y = g(t)$$

3.2 System of ordinary Differential equation

Definition 3.2 System of ordinary differential equations

A system of ordinary differential equation is defined as

$$\frac{dX}{dt} = F(X(t), t)$$

Where $X(T) = (x_1(t), x_2(t), x_3(t), \dots x_n(t))^T, F = (f_1, f_2, f_3, \dots . f_n)^T$ and

$$f_i = (f_i\big(x_1(t), x_2(t), \dots, x_n(t)\big), i = 1,2,3, \dots, n$$

3.3 Autonomous System of ordinary differential equation

Let us consider of an n-dimensional autonomous system of the form:

$$x' = f(x(t))$$

$$x(t_0) = x_0 \qquad 3.1$$

Definition 3.3(Well-posedness)

An initial value problem (IVP) given (3.1) is mathematically said to be well-posed if the following conditions hold:

(i) . Its solution exists

(ii).Its solution is unique

(iii).Its solution continuously depends on the initial conditions.Since in epidemiology we are dealing with population, the following two conditions are important.

(iv).The solution should be non-negative over time.

(v). The solution should be bounded.

Existence and uniqueness of the solution

We use Jorge.T,Telima.S[24] to prove the existence and uniqueness of the solution of system equation$(4.4 - 4.8)$. Consider the system of equation

$$x_1' = f_1(t, x_1, x_2, \dots, x_n), (x_1(t_0) = x_{01}$$
$$x_2' = f_2(t, x_1, x_2, \dots, x_n), (x_2(t_0) = x_{02},$$
$$\vdots\vdots\vdots \qquad\qquad 3.2$$
$$x_n' = f_n(t \quad , x_1, x_2, \dots, x_n), (x_n(t_0) = x_{0n}$$

The system of equation 4.2.1 is written as a compact form

$$x' = f(t, x), x(t_0) = x_0.$$

Now we state the following theorem

Theorem 3.1let$D = \{(S, A, C. Q, R)\epsilon R_+^5\}$ denote the region

$$|t - t_0| \le a, \left||x - x_0|\right| \le b, x = (x_1, x_2, \dots, x_n), x_0 = x_{01}, x_{02}, \dots, x_{0n} \qquad 3.3$$

and suppose that f(t,x) satisfies the Lipschitz condition

$$\left||f(t, x_1) - f(t, x_2)|\right| \le \left||x_1 - x_2|\right| \qquad 3.4$$

when the pairs(t, x_1) and(t, x_2) belong to D where r is a positive constant. Then, there exists a constant $\delta > 0$ such that there exists a unique continuous vector solution $x(t)$ of condition 3.2 in the interval $|t - t_0| < \delta$.. It is important to note that condition 3.2 is satisfied by the requirement that $\frac{\partial f_i}{\partial x_j}, i, j = 1,2,3, \dots, n$ is continuous and bounded in D.

Definition 3.4 (Positivity of solution)

The solution of given autonomous system of (3.1) is said to be positive ,if all trajectories $x(t)$ is positive for any $t \geq 0$

Definition 3.5 (Boundedness of solution)

The positive solution given in (3.2) of an autonomous system of (3.1) is said to be bounded if any solution of $x(t, t_0, x_0)$ in (3.1) satisfies

$$||x(t, t_0, x_0)|| \leq C(\left||x_0|\right|, t_0)$$

For all $t \geq t_0$ where, $C: \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ is constant that depends on t_0 *and* x_0

Theorem 3.2 Let a linear ordinary differential equation is given by

$$y^{\cdot} + p(x)y = q(x) \qquad 3.5$$

Then using integrating factor method the integrating factor and its general solution respectively given by

$$I(x) = e^{\int p(x)dx} \qquad 3.5a$$

$$y(x) = e^{-\int p(x)dx}[C + \int_0^t q(x)e^{\int p(x)}dx] \qquad 3.5b$$

3.4 Equilibrium points

Definition 3.4 Equilibrium solution in one dimension:

A point x^* in R is said to be an equilibrium solution of the one-dimensional differential equation $\frac{dx}{dt} = f(x)$ if it satisfies $x^{\cdot} = 0$ at$x = x^*$.

The equilibrium solutions or equilibriax^* are found by solving the equations$f(x^*) = 0$.

Definition 3.4.1 Equilibrium solution in two dimensions:

The equilibrium solutions to a system of first-order differential equations are the points at which each differential equation is equal to zero.

A point $(x,^* y^*)$ is said to be the equilibrium solution of the two-dimensional dynamical system

$\frac{dx}{dt} = f(x, y)$

$\frac{dy}{dt} = g(x, y)$ if it satisfies $f(x^*, y^*) = 0$ and $g(x^*, y^*) = 0$.

Definition 3.4.2 Equilibrium points in n dimensions:

The equilibrium points to a system of first-order differential equations are the points at which each differential equation is equal to zero.

A point $(x_1, x_2, ., ., .x_n)$ is said to be the equilibrium point of the n-dimensional dynamical system

$$\frac{dx}{dt} = f(x_1, x_2, \dots, x_n)$$

$$\frac{dx}{dt} = f_2(x_1, x_2, \dots, x_n)$$

$$\frac{dx_n}{dt} = f_n(x_1, x_2, \dots, x_n),$$

If it satisfies $f_1(x_1{}^*, x_2{}^*, \dots, x_n{}^* = \cdots = f_n\left(x^*{}_{1,}x^*{}_{2,\dots,}x^*{}_n\right) = 0$

3.5 Basic Reproduction Number, R_0

The basic reproduction number, denoted as R_0, plays a crucial role in the control and eradication of epidemics. It represents the average number of secondary cases produced by a single infected individual in a completely susceptible population.

The key points regarding the basic reproduction number R_0, are:

If each $R_0 < 1$ infected individual produces, on average less than one new infected individual. In this case, the outbreak will become extinct and the epidemic will die out over time.

When each $R_0 > 1$ infected individual produces, on average, more than one new infected individual. This means the crime can invade the susceptible population and spread without any control measures. The larger the value of R_0, the harder it is to control the crime.

To compute the basic reproduction number R_0, using the next generation matrix method, identify the number of compartments (n) and the number of infected/criminal compartments (m). Define the functions $f_i(x)$ as the rate of appearance of new infections/criminals in compartment $\boldsymbol{i}$, and $\boldsymbol{v_i}$ as the difference between the rate of transfer of individuals out of and into compartment $\boldsymbol{i}$. Construct the matrices $F\ and\ V$ and where F is the Jacobian of a $f_i(x)$ and is V the Jacobian of

$\boldsymbol{v_i}$, both evaluated at the crime-free equilibrium. The basic reproduction number R_0, is then calculated as the spectral radius of the next generation matrix FV^{-1}.

The compartment model can be written as

$$\frac{\mathrm{dx_i}}{\mathrm{dt}} = \mathrm{F_i} - \mathrm{V_i}(\mathrm{x}, \mathrm{y}), \mathrm{i} = 1,2,3, \dots, \mathrm{n}$$

$$\frac{\mathrm{dy_i}}{\mathrm{dt}} = \mathrm{G_j}(\mathrm{x}, \mathrm{y}), \mathrm{j} = 1,2,3, \dots, \mathrm{m}$$

Denoting F and V the matrices,

$$\mathrm{F} = \left(\frac{\partial \mathrm{F_i}}{\partial \mathrm{X_j}}(0, \mathrm{y}^*)\right), 1 \leq \mathrm{i}, \mathrm{j} \leq \mathrm{n}$$

$$\mathrm{V} = \left(\frac{\partial \mathrm{V_i}}{\partial \mathrm{X_j}}(0, \mathrm{y}^*)\right), 1 \leq \mathrm{i}, \mathrm{j} \leq \mathrm{n}$$

Then FV^{-1} is called the next generation matrix. The basic reproduction number is the spectral radius (dominant eigenvalues) of the matrix FV^{-1}.That is $R_0 = \rho(FV^{-1})$.

3.6 Stability of the equilibrium points

Definition 3.6 (Stable, unstable and center subspace)

Based on the algebraic signs of eigenvalues of the Jacobian matrix and we can determine the types of stability of equilibrium points.

- ✓ If eigenvalues are real, distinct and negative then the equilibrium point is stable
- ✓ If eigenvalues are real, distinct and positive then the equilibrium point is unstable
- ✓ If eigenvalues are real, distinct and opposite in sign then the equilibrium point is saddle point which is unstable
- ✓ If the eigenvalues are complex with negative real parts, then the equilibrium point is stable focus
- ✓ If the eigenvalues are complex conjugate with positive real part, then the equilibrium point is unstable focus
- ✓ If the eigenvalues are complex pure imaginary, then the equilibrium point is canter

3.7 Linearization Technique

For most dynamical systems, the equilibrium point of the system of non-linear differential equations plays an important role in the analysis of the model.

Let $f: R^n \to R^n$ be a mapping, and suppose that $x^* \epsilon R^n$ is a point that satisfies $f(x^*) = 0$. Then x^* is said to be an equilibrium point for the system of differential equation $\frac{dx}{dt} = f(x(t))$.

The linear part of f at is x^* denoted by $Df(x^*)$, which is the matrix of the partial derivatives at x^*.

That is for any $x \epsilon R^n$ we write

$$f(x) = \begin{pmatrix} f_1(x) \\ f_2(x) \\ \vdots \\ f_n(x) \end{pmatrix}$$

The function f_i where $i = 1,2,3, \dots, n$ are called components of f. We define

$$Df(x^*) = \begin{pmatrix} \frac{\partial f_1(x^*)}{\partial x_1} & \frac{\partial f_1(x^*)}{\partial x_2} & \dots . & \frac{\partial f_1(x^*)}{\partial x_n} \\ \frac{\partial f_2(x^*)}{\partial x_2} & \frac{\partial f_2(x^*)}{\partial x_2} & \dots . & \frac{\partial f_2(x^*)}{\partial x_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n(x^*)}{\partial x_n} & \frac{\partial f_2(x^*)}{\partial x_n} & & \frac{\partial f_n(x^*)}{\partial x_n} \end{pmatrix}$$

which is called the Jacobian matrix of the system. To find the eigenvalues of $Df(x^*)$ we solve the characteristic polynomial equation $|Df(x^*) - \lambda I| = 0$, where I is $n \times n$ identity matrix. Therefore, the characteristic polynomial equation is

$$Det(J) = Det(Df(x^*) - \lambda I) = \begin{vmatrix} \frac{\partial f_1(x^*)}{\partial x_1} - \lambda & \frac{\partial f_1(x^*)}{\partial x_2} & \dots . & \frac{\partial f_1(x^*)}{\partial x_n} \\ \frac{\partial f_2(x^*)}{\partial x_2} & \frac{\partial f_2(x^*)}{\partial x_2} - \lambda & \dots . & \frac{\partial f_2(x^*)}{\partial x_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n(x^*)}{\partial x_n} & \frac{\partial f_2(x^*)}{\partial x_n} & & \frac{\partial f_n(x^*)}{\partial x_n} - \lambda \end{vmatrix} = 0$$

Let $\lambda_1, \lambda_{2,} \lambda_3, \dots, \lambda_n$ bethedistincteigenvaluesoftheJacobianmatrixofthe n-dimensional autonomous linear system with $Det(J) \neq 0$. Then the equilibrium point x^* is stable if all the eigenvalues have negative real part and unstable if any of the eigenvalues have positive real parts. Solving the higher degree polynomial equation is might be difficult, but for the stability, we only need the sign of eigenvalues. We know that a steady state is stable if $R_e(\lambda) < 0$ for all λ and the stability condition is established by Routh-Hurwitz stability criteria. Generally the stability of the non-linear system can be studied using different approaches if the algebraic sign of the eigenvalues of the Jacobian matrix is easily identified. But if the algebraic sign of the eigenvalues are not determined easily, we can use and restrict our self to Routh-Hurwitz stability criterion.

3.7.1Routh-Hurwitz stability criterion

We use the Routh-Hurwitz Criterion (RHC) to show the local stability of crime persistent equilibrium point. Routh Hurwitz stability criterion is a method for determining the stability condition of the equilibrium point of a non-linear system of differential equation by examining the location of the roots of the characteristic equation of the system. Routh-Hurwitz stability criterion deals with necessary and sufficient conditions of stability. Necessary conditions of stability refer to conditions that must be hold for a polynomial to be Routh- Hurwitz, while sufficient conditions refer to conditions that it meet imply that the equilibrium point of a dynamical system is stable.

Consider an n^{th} degree characteristic polynomial equation

$$a_n\lambda^n + a_{n-1}\lambda^{n-1} + a_{n-2}\lambda^{n-2} + \cdots + a_1\lambda + a_0 = 0$$

We have two conditions.

I. **Necessary conditions for stability:**
   - ✓ All the coefficients of the characteristic polynomial equation must be the same sign (either all positive or all negative) and real.
   - ✓ All the coefficients must be non-zero.

II. **Sufficient conditions for stability:**
   - ✓ The first row will consists all even term of the coefficients of the characteristic equation which is arranged from the first even term to the last even term (if n is even).
   - ✓ The second row term will consists of all odd-term of the characteristic equation which is arranged from the first odd term to the last term (if n is odd).

   Thus if these conditions are satisfied, then the Routh Hurwitz array is presented as follow

$$
\begin{array}{c|cccc}
\lambda^n & a_n & a_{n-2} & a_{n-4} & \cdots \\
\lambda^{n-1} & a_{n-1} & a_{n-3} & a_{n-5} & \cdots \\
\lambda^{n-2} & b_1 & b_2 & b_3 & \cdots \\
. & c_1 & c_2 & c_3 & \cdots \\
\vdots & d_1 & d_2 & d_3 & \cdots \\
\lambda^0 & . & . & . & . \\
 & . & . & . & .
\end{array}
$$

Where $a_i = 1,2,3,\dots,n$are the characteristic polynomial coefficients and the elements in the rest of the array are computed using the following ways.

$$b_1 = \frac{-1}{\mathrm{an}-1} = \begin{vmatrix} \mathrm{a_n} & \mathrm{a_{n-2}} \\ \mathrm{a_{n-1}} & \mathrm{a_{n-3}} \end{vmatrix} = \frac{a_{n-1}a_{n-2} - a_n a_{n-3}}{a_{n-1}}$$

$$b_2 = \frac{-1}{\mathrm{a_{n-1}}} = \begin{vmatrix} \mathrm{a_n} & \mathrm{a_{n-4}} \\ a_{n-1} & \mathrm{a_{n-5}} \end{vmatrix} = \frac{a_{n-1}a_{n-4} - a_{n-1}a_{n-5}}{a_{n-1}}$$

$$b_3 = \frac{-1}{\mathrm{a_{n-1}}} = \begin{vmatrix} \mathrm{a_n} & \mathrm{a_{n-6}} \\ \mathrm{a_{n-1}} & \mathrm{a_{n-7}} \end{vmatrix} = \frac{a_{n-1}a_{n-6} - \mathrm{a_n a_{n-7}}}{\mathrm{a_{n-1}}}$$

$$b_4 = \frac{-1}{\mathrm{a_{n-1}}} = \begin{vmatrix} \mathrm{a_n} & \mathrm{a_{n-8}} \\ \mathrm{a_{n-1}} & \mathrm{a_{n-9}} \end{vmatrix} = \frac{a_{n-1}a_{n-8} - a_n a_{n-9}}{a_{n-1}}$$

and we continue in similar way

$$c_1 = \frac{-1}{\mathrm{b1}} = \begin{vmatrix} \mathrm{a_{n-1}} & \mathrm{a_{n-3}} \\ \mathrm{b_1} & \mathrm{b_2} \end{vmatrix} = \frac{a_{n-3}b_1 - a_{n-1}b_2}{b_1}$$

$$c_2 = \frac{-1}{\mathrm{b_1}} = \begin{vmatrix} \mathrm{a_{n-1}} & \mathrm{a_{n-5}} \\ \mathrm{b_1} & \mathrm{b_3} \end{vmatrix} = \frac{a_{n-5}b_1 - a_{n-1}b_3}{b_1}$$

$$c_3 = \frac{-1}{\mathrm{b_1}} = \begin{vmatrix} \mathrm{a_{n-1}} & \mathrm{a_{n-7}} \\ \mathrm{b_1} & \mathrm{b_4} \end{vmatrix} = \frac{a_{n-7}b_1 - a_{n-1}b_4}{b_1}$$

$$c_4 = \frac{-1}{\mathrm{b_1}} = \begin{vmatrix} \mathrm{a_{n-1}} & \mathrm{a_{n-9}} \\ \mathrm{b_1} & \mathrm{b_5} \end{vmatrix} = \frac{a_{n-1}b_1 - a_{n-1}b_5}{b_1}$$

and we continue in a similar way

$$d_1 = \frac{-1}{\mathrm{c_1}} \begin{vmatrix} \mathrm{b_1} & \mathrm{b_2} \\ \mathrm{c_1} & \mathrm{c_2} \end{vmatrix} = \frac{b_2 c_1 - \mathrm{b_1 c_2}}{\mathrm{c_1}}$$

$$d_2 = \frac{-1}{\mathrm{c1}} \begin{vmatrix} \mathrm{b_1} & \mathrm{b_3} \\ \mathrm{c_1} & \mathrm{c_2} \end{vmatrix} = \frac{b_3 c_1 - b_1 c_3}{c_1}$$

$$d_3 = \frac{-1}{c1}\begin{vmatrix} b_1 & b_4 \\ c_1 & c_4 \end{vmatrix} = \frac{b_4c_1 - b_1c_4}{c_1}$$

$d_4 = \frac{-1}{c_1}\begin{vmatrix} b_1 & b_5 \\ c_1 & c_5 \end{vmatrix} = \frac{b_5c_1 - b_1c_5}{c_1}$ and continue in a similar way

The stability criteria is that

(a) If the first column of the Routh Hurwitz array has no sign change, therefore the system will be stable

(b) If the first column of the Routh Hurwitz array has sign change, therefore the system will be unstable

Defination3.7 (A Metzler matrix)

A Metzler matrix is a matrix in which all the off diagonals elements are non negative. That is, square matrix $A = (a_{ij})$ of order $n \times n$ is Metzler matrix if for all $i \neq j$ then $a_{ij} \geq 0$ which is used to show the global stability of the crime free equilibrium point.

Theorem3.3(Castillo-Chavez theorem)

Assume that (3.1) can be written in the form of

$$\frac{dZ_1}{dt} = F(Z_1, Z_2)$$

$$\frac{dZ_2}{dt} = G(Z_1, Z_2)$$

Where $Z_1 \epsilon R^m$ (represents the classes of non-criminal individuals) and$Z_2 \epsilon R^{n-m}$(represents class of criminal individuals). Assume that $G(Z_1, 0) = 0$ and $E_0 = ({Z_1}^*, 0)$ be a steady state of (3.1)(the disease free equilibrium point). If the following conditions are satisfied:

a. For the system $\frac{dZ_1}{dt} = F(Z_1, 0)$the steady state Z^* is globally asymptotically stable

b. $G(Z_2, Z_2) = AZ_2 - \tilde{G}(Z_1, Z_2)$, $\tilde{G}(Z_1, Z_2) \geq 0\ for Z_1, Z_2 \epsilon\ \Omega$ where A is a Metzler matrix(the off diagonal elements of matrix A are non-negative and the region Ω is where the model makes biological sense. Then the steady state $E_0 = ({Z_1}^*, 0)$ is globally asymptotically stable for the system $\frac{dZ_1}{dt} = F(Z_1, 0)$and $G(Z_2, Z_2) = AZ_2 - \tilde{G}(Z_1, Z_2)$Provided that the basic reproduction number is less than one.

3.8 Sensitivity analysis

Sensitivity analysis is usually performed as a series of test in which one can use different sets of input parameters to observe how a change in the predictor parameter values affects the dynamical behavior of the system. It is useful tool to identify how closely input parameters are related to a predictor parameter. Its helps to determine the level of change necessary for an input parameter to reach the desired value of a predictor parameter. Changing the values of the most sensitivity parameters will be the most effective strategy in changing the results of the model.

Definition 3.8 The absolute sensitivity coefficient of a quantity M with respect to parameter I is the rate of change of M with respect to I . it is denoted by $\frac{\partial M}{\partial I}$. This sensitivity coefficient informs whether a quantity increases or decreases as a parameter varies. However, it does not tell how fast the parameters influence the quantity M.

Theorem 3.9 The relative sensitivity coefficient (or normalized forward sensitivity index) of a quantity M with respect to a parameter I is defined as :

$$M_I = \frac{\partial M}{\partial I} \times \frac{I}{M}$$

3.9. Numerical method

We use numerical methods to obtain approximate solutions when analytical methods are difficult. Numerical methods are used to approximate solutions of equations when exact solutions cannot be determined via algebraic methods and construct successive approximations that converge to the exact solution of an equation or system of equations. The basic method is to divide continuous time into discrete intervals, and to estimate the state of the system at the start of each interval. Thus the approximate solution changes through a series of steps. The crudest method for calculating the steps is to multiply the step length by the derivative at the start of the interval. This is called Euler’s method.

CHAPTER FOUR

4. MATHEMATICAL MODEL FORMULATION AND ANALYSIS

In this section, we propose a deterministic mathematical model, which comprises human populations, and formulated as a system of ordinary differential equations. Some fundamental Properties of the model which include; the invariant region, existence and uniqueness, positivity of the solution are presented. The equilibrium point and basic reproduction number R_0 is computed. Stability of the crime-free and the persistent equilibrium in terms of the basic reproduction number R_0 are also derived.

4.1 The Existing Mathematical Model

The model is divided in to three differentcompartmentssuch as non-criminal individuals (N_p),criminal individuals$\left(C_p\right)$, and recovered individuals(R_p).

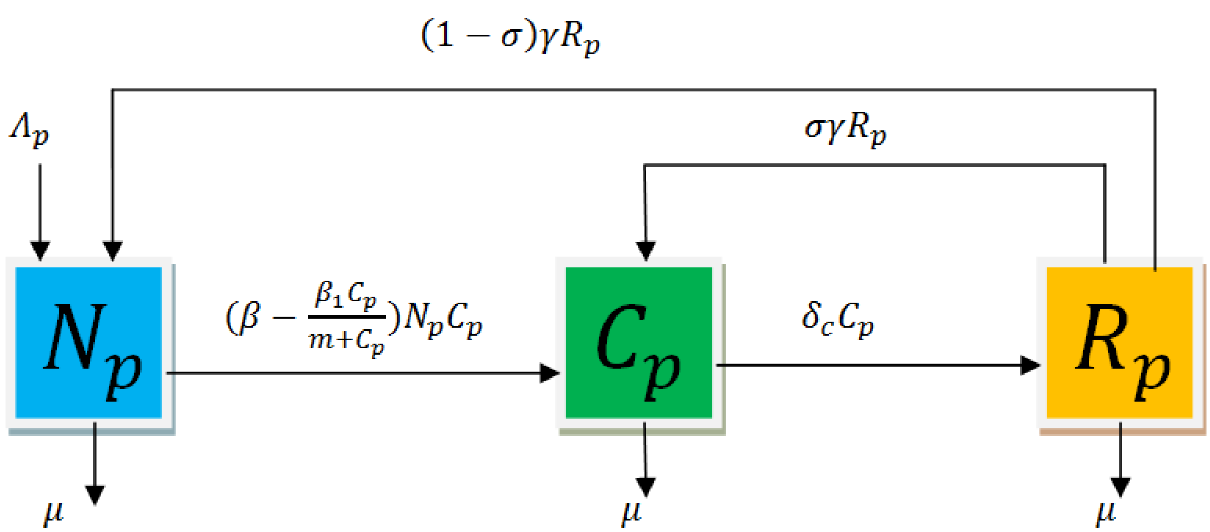


Figure 4.1The flow chart of existing mathematical model

The mathematical model that describes $N_p C_p R_p$ can be expressed as the following systems of non-linear ordinary differential equations.

$$\frac{dN_p}{dt} = \Lambda_p - \left(\beta - \frac{\beta_1 C_p}{m + C_p}\right) N_p C_p - (1-\sigma)\gamma R_p - \mu N_p \tag{4.1}$$

$$\frac{dC_p}{dt} = \left(\beta - \frac{\beta_1 C_p}{m + C_p}\right) N_p C_p - (\mu + \delta_c)C_p + \sigma\gamma R_p \tag{4.2}$$

$$\frac{dR_p}{dt} = \delta_c C_p - (\gamma + \mu) R_p \tag{4.3}$$

4.2 The Modified Mathematical Model

The modified mathematical model divides total human population N into five compartments: crime unaware susceptible (S), crime aware susceptible (A), criminal (C), Quarantined or jailed (Q) and recovered(R) has been incorporated explicitly in the modeling process. We divided the total susceptible population into two sub-classes: crime unaware susceptible (S) and crime aware susceptible (A). All these susceptible population represents the number of individuals that are not committed any crime but prone to commit crime. A criminal class (C) is one that are currently involved in various criminal activities and capable of influencing a susceptible individual to become criminal. A quarantined or jailed is an individual who is essentially a criminal residing in the jail or rehabilitation centers as a punishment. The recovered individual is one that are not currently involved in any criminal activities under any situation.

The transfer rates between the sub-classes are composed of several parameters. We assume that there is a positive recruitment rate Λ into the susceptible class, of which a certain proportion (δ) joins to crime unaware susceptible class (S), while the remaining proportion (1 −δ) join to crime aware susceptible class (A). It is assumed that an individual in the susceptible class becomes a criminal due to the interaction with active criminals and a positive natural death rate μ for all time under the study. Crime unaware susceptible individuals (S) become criminal through contact with criminals by the conversion rate βSC, where β represents the transmission rate of criminality and they join to the crime aware susceptible class (A) by a rate η due to the moment in which people act accordingly. The crime aware susceptible individual (A) becomes criminal by the conversion through contact with $\left(1 - \frac{\omega}{m+\omega}\right) AC$ criminal individuals, where, ω is measures efficacy of media coverage &m is media coverage and then recovered class R at a rate α due to moral/religious or media. The contact of susceptible individuals with criminals (C) by a rate βfor crime unaware and $(1 - \frac{\omega}{m+\omega})$for crime aware leads to the increasing of criminal population.

Effective use of media coverage (m) as well as crime-associated death rate (ρ) decreases the criminal population. The prisoner individual leave from the jail at a rate (γ). It is considered that a certain proportion (p) of α becomes fully rehabilitated (stop getting involved any criminal activities) and join to R, however the remaining proportion $(1-p)$ of α are susceptible to criminal (prone to commit crime) and join to a recovered class (R) increases due to moral/religious or use of media coverage and fully rehabilitation after prison time (θR) .

In addition to the above, we also considerd the following assumptions:

i. An individual can be criminal only through contacts with criminal individuals. That is, social interaction plays a significant role in the spread of crime.

ii. The non-criminal individuals are not prone to any criminal activities because of the Individuals know the depressing of crime.

iii. The unaware susceptible individuals will either get awareness or move to A class or get involved with a crime. Hence, we assume that there is no transition from $S$to become forever a noncriminal.

iv. The transfer from jail to non-criminal class is permanent as fully recovered (rehabilitated) due to good treatment in jail.

v. The prisoner individual would leave from the jail because the individual has finished his or her prison sentence or those individuals has shown change of behavior, and then become free.

vi. The criminal individuals moves to either quarantined population or recovered population.

Variable	Description of state variable
$S(t)$	Susceptible individuals those are un aware about crime
$A(t)$	Susceptible individuals those are aware about crime
$C(t)$	Criminal individuals who are currently involved in various criminal activities
$Q(t)$	Quarantined or jailed individuals
$R(t)$	Recovered individuals

TABLE 4.1 DESCRIPTION OF STATE VARIABLE FOR THE MODIFIED MATHEMATICAL MODEL

Parameters	Description of parameters
Λ	Constant recruitment rate of individuals
β	Transmission rate of criminality(crime unaware susceptible to criminal)

δ	Proportion of crime unaware susceptible
μ	Natural death rate
γ	Prison leaving rate
η	Awareness creation rate
θ	Rate of recovered become aware susceptible
α	Incarceration rate
ω	Rate of efficacy of media coverage
ρ	Death rate due to crime
m	Media coverage
p	Proportion of quarantined individuals

TABLE 4.2 PARAMETER OF THE MODIFIED MATHEMATICAL MODEL

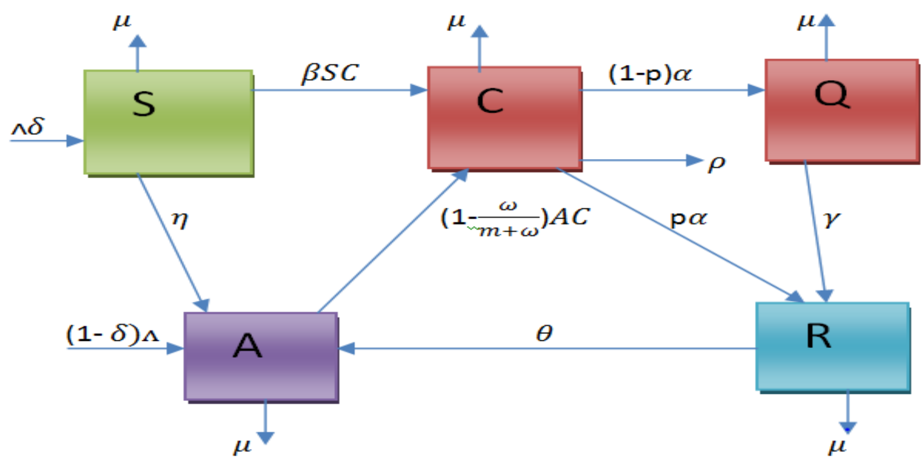


Figure4.2 The flow chart of modified mathematical model

Considering the assumption the modified mathematical model has the form

$$\frac{dS}{dt} = \delta\Lambda - \beta SC - (\mu + \eta)S \tag{4.4}$$

$$\frac{dA}{dt} = (1-\delta)\Lambda + \eta S + \theta R - \mu A - \left(1 - \frac{\omega}{m+\omega}\right) AC \tag{4.5}$$

$$\frac{dC}{dt} = \beta SC + \left(1 - \frac{\omega}{m+\omega}\right) AC - (\alpha + \mu + \rho)C \tag{4.6}$$

$$\frac{dQ}{dt} = (1-p)\alpha C - (\mu + \gamma)Q \quad (4.7)$$

$$\frac{dR}{dt} = \alpha p C + \gamma Q - (\theta + \mu)R \quad (4.8)$$

4.3 Existence, positivity and boundednessof solution of the modified mathematical model

The modified mathematical model is meaningful if the solutions exist, positive and bounded. So, in this section we will discuss about positivity, existence and boundedness of solution of the model system.

Existence and uniqueness of the solution

We use Jorge.T,Telima.S[24] to prove the existence and uniqueness of the solution of system equation$(4.4 - 4.8)$. Consider the system of equation

$$\begin{aligned} x_1' &= f_1(t, x_1, x_2, \dots, x_n), (x_1(t_0) = x_{01}, \\ x_2' &= f_2(t, x_1, x_2, \dots, x_n), (x_2(t_0) = x_{02}, \\ &\vdots\vdots\vdots \qquad\qquad 4.3.1 \\ x_n' &= f_n(t, x_1, x_2, \dots, x_n), (x_n(t_0) = x_{0n}, \end{aligned}$$

The system of equation 4.2.1 is written as a compact form

$$x' = f(t, x), x(t_0) = x_0.$$

Now we state the following theorem

Theorem 4.2 let$D = \{(S, A, C. Q, R)\epsilon R_+^5\}$ denote the region

$$|t - t_0| \le a, \left||x - x_0|\right| \le b, x = (x_1, x_2, \dots, x_n), x_0 = x_{01}, x_{02}, \dots, x_{0n} \quad 4.3.2$$

and suppose that f(t,x) satisfies the Lipschitz condition

$$\left||f(t, x_1) - f(t, x_2)|\right| \le \left||x_1 - x_2|\right| \quad 4.3.3$$

when the pairs(t, x_1) and(t, x_2) belong to D where r is a positive constant. Then, there exists a constant $\delta > 0$ such that there exists a unique continuous vector solution $x(t)$ of condition 4.2.2 in the interval $|t - t_0| < \delta$.. It is important to note that condition 4.2.2 is satisfied by the requirement that $\frac{\partial f_i}{\partial x_j}, i, j = 1,2,3, \dots, n$ is continuous and bounded in D.

Proof: Let D denote the region above. It suffices to show that $\frac{\partial f_i}{\partial x_j}, i, j = 1,2,3,4,5$ are continuous and bounded in D. Now, from 4.2.2 $D = (S, A, C, Q, R)$ where

$$|S - S_0| \leq a_1, |A - A_0| \leq a_2, |C - C_0| \leq a_3, |Q - Q_0| \leq a_4, |R - R_0| \leq a_5$$

Let

$$f_1(S, A, C, Q, R) = \frac{\mathrm{dS}}{\mathrm{dt}} = \delta\Lambda - \beta SC - (\mu + \eta)S$$

$$f_2(S, A, C, Q, R) = \frac{dA}{dt} = (1 - \delta)\Lambda + \ \eta S + \theta R - \mu A - \left(1 - \frac{\omega}{m + \omega}\right)AC$$

$$f_3(S, A, C, Q, R) = \frac{dC}{dt} = \beta SC + \left(1 - \frac{\omega}{m + \omega}\right)AC - (\alpha + \mu + \rho)C$$

$$f_4(S, A, C, Q, R) = \frac{dQ}{dt} = (1 - p)\alpha C - (\mu + \gamma)Q$$

$$f_5(S, A, C, Q, R) = \frac{dR}{dt} = \alpha pC + \gamma Q - (\theta + \mu)R$$

Consider the partial derivatives with respect to each state variable

$\frac{\partial \mathrm{f}_1}{\partial \mathrm{S}} = -\beta C - \mu - \eta, \frac{\partial \mathrm{f}_1}{\partial \mathrm{A}} = 0, \quad \frac{\partial \mathrm{f}_1}{\partial \mathrm{C}} = -\beta S, \frac{\partial \mathrm{f}_1}{\partial \mathrm{Q}} = 0 \, \frac{\partial \mathrm{f}_1}{\partial \mathrm{R}} = 0$

$\frac{\partial \mathrm{f}_2}{\partial \mathrm{S}} = \eta, \ \frac{\partial \mathrm{f}_2}{\partial \mathrm{A}} = -\mu - \left(1 - \frac{\omega}{m+\omega}\right)C \ , \ \frac{\partial \mathrm{f}_2}{\partial \mathrm{C}} = -1\left(-\frac{\omega}{m+\omega}\right)A), \frac{\partial \mathrm{f}_2}{\partial \mathrm{Q}} = 0, \frac{\partial \mathrm{f}_2}{\partial \mathrm{R}} = \theta$

$\frac{\partial \mathrm{f}_3}{\partial \mathrm{S}} = \beta C, \frac{\partial \mathrm{f}_3}{\partial \mathrm{A}} = (1 - \frac{\omega}{m+\omega})C, \frac{\partial \mathrm{f}_3}{\partial \mathrm{C}} = \beta S + \left(1 - \frac{\omega}{m+\omega}\right)A - (\alpha + \mu + \rho), \frac{\partial \mathrm{f}_3}{\partial \mathrm{Q}} = 0, \frac{\partial \mathrm{f}_3}{\partial \mathrm{R}} = 0$

$\frac{\partial \mathrm{f}_4}{\partial \mathrm{S}} = 0, \frac{\partial \mathrm{f}_4}{\partial \mathrm{A}} = 0, \frac{\partial \mathrm{f}_4}{\partial \mathrm{C}} = (1 - p)\alpha, \frac{\partial \mathrm{f}_4}{\partial \mathrm{Q}} = -\mu - \gamma \ , \frac{\partial \mathrm{f}_4}{\partial \mathrm{R}} = 0$

$\frac{\partial \mathrm{f}_5}{\partial \mathrm{S}} = 0, \frac{\partial \mathrm{f}_5}{\partial \mathrm{A}} = 0, \frac{\partial \mathrm{f}_5}{\partial \mathrm{C}} = \alpha p, \frac{\partial \mathrm{f}_5}{\partial \mathrm{Q}} = \gamma, \frac{\partial \mathrm{f}_5}{\partial \mathrm{R}} = -\theta - \mu =$

Now by substituting 4.5 in to 4.6 we have get

$$\left|\frac{\partial \mathrm{f}_1}{\partial \mathrm{S}}\right| = |\beta C - \mu - \eta| = \beta C + \mu + \eta \leq \beta(C_0 + a_3) + \mu + \eta < \infty$$

$$\left|\frac{\partial \mathrm{f}_1}{\partial \mathrm{A}}\right| = 0 < \infty$$

$\left|\frac{\partial \mathrm{f}_1}{\partial \mathrm{C}}\right| = | - \beta S| \leq \beta(S_0 + a_1 < \infty,$

$$|\frac{\partial \mathrm{f}_1}{\partial \mathrm{Q}}| = 0 < \infty \left|\frac{\partial \mathrm{f}_1}{\partial \mathrm{R}}\right| = 0 < \infty$$

$|\frac{\partial \mathrm{f}_2}{\partial \mathrm{S}}| = |\eta| = \eta < \infty|,$

$$|\frac{\partial \mathrm{f}_2}{\partial \mathrm{A}}| = \left|-\mu - \left(1 - \frac{\omega}{m+\omega}\right) C\right| = \mu + \left(1 + \frac{\omega}{m+\omega}\right) C \le \mu + (1 + \frac{\omega}{m+\omega})(C_0 + a_3) < \infty$$

$\left|\frac{\partial f_2}{\partial C}\right| = \left|-\left(1 - \frac{\omega}{m+\omega}\right) A\right| = 1 - \frac{\omega}{m+\omega})A \le 1 - \frac{\omega}{m+\omega})(A_0 + a_2) < \infty,$

$|\frac{\partial f_2}{\partial Q}| = 0 < \infty,$

$\left|\frac{\partial f_2}{\partial R}\right| = |\theta| = \theta < \infty,$

We continue in similar way for $f_3, f_4 and f_5$.Thus, the system of equation $(4.4 - 4.8)$ are differentiable with respect to all the state variables and all the partial derivatives of system $(4.4 - 4.8)$ are continuous and bounded. Therefore, by the existence and uniqueness theorem the modified mathematical model of the equation system $(4.4 - 4.8)$hasa unique solution.

Positivity of the solution

To show the positivity of the solution for the dynamical system, it is necessary to prove that solutions of system $(4.4 - 4.8)$ with positive initial conditions remain positive for all time$t \ge 0$. This fact has been stated by the following theorem.

Theorem 4.1:- Let the initial values $\{(S(0), A(0), C(0), Q(0), R(0)) \ge 0\}\epsilon\Omega_+^5$, then the solution set $\{S(t), A(t), C(t), Q(t), R(t)\}$ of the dynamic system $(4.4 - 4.8)$ is non-negative for all $t \ge 0$.

Proof: From the equation (4.4) we have,

$$\frac{\mathrm{dS}}{\mathrm{dt}} = \delta\Lambda - \beta SC - (\mu + \eta)S$$

$\frac{\mathrm{dS}}{\mathrm{dt}} + (\beta C + \mu + \eta)S = \delta\Lambda$Which is first order linear ordinary differential equation of the form $\frac{dS}{dt} + P(t)S = Q(t)$. Where $P(t) = \beta C + \mu + \eta \; and \; Q(t) = \delta\Lambda$ and the integrating factor is $I(t) = e^{\int P(t)dt}$

$\Rightarrow I(t) = e^{(\beta C + \mu + \eta)t}$

Then the solution of the equation is given by$S(t) = \frac{1}{I(t)}[\int_0^t I(t)Q(t)dt + c]$

$$S(t) = e^{-(\beta C + \mu + \eta)}[S(0) + \int_0^t e^{(\beta C + \mu + \eta)t}\Lambda\delta dt]$$

$$S(t) = e^{-(\beta C + \mu + \eta)}[S(0) + \Lambda\delta \int_0^t e^{(\beta C + \mu + \eta)t}dt] \ge 0 \qquad (4.9)$$

Since $\Lambda, \delta > 0$ and all exponential functions are positive.

Therefore, $S(t) > 0$.

From the (4.7) we have,

$$\frac{dQ}{dt} = (1-p)\alpha C - (\mu+\gamma)Q$$

$\frac{dQ}{dt} + (\mu+\gamma)Q = (1-p)\alpha C$This is the first order ordinary differential equation, so that the general solution is given by

$$Q(t) = e^{-(\mu+\gamma)t}[Q(0) + \int_0^t e^{(\mu+\gamma)t}(1-p)\alpha C dt]$$

$$\Rightarrow Q(t) = e^{-(\mu+\gamma)t}[Q(0) + (1-p)\alpha \int_0^t C e^{(\mu+\gamma)t} dt] \geq 0 \qquad (4.10)$$

Thus, $Q(t) \geq 0$ provided that$0 < p \leq 1$,

From (4.8) we have

$$\frac{dR}{dt} = \alpha pC + \gamma Q - (\theta+\mu)R$$

$\frac{dR}{dt} + (\theta+\mu)R = \alpha pC + \gamma Q$ which is the first order ordinary differential equation, so that the general solution is given by $R(t) = \frac{1}{I(t)}[C + \int_0^t I(t)Q(t)dt]$

$$R(t) = e^{-(\theta+\mu)t}[R(0) + \int_0^t e^{(\theta+\mu)t}(\alpha pC + \gamma Q)dt]$$

$$R(t) = e^{-(\theta+\mu)t}[R(0) + \alpha p \int_0^t e^{(\theta+\mu)t} C dt + \gamma \int_0^t e^{(\theta+\mu)t} Q dt] \geq 0 \qquad 4.11$$

Therefore,$R(t) \geq 0$

From (4.5) we have

$$\frac{dA}{dt} = (1-\delta)\Lambda + \eta S + \theta R - \mu A - \left(1 - \frac{\omega}{m+\omega}\right)AC$$

$\frac{dA}{dt} + \left(\mu + \left(1 - \frac{\omega}{m+\omega}\right)C\right)A = (1-\delta)\Lambda + \eta S + \theta R$ .Now let $f = \mu + \left(1 - \frac{\omega}{m+\omega}\right)C$

$\frac{dA}{dt} + fA = (1-\delta)\Lambda + \eta S + \theta R$ This is the first order ordinary differential equation, so that the general solution is given by

$$A(t) = e^{-(f)t}[A(0) + \int_0^t \left(e^{(f)t}(1-\delta)\Lambda + \eta S + \theta R\right)dt]$$

$$A(t) = e^{-(f)t}[A(0) + (1-\delta)\Lambda\int_0^t e^{(f)t}dt + \eta\int_0^t e^{(f)t}dt + \theta\int_0^t e^{(f)t}dt] \geq 0 \qquad 4.12$$

Therefore, $A(t) \geq 0$ provided that $0 < \delta \leq 1$

From equation (4.6) we have , $\frac{dC}{dt} = \beta SC + \left(1 - \frac{\omega}{m+\omega}\right)AC - (\alpha + \mu + \rho)C$

$\frac{dC}{dt} + \left(\beta S + \left(1 - \frac{\omega}{m+\omega}\right)A + (\alpha + \mu + \rho)\right)C = 0$Which is first order ordinary differential equation and the solution is given by

$$C(t) = e^{-(\beta S + \left(1 - \frac{\omega}{m+\omega}\right)A + (\alpha+\mu+\rho))t}[C(0)] \geq 0 \qquad 4.13$$

Thus,$C(t) \geq 0$

By the above proof we conclude that the solution of the equation$(4.4 - 4.8)$ are positive for all $t \geq 0$.

Invariant region

The model under consideration monitors population as such; we assume that all the variables and parameters of the model are positive for all $t > 0$. Thus, the models should be considered in feasible region where non-negative is preserved.

Theorem 4.2:- There exists a positively invariant region Ω in which the solution $(S(t), A(t), C(t), R(t), Q(t))$ of the dynamic system $(4.4 - 4.8)$ is bounded.

Proof: The positivity has already been established by theorem 4.1. For this model the total population is $N(t) = S(t) + A(t) + C(t) + Q(t) + R(t)$

$$\frac{dN}{dt} = \delta\Lambda - \beta SC - (\mu+\eta)S + (1-\delta)\Lambda + \eta S + \theta R - \mu A - \left(1 - \frac{\omega}{m+\omega}\right)\mathrm{AC} + \beta SC$$
$$+ \left(1 - \frac{\omega}{m+\omega}\right)AC - (\alpha + \mu + \rho)C + \alpha pC + \gamma Q - (\theta+\mu)R + (1-p)\alpha C$$
$$- (\mu+\gamma)Q$$

$= \Lambda - \rho C - S\mu - \mu A - \mu C - \mu Q - \mu R$

$= \Lambda - \rho C - \mu(S + A + C + Q + R)$

$= \Lambda - \rho C - \mu N,$

$\leq \Lambda - \mu N$ Since $\rho C > 0$ by theorem 4.1

$$\frac{dN}{dt} \leq \Lambda - \mu N$$

$\frac{dN}{dt} + \mu N \leq \Lambda$ Then by using integrating factor

$$N(t) \leq e^{-\mu t}(\int e^{\mu t}\Lambda dt + C)$$

$$\Rightarrow N(t) \leq e^{-\mu t}\left(\frac{\Lambda}{\mu}e^{\mu t} + C\right)$$

$$\Rightarrow N(t) \leq \frac{\Lambda}{\mu} + Ce^{-\mu t}$$

$$N(0) = \frac{\Lambda}{\mu} + C, at\ t = 0$$

$$\Rightarrow C = N(0) - \frac{\Lambda}{\mu}$$

$$N(t) \leq \frac{\Lambda}{\mu} + (N(0) - \frac{\Lambda}{\mu})e^{-\mu t}$$

If $N(0) \leq \frac{\Lambda}{\mu}$ then we obtain $0 \leq N(t) \leq \frac{\Lambda}{\mu}$, for all $t > 0$. If $N(0) \geq \frac{\Lambda}{\mu}$ then we have $0 \leq N(t) \leq N(0)$ for all $t > 0$ .Thus the feasible solution set of the system$(4.4 - 4.8)$ remain in the region , $\Omega = \left\{(S, A, C, R, Q)\epsilon\mathcal{R}_+^5 : 0 \leq N(t) \leq \max(N(0), \frac{\Lambda}{\mu})\right\}$

If we start with initial data $N(0)\ \epsilon\ \Omega$ then the solution $N(t)\ \epsilon\ \Omega$ for every $t > 0$. This shows the positively invariance of Ω. Thus the solution of dynamic system $(4.4 - 4.8)$ is bounded.

4.4 Equilibrium points

The equilibrium point of the model system is obtained by setting the left side of the system equation $(4.4 - 4.8)$ is equal to zero. The model exhibits two equilibrium points: the crime-free equilibrium point and the crime-persistent equilibrium point.

4.4.1 Crime Free Equilibrium Point (CFE)

The crime free equilibrium point is the steady state solution, where there is no crime in the society. Thus, the crime free equilibrium point is obtained as follow:

$$\frac{dS}{dt} = 0$$

$$\Lambda\delta - \beta SC - (\mu + \eta)S = 0$$

$$\Lambda\delta - (\mu + \eta)S = 0$$

$$S = \frac{\Lambda\delta}{(\mu + \eta)}$$

$$\frac{dC}{dt} = 0$$

$$C = 0$$

$$\frac{dQ}{dt} = 0$$

$$Q = 0$$

$$\frac{dR}{dt} = 0$$

$$\alpha pC + \gamma Q - (\theta + \mu)R = 0$$

$$R = 0$$

$$\frac{dA}{dt} = 0$$

$$(1-\delta)\Lambda + \eta S + \theta R - \mu A - \left(1 - \frac{\omega}{m+\omega}\right)AC = 0$$

$$(1-\delta)\Lambda + \eta S - \mu A = 0$$

$$\mu A = (1-\delta)\, \Lambda + \eta S$$

$$\mu A = (1-\delta)\, \Lambda + \eta (\frac{\Lambda\delta}{(\mu + \eta)}$$

$$A = \frac{(1-\delta)\, \Lambda(\mu + \eta) + \eta\Lambda\delta}{\mu(\mu + \eta)}$$

Thus, the Crime Free Equilibrium (CFE) point of the modified mathematical model is given by

$$E_0 = (S_0, A_0, C_0, Q_0, R_0)$$

$$E_{0=}\left(\frac{\Lambda\delta}{(\mu + \eta)}, \frac{(1-\delta)\, \Lambda(\mu + \eta) + \eta\Lambda\delta}{\mu(\mu + \eta)}, 0,0,0\right) \qquad (4.14)$$

4.4.3 Crime-Persistent Equilibrium Point (CPE)

Crime persistent equilibrium point is steady state solution where the crime persists in the population. The crime will persists in the population and the persistent equilibrium point of the modified mathematical model is given by $E^* = (S^*, A^*, C^*, Q^*, R^*)$. To solve the persistent equilibrium point of the system of deferential equation$(4.4)-(4.8)$ of dynamical system we equate the system equal to zero.

From system of equation $(4.4-4.8)$ we have

$$\delta\Lambda-\beta S^{*}C^{*}-(\mu+\eta)S^{*}=0 \tag{4.15}$$

$$(1-\delta)\Lambda+\ \eta S+\theta R-\mu A-\left(1-\frac{\omega}{m+\omega}\right)A^{*}C^{*}=0 \tag{4.16}$$

$$\beta SC+\left(1-\frac{\omega}{m+\omega}\right)A^{*}C^{*}-(\alpha+\mu+\rho)C^{*}=0 \tag{4.17}$$

$$(1-p)\alpha C^{*}-(\mu+\gamma)Q^{*}=0 \tag{4.18}$$

$$\alpha pC^{*}+\gamma Q^{*}-(\theta+\mu)R^{*}=0 \tag{4.19}$$

From equation(4.15) we get,

$S^{*}=\frac{\delta\Lambda}{\beta\mathrm{C}+\mu+\eta}$ and from equation (4.18) we get,

$$Q^{*}=\frac{(1-p)\alpha C}{\mu+\gamma}$$

From equation (4.16) we have

$A^{*}=\frac{(1-\delta)\Lambda+\eta S^{*}+\theta R^{*}}{\mu+\left(1-\frac{\omega}{m+\omega}\right)C^{*}}$ and from equation (4.19)

$R^{*}=\frac{\alpha pC^{*}+\gamma Q^{*}}{\theta+\mu}$ then substituting the value of Q we get

$R^{*}=\frac{C^{*}(\alpha pC)(\mu+\gamma)+\gamma\alpha(1-p)C^{*}}{(\mu+\gamma)(\theta+\mu)}=C^{*}(\frac{\alpha p(\mu+\gamma)+\alpha\gamma(1-p)}{(\mu+\gamma)(\theta+\mu)})$

From $\beta SC^{*}+\left(1-\frac{\omega}{m+\omega}\right)A^{*}C^{*}-(\alpha+\mu+\rho)\,C^{*}=0$

$$C^{*}\left(\beta S^{*}+\left(1-\frac{\omega}{m+\omega}\right)A^{*}-(\alpha+\mu+\rho)\right)=0$$

$\beta S^{*}+\left(1-\frac{\omega}{m+\omega}\right)A^{*}-(\alpha+\mu+\rho)=0$, Since $C^{*}$is positive

$$\frac{\beta\delta\Lambda}{\beta C^* + \mu + \eta} + \left(1 - \frac{\omega}{m+\omega}\right)\left[\frac{\left((1-\delta)\Lambda + \frac{\eta\delta\Lambda}{\beta C^*+\mu+\eta} + \frac{\theta(\alpha p(\mu+\gamma)+\alpha\gamma(1-p)C^*)}{(\mu+\gamma)(\theta+\mu)}\right)}{\mu + \left(1-\frac{\omega}{m+\omega}\right)C^*}\right] - (\alpha + \mu + \rho)C^*$$

$$= 0$$

Let $= \frac{\beta\delta\Lambda}{\beta C+\mu+\eta}$, $g = 1 - \frac{\omega}{m+\omega}$, $k = \frac{(\alpha p(\mu+\gamma)+\alpha\gamma(1-p))}{(\mu+\gamma)(\theta+\mu)}$

$\Rightarrow e + \frac{g\Lambda(1-\delta)}{\mu+gC^*} + \frac{g\delta\Lambda\eta}{(\beta+C^*\mu+\eta)(\mu+g)C^*} + \frac{g\theta k C^*}{\mu+gC^*} = 0$

$$e(\beta C^* + \mu + \eta)(\mu + gC^*) + g\Lambda(1-\delta)(\beta C^* + \mu + \eta) + g\theta k C^*(\beta C^* + \mu + \eta) + g\delta\eta\Lambda - (\alpha + \mu + \rho)(\beta C^* + \mu + \eta)(\mu + g)C^* = 0$$

$\Rightarrow \quad e\beta C^*\mu + e\mu^2 + e\mu\eta + e\beta g C^{*2} + e\mu g + e\eta g C^* + g\beta\Lambda C^* - g\Lambda\beta\delta C^* + g\mu\Lambda - g\Lambda\mu\delta + g\eta\Lambda - g\Lambda\delta\eta + g\theta\beta k C^{*2} + g\theta\mu k C^* + g\theta\eta k C^* + g\eta\delta\Lambda - \alpha\mu\beta C^* - \alpha\mu^2 - \alpha\eta\mu - \mu^2\beta C^* - \mu^2 - \mu^2\eta - \mu\rho\beta C^* - \rho\mu^2 - \rho\eta\mu - \alpha\beta g C^{*2} - \alpha\mu g C^* - \alpha\eta g C^* - \mu\beta\beta g C^{*2} - \mu^2 g C^* - \mu\eta g - \rho\beta g C^{*2} - \rho\mu g C^* - \rho\eta g C^* = 0$

$\Rightarrow \quad C^{*2}(e\beta g + g\theta k\beta - \alpha\beta g - \mu\beta g - \rho\beta g) + C^*(e\beta\mu + e\mu g + e\eta g + g\beta\Lambda + g\theta\mu k + g\theta\eta k - g\Lambda\delta\beta - \mu\alpha\beta - \mu^2\beta - \mu\rho\beta - \mu\alpha g - \alpha\eta g - \mu^2 g - \mu\eta g - \rho\mu g - \rho\eta g) + (e\mu^2 + e\eta\mu + g\mu\Lambda - g\Lambda\delta\mu + g\eta\Lambda - \alpha\mu^2 - \alpha\eta\mu - \mu^2 - \mu^2\eta - \rho\mu^2 - \rho\eta\mu) = 0$

$\Rightarrow aC^{*2} + bC^* + d = 0$

Where

$$a = e\beta g + g\theta k\beta - \alpha\beta g - \mu\beta g - \rho\beta g = \beta g[(e + \theta k) - (\alpha + \mu + \rho)]$$

$$b = e\beta\mu + e\mu g + e\eta g + g\beta\Lambda + g\theta\mu k + g\theta\eta k - (g\Lambda\delta\beta + \mu\alpha\beta + \mu^2\beta + \mu\rho\beta + \mu\alpha g + \alpha\eta g + \mu^2 g + \mu\eta g + \rho\mu g + \rho\eta g)$$

$d = e\mu^2 + e\eta\mu + g\mu\Lambda + g\eta\Lambda - (g\Lambda\delta\mu + \alpha\mu^2 + \alpha\eta\mu + \mu^2 + \mu^2\eta + \rho\mu^2 + \rho\eta\mu)$

$C^* = \frac{-b + \sqrt{b^2 - 4ad}}{2a}$

$$C^* > 0 if \frac{-b + \sqrt{b^2 - 4ad}}{2a} > 0 \Rightarrow -b + \sqrt{b^2 - 4ad} > 0$$

$\Rightarrow \sqrt{b^2 - 4ad} > b$ *then squaring both sides we get,* $b^2 - 4ad > b^2$

$-4ad > 0 \ \Rightarrow ad < 0$ *if and only if,*

casei. if$a > 0$ $and d < 0$ since, $a = \beta g[(e + \theta k) - (\alpha + \mu + \rho)]$ and $d = e\mu^2 + e\eta\mu + g\mu\Lambda + g\eta\Lambda - (g\Lambda\delta\mu + \alpha\mu^2 + \alpha\eta\mu + \mu^2 + \mu^2\eta + \rho\mu^2 + \rho\eta\mu)$

$$\Rightarrow \beta g[(e + \theta k) - (\alpha + \mu + \rho)] > 0$$

and $e\mu^2 + e\eta\mu + g\mu\Lambda + g\eta\Lambda - (g\Lambda\delta\mu + \alpha\mu^2 + \alpha\eta\mu + \mu^2 + \mu^2\eta + \rho\mu^2 + \rho\eta\mu) < 0$

$\Rightarrow e + \theta k > \alpha + \mu + \rho$ and $e\mu^2 + e\eta\mu + g\mu\Lambda + g\eta\Lambda < g\Lambda\delta\mu + \alpha\mu^2 + \alpha\eta\mu + \mu^2 + \mu^2\eta + \rho\mu^2 + \rho\eta\mu$

caseii. if a < 0 and $d > 0$ Since $a = \beta g[(e + \theta k) - (\alpha + \mu + \rho)]$ and $d = e\mu^2 + e\eta\mu + g\mu\Lambda + g\eta\Lambda - (g\Lambda\delta\mu + \alpha\mu^2 + \alpha\eta\mu + \mu^2 + \mu^2\eta + \rho\mu^2 + \rho\eta\mu)$then,

$\Rightarrow \beta g[(e + \theta k) - (\alpha + \mu + \rho)] < 0$

and $e\mu^2 + e\eta\mu + g\mu\Lambda + g\eta\Lambda - (g\Lambda\delta\mu + \alpha\mu^2 + \alpha\eta\mu + \mu^2 + \mu^2\eta + \rho\mu^2 + \rho\eta\mu) > 0$

$\Rightarrow e + \theta k < \alpha + \mu + \rho$ and

$e\mu^2 + e\eta\mu + g\mu\Lambda + g\eta\Lambda > g\Lambda\delta\mu + \alpha\mu^2 + \alpha\eta\mu + \mu^2 + \mu^2\eta + \rho\mu^2 + \rho\eta\mu$

Thus, C^* is positive in the above two cases.

$$S^* = \frac{\delta\Lambda}{\beta C^* + \mu + \eta}$$

$Q^* = \frac{(1-p)\alpha C^*}{\mu+\gamma}$, provided that $0 < p \leq 1$,

$$R^* = \frac{C^*\big(\alpha p(\mu + \gamma) + \gamma\alpha(1 - p)\big)}{(\mu + \gamma)(\theta + \mu)} = \frac{\alpha C^*(\mu p + \gamma)}{(\theta + \mu)(\mu + \gamma)}$$

$$A^* = \frac{(1 - \Lambda)\delta + \eta S^* + \theta R^*}{\mu + (1 - \frac{\omega}{m+\omega})C^*} = \frac{(1 - \Lambda)\delta + \eta\left(\frac{\delta\Lambda}{\beta C^* + \mu + \eta}\right) + \theta\left(\frac{\alpha C^*(\mu p + \gamma)}{(\theta+\mu)(\mu+\gamma)}\right)}{\mu + (1 - \frac{\omega}{m+\omega})C^*}$$

$A^* = \frac{eC^{*2} + fC^* + g}{h}$ Where,

$e = (m + \omega)(\theta\alpha\mu\beta p + \theta\alpha\gamma\beta)$

$$f = (m + \omega)(\theta\alpha\mu^2 p + \theta\alpha\mu\eta p + \theta\alpha\gamma\mu + \delta\beta\theta\mu + \delta\beta\theta\gamma + \beta\mu\gamma\delta + \beta\delta\mu^2 - (\beta\theta\mu\delta\Lambda + \beta\theta\gamma\delta\Lambda + \beta\mu\gamma\delta\Lambda + \beta\mu^2\delta\Lambda)$$

$$g = (m + \omega)(\mu^2\theta\delta + \mu\theta\gamma\delta + \mu^2\gamma\delta + \mu\eta\theta\delta + \eta\theta\gamma\delta + \eta\delta\mu^2 + \eta\mu\gamma\delta - (\mu^2\theta\delta\Lambda + \mu\theta\delta\Lambda + \mu\theta\gamma\delta\Lambda + \mu^2\gamma\delta\Lambda))$$

$h = (mC^* + \mu(m + \omega)(\beta C^* + \mu + \eta)(\theta + \mu)(\mu + \gamma)$

Therefore, the crime persistent equilibrium points are:

$$E^* = (S^*, A^*, C^*, Q^*, R^*)$$

$$E^* = \left(\frac{\delta\Lambda}{\beta C^* + \mu + \eta}, \frac{eC^{*2} + fC^* + g}{h}, C^*, \frac{(1-p)\alpha C^*}{\mu + \gamma}, \frac{\alpha C^*(\mu p + \gamma)}{(\theta + \mu)(\mu + \gamma)}\right)$$

4.4.2 Basic Reproduction Number (R_0)

Rewriting the system of modified mathematical model starting with the criminal compartments for both populations; C, Q and then followed by non-criminal classes,S, A, R, from the two populations, the system of equation gives

$$\frac{dS}{dt} = \delta\Lambda - \beta SC - (\mu + \eta)S$$

$$\frac{dA}{dt} = (1-\delta)\Lambda + \eta S + \theta R - \mu A - \left(1 - \frac{\omega}{m + \omega}\right) AC$$

$$\frac{dC}{dt} = \beta SC + \left(1 - \frac{\omega}{m + \omega}\right) AC - (\alpha + \mu + \rho)C$$

$$\frac{dQ}{dt} = (1-p)\alpha C - (\mu + \gamma)Q$$

$$\frac{dR}{dt} = \alpha pC + \gamma Q - (\theta + \mu)R$$

The method of next generation matrix has been used to show the rate of appearance new criminal in compartments C.

$$f_i = \begin{pmatrix} C \\ Q \end{pmatrix} = \begin{pmatrix} \beta SC + (1 - \frac{\omega}{m + \omega})\mathrm{AC} \\ 0 \end{pmatrix} \tag{4.20}$$

The Jacobian matrix of f of the crime free equilibrium point$E_{0,}$

$$F = \left(\frac{\partial f_i E_0}{\partial \mathrm{x_j}}\right) = \begin{pmatrix} \beta S + (1 - \frac{\omega}{m + \omega})\mathrm{A} & 0 \\ 0 & 0 \end{pmatrix} \tag{4.21}$$

Calculating the transfer of individuals of the compartments of the system 4.20 by all other means.

$v_i = v_i^- - v_i^+$, where v_i^- is the rate of movement out of each compartement and v_i^+ is the rate of change of movement in to each compartement and is calculated as

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} (\alpha + \mu + \rho)C \\ (\mu + \gamma)Q - (1-p)\alpha C \end{pmatrix}$$

The Jacobian matrix of V at the CFE $E_0$is given by

$$V = \left(\frac{\partial \mathrm{f_i(E_0)}}{\partial x_j}\right) = \begin{pmatrix} \alpha + \mu + \rho & 0 \\ -(1-p)\alpha & \mu + \gamma \end{pmatrix}$$

Now let us find the inverse of matrix V, V^{-1}

$$V^{-1} = \frac{AdjV}{|V|}$$

$$Adj(V) = \begin{pmatrix} \mu + \gamma & 0 \\ (1-p)\alpha & \alpha + \mu + \rho \end{pmatrix}$$

$|V| = (\alpha + \mu + \rho)\ (\mu + \gamma)$Then the inverse of V becomes

$$V^{-1} = \frac{AdjV}{|V|}$$

$$\Rightarrow \frac{1}{(\alpha + \mu + \rho)(\mu + \gamma)} \begin{pmatrix} \mu + \gamma & 0 \\ (1-p)\alpha & \alpha + \mu + \rho \end{pmatrix}$$

$$V^{-1} = \begin{pmatrix} \dfrac{1}{\alpha + \mu + \rho} & 0 \\ \dfrac{(1-p)\alpha}{\alpha + \mu + \rho)\ (\mu + \gamma)} & \dfrac{1}{\mu + \gamma} \end{pmatrix}$$

We calculate the product of the matrices

$$FV^{-1} = \begin{pmatrix} \beta S + (\ 1 - \dfrac{\omega}{m + \omega})\mathrm{A} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \dfrac{1}{\alpha + \mu + \rho} & 0 \\ \dfrac{(1-p)\alpha}{\alpha + \mu + \rho)\ (\mu + \gamma} & \dfrac{1}{\mu + \gamma} \end{pmatrix}$$

$$FV^{-1} = \begin{pmatrix} \dfrac{\beta S + \left(1 - \frac{\omega}{m+\omega}\right)A}{\alpha + \mu + \rho} & 0 \\ 0 & 0 \end{pmatrix}$$

The reproduction number R_0 is the dominant eigenvalues of or the FV^{-1}spectral radius ofFV^{-1} that is $R_0 = \rho(FV^{-1})$ hence ,

$R_0 = \frac{\beta S\ \ + \left(1 - \frac{\omega}{m+\omega}\right)A}{\alpha + \mu + \rho}$Substituting $A = \frac{(1-\delta)\ \Lambda(\mu+\eta) + \eta\Lambda\delta}{\mu(\mu+\eta)}$and $S = \frac{\Lambda\delta}{(\mu+\eta)}$ then R_0 becomes,

$$R_0 = \frac{\mu\beta\delta\Lambda(m + \omega) + m\Lambda(\mu + \eta - \delta\mu)}{(\mu + \eta)(m + \omega)(\alpha + \mu + \rho)\mu} \qquad (4.22)$$

4.5 Stability Analysis

In this section we briefly describe the existence and stability of the equilibrium points. According to linear stability analysis, an equilibrium state is stable if the eigenvalues of the Jacobian at the equilibrium point have negative real parts. We will discuss the stability behavior of the equilibria E_0 and E^* in detail. Local stability of the crime free equilibrium point The local stability of the crime-free equilibrium can be discussed by examining the linearized form of the system equation $(4.4 - 4.8)$ at the steady state.

4.5.1 Local stability of crime free equilibrium point

Theorem4.4. The equilibrium solution E_0 of the non linear system of equation $(4.4 - 4.8)$ is locally asymptotically stable if R_0< 1 and unstable ifR_0>1 .

Proof. To determine the stability of the crime-free equilibrium point, first construct the Jacobian matrix of the model system.

We recall that the system has the form

$$\frac{dS}{dt} = \delta\Lambda - \beta SC - (\mu + \eta)S$$

$$\frac{dA}{dt} = (1-\delta)\Lambda + \eta S + \theta R - \mu A - (1 - \frac{\omega}{m+\omega})AC$$

$$\frac{dC}{dt} = \beta SC + \left(1 - \frac{\omega}{m+\omega}\right)AC - (\alpha + \mu + \rho)C$$

$$\frac{dQ}{dt} = (1-p)\alpha C - (\mu + \gamma)Q$$

$$\frac{dR}{dt} = \alpha p C + \gamma Q - (\theta + \mu)R$$

And the crime free crime equilibrium point as we discussed before is

$$E_0 = (S_0, A_0, C_0, Q_0, R_0)$$

$$\Rightarrow E_0 = \left(\frac{\Lambda\delta}{(\mu + \eta)}, \frac{(1-\delta)\Lambda(\mu+\eta) + \eta\Lambda\delta}{\mu(\mu+\eta)}, 0,0,0\right)$$

The Jacobian matrix is given by

$$J(S,A,C,Q,R)=\begin{pmatrix} \frac{\partial f_1}{\partial S} & \frac{\partial f_1}{\partial A} & \frac{\partial f_1}{\partial C} & \frac{\partial f_1}{\partial Q} & \frac{\partial f_1}{\partial R} \\ \frac{\partial f_2}{\partial S} & \frac{\partial f_2}{\partial A} & \frac{\partial f_2}{\partial C} & \frac{\partial f_2}{\partial Q} & \frac{\partial f_2}{\partial R} \\ \frac{\partial f_3}{\partial S} & \frac{\partial f_3}{\partial A} & \frac{\partial f_3}{\partial C} & \frac{\partial f_3}{\partial Q} & \frac{\partial f_3}{\partial R} \\ \frac{\partial f_4}{\partial S} & \frac{\partial f_4}{\partial A} & \frac{\partial f_4}{\partial C} & \frac{\partial f_4}{\partial Q} & \frac{\partial f_4}{\partial R} \\ \frac{\partial f_5}{\partial S} & \frac{\partial f_5}{\partial A} & \frac{\partial f_5}{\partial C} & \frac{\partial f_5}{\partial Q} & \frac{\partial f_5}{\partial R} \end{pmatrix}$$

Where $f_1(S,A,C,Q,R)=\frac{\mathrm{dS}}{\mathrm{dt}}=\delta\Lambda-\beta SC-(\mu+\eta)S$

$$f_2(S,A,C,Q,R)=\frac{dA}{dt}=(1-\delta)\Lambda+\ \eta S+\theta R-\mu A-\left(1-\frac{\omega}{m+\omega}\right)AC$$

$$f_3(S,A,C,Q,R)=\frac{dC}{dt}=\beta SC+\left(1-\frac{\omega}{m+}\right)AC-(\alpha+\mu+\rho)C$$

$$f_4(S,A,C,Q,R)=\frac{dQ}{dt}=(1-p)\alpha C-(\mu+\gamma)Q$$

$$f_5(S,A,C,Q,R)=\frac{dR}{dt}=\alpha pC+\gamma Q-(\theta+\mu)R$$

The Jacobian matrix of the system is given by

J(S,A,C,Q,R)=

$$\begin{bmatrix} -\beta C-\mu-\eta & 0 & -\beta S & 0 & 0 \\ \eta & -\mu-\left(1-\frac{\omega}{m+\omega}\right)C & -\left(1-\frac{\omega}{m+\omega}\right)A & 0 & \theta \\ \beta C & \left(1-\frac{\omega}{m+\omega}\right)C & \beta S+\left(1-\frac{\omega}{m+\omega}\right)A-(\alpha+\mu+\rho) & 0 & 0 \\ 0 & 0 & (1-p)\alpha & -\mu-\gamma & 0 \\ 0 & 0 & \alpha p & \gamma & -\mu-\theta \end{bmatrix}$$

Now Jacobian matrix at the equilibrium point

$E_0=(\frac{\Lambda\delta}{\mu+\eta},\frac{(1-\delta)(\mu+\eta)+\eta\delta\Lambda}{\mu(\mu+\eta)},0,0,0)$ is given by

$J(E_0)$=

$$\begin{bmatrix} -(\mu+\eta) & 0 & -\beta S & 0 & 0 \\ \eta & -\mu & -(1-\frac{\omega}{m+\omega})A & 0 & \theta \\ 0 & 0 & \beta S+(1-\frac{\omega}{m+\omega})A-(\alpha+\mu+\rho) & 0 & 0 \\ 0 & 0 & (1-p)\alpha & -(\mu+\gamma) & 0 \\ 0 & 0 & \alpha p & \gamma & -(\mu+\theta) \end{bmatrix}$$

$$J(E_0) = \begin{bmatrix} -k & 0 & -\beta S & 0 & 0 \\ \eta & -h & -(1-\frac{\omega}{m+\omega})A & 0 & \theta \\ 0 & 0 & f & 0 & 0 \\ 0 & 0 & (1-p)\alpha & -m & 0 \\ 0 & 0 & \alpha p & \gamma & -n \end{bmatrix}$$

Where $k = \mu + \eta,\ h = \mu, f = \frac{\beta(\Lambda\delta)}{\mu+\eta} + \left(1 - \frac{\omega}{m+\omega}\right)\frac{(1-\delta)\Lambda(\mu+\eta)\eta\delta\Lambda}{\mu(\mu+\eta)} - (\alpha+\mu+\rho),\ m = \mu + \gamma$, $n = \mu + \theta$

The characteristics equation of the Jacobian matrix is defined by

$$|J(E_0) - \lambda I| = 0$$

Here I is 5 by 5 identity matrix and λ is the eigenvalues

$$|J(E_0) - \lambda I| = \begin{vmatrix} -k-\lambda & 0 & -\beta S & 0 & 0 \\ \eta & -h-\lambda & -(1-\frac{\omega}{m+\omega})A & 0 & \theta \\ 0 & 0 & f-\lambda & 0 & 0 \\ 0 & 0 & (1-p)\alpha & -m-\lambda & 0 \\ 0 & 0 & \alpha p & \gamma & -n-\lambda \end{vmatrix}$$

Computing the determinant, we obtain the eigenvalues of the Jacobian matrix as

$$\lambda_i = \begin{pmatrix} -k \\ -h \\ f \\ -m \\ -n \end{pmatrix} = \begin{pmatrix} \mu+\eta \\ \mu \\ \frac{\beta(\Lambda\delta)}{\mu+\eta} + \left(1 - \frac{\omega}{m+\omega}\right)\frac{(1-\delta)\Lambda(\mu+\eta)\eta\delta\Lambda}{\mu(\mu+\eta)} - (\alpha+\mu+\rho) \\ (\mu+\gamma) \\ \mu+\theta \end{pmatrix}$$

For $= 1,2,3,4,5$. $\lambda_i < 0 for\ i = 1,2,4,5$

Now all eigenvalue become negative and the system $(4.4-4.8)$is stable if we are able to show λ_3 is negative. But $\lambda_3 = \frac{\beta(\Lambda\delta)}{\mu+\eta} + \left(1 - \frac{\omega}{m+\omega}\right)\frac{(1-\delta)\Lambda(\mu+\eta)\eta\delta\Lambda}{\mu(\mu+\eta)} - (\alpha+\mu+\rho)$

Now $\lambda_3 < 0$ if $\frac{\beta(\Lambda\delta)}{\mu+\eta} + \left(1 - \frac{\omega}{m+\omega}\right)\frac{(1-\delta)\Lambda(\mu+\eta)\eta\delta\Lambda}{\mu(\mu+\eta)} < (\alpha+\mu+\rho)$ now dividing both sides by $(\alpha+\mu+\rho)$ we get $\frac{\mu\beta\delta\Lambda(m+\omega)+m\Lambda(\mu+\eta-\delta\mu)}{(\mu+\eta)(m+\omega)(\alpha+\mu+\rho)\mu} < 1$ but $\frac{\mu\beta\delta\Lambda(m+\omega)+m\Lambda(\mu+\eta-\delta\mu)}{(\mu+\eta)(m+\omega)(\alpha+\mu+\rho)\mu} = R_0$

Finally, we get $R_0 < 1$.

This shows that the crime free equilibrium point is locally asymptotically stable when basic reproduction number $R_0 < 1$ and unstable when $R_0 > 1$.

4.5.2 Global Stability of the Crime Free Equilibrium Point

To investigate the global stability of the Crime free equilibrium point (E_0) the global stability can be described the solution behavior in the whole domain that is in the region ρ from (4.4-4.8). We use the Castillo-Chaves theorem to prove the global stability disease free equilibrium point.

Theorem 4.5 For $R_0 < 1$ the crime free equilibrium point E_0 of the system (4.4-4.8) is globally asymptotically stable if $S_0 \geq S$.

Prove: let us write our model system (4.4-4.8) as

$\frac{\mathrm{dZ_1}}{\mathrm{dt}} = \mathrm{F(Z_1, Z_2)}$,

$\frac{\mathrm{dZ_2}}{\mathrm{dt}} = \mathrm{G(Z_1, Z_2)}$and

Where $\mathrm{Z_1} = (\mathrm{S, A, R}) \epsilon \mathrm{R^3}$ represent the class of non-criminal individuals,

$\mathrm{Z_2} = (\mathrm{C, Q}\ )\epsilon R^2$represent the class of Criminal individuals.

The disease free equilibrium of the model is denoted by $E_0 = ({Z_1}^*, 0)$ where

${Z_1}^* = (\frac{\Lambda\delta}{\mu+\eta}, \frac{(1-\delta)(\mu+\eta)+\eta\delta\Lambda}{\mu(\mu+\eta)}, 0)$Since the crime free equilibrium point is locally asymptotically stable, to prove global stability we will apply Castillo-Chaves theorem.

We have $\frac{dZ_1}{dt} = F(Z_1, Z_2) = \begin{bmatrix} \delta\Lambda - \beta SC - (\mu+\eta)S \\ (1-\delta)\Lambda + \ \eta S + \theta R - \mu A - \left(1 - \frac{\omega}{m+\omega}\right) AC \\ \alpha pC + \gamma Q - (\theta+\mu)R \end{bmatrix}$

$$\frac{dZ_2}{dt} = G(Z_1, Z_2) = \begin{bmatrix} \beta SC + \left(1 - \frac{\omega}{m+}\right) AC - (\alpha+\mu+\rho)C \\ (1-p)\alpha C - (\mu+\gamma)Q \end{bmatrix}$$

Firstly, to show ${Z_1}^*$is globally asymptotically stable for the system $\frac{dZ_1}{dt} = \ F(Z_1, 0)$

Let us consider the reduced system

$$\frac{dZ_1}{dt} = \ F(Z_1, 0) = \begin{bmatrix} \delta\Lambda - (\mu+\eta)S \\ (1-\delta)\Lambda + \ \eta S + \theta R - \mu A \\ -(\theta+\mu)R \end{bmatrix}$$

We can rewrite this system as,

$$\frac{dS}{dt} = \delta\Lambda - (\mu+\eta)S$$

$$\frac{dA}{dt} = (1-\delta)\Lambda + \ \eta S + \theta R - \mu A$$

$$\frac{dR}{dt} = -(\theta+\mu)R$$

And this system is a non-homogeneous linear system of ordinary differential equations. Then let us find the solution for them.

$$\frac{dS}{dt} = \delta\Lambda - (\mu+\eta)S$$

$$\frac{dS}{dt} + (\mu+\eta)S = \delta\Lambda$$

$$S(t) = e^{-(\mu+\eta)t}(\int \delta\Lambda\, e^{(\mu+\eta)t} dt + C)$$

$$S(t) = \frac{\delta\Lambda}{(\mu+\eta)} + Ce^{-(\mu+\eta)t}$$

$$S(0) = \frac{\delta\Lambda}{(\mu+\eta)} + C \Rightarrow C = S(0) - \frac{\delta\Lambda}{(\mu+\eta)}$$

$$S(t) = \frac{\delta\Lambda}{(\mu+\eta)} + \left(S(0) - \frac{\delta\Lambda}{(\mu+\eta)}\right) e^{-(\mu+\eta)t}$$

As $t \to \infty, S(t) \to \frac{\delta\Lambda}{(\mu+\eta)} = S_0$

Similarly, $\frac{dR}{dt} = -(\theta+\mu)R$

$$\frac{dR}{dt} + (\theta+\mu)R = 0$$

$$R(t) = e^{-(\theta+\mu)t}(\int 0e^{(\theta+\mu)t}dt + C)$$

$$R(t) = Ce^{-(\theta+\mu)t} \Rightarrow R(0) = C \Rightarrow R(t) = R(0)e^{-(\theta+\mu)t}$$

As $t \to \infty,\ R(t) \to 0 = R_0$

Similarly,

$$\frac{dA}{dt} = (1-\delta)\Lambda + \eta S + \theta R - \mu A$$

$$\frac{dA}{dt} + \mu A = (1-\delta)\Lambda + \frac{\eta\delta\Lambda}{\mu+\eta}$$

$$A(t) = e^{-\mu t}[\int e^{\mu t}\left((1-\delta)\Lambda + \frac{\eta\delta\Lambda}{\mu+\eta}\right)dt + C]$$

$$A(t) = e^{-\mu t}[\frac{e^{\mu t}}{\mu}\left(\Lambda(1-\delta) + \frac{\eta\delta\Lambda}{\mu+\eta}\right) + C]$$

$$A(t) = \frac{1}{\mu}\left[\Lambda(1-\delta) + \frac{\eta\delta\Lambda}{\mu+\eta}\right] + Ce^{-\mu t}$$

$$A(0) = \frac{(1-\delta)\Lambda(\mu+\eta)\eta\delta\Lambda}{\mu(\mu+\eta)} + C \text{ , then } C = A(0) - \frac{(1-\delta)\Lambda(\mu+\eta)+\eta\delta\Lambda}{\mu(\mu+\eta)}$$

$$A(t) = \frac{(1-\delta)\Lambda(\mu+\eta)+\eta\delta\Lambda}{\mu(\mu+\eta)} + e^{-\mu t}(A(0) - \frac{(1-\delta)\Lambda(\mu+\eta)+\eta\delta\Lambda}{\mu(\mu+\eta)})$$

$$\text{As } t \to \infty \text{ , } A(t) \to \frac{(1-\delta)\Lambda(\mu+\eta)+\eta\delta\Lambda}{\mu(\mu+\eta)} = A_0$$

Hence $\big(S(t), A(t), R(t)\big) \to \left(\frac{\delta\Lambda}{(\mu+\eta)}, \frac{(1-\delta)\Lambda(\mu+\eta)+\eta\delta\Lambda}{\mu(\mu+\eta)}, 0\right) = Z_1^*$

Therefore, Z_1^* is globally asymptotically stable for the system $\frac{dZ_1}{dt} = F(Z_1, 0)$

Secondly, we will show that $G(Z_1, Z_2) = AZ_2 - \tilde{G}(Z_1, Z_2)$ and $\tilde{G}(Z_1, Z_2) \geq 0$ for $(Z_1, Z_2) \epsilon\, \Omega$, where $A = \frac{dG}{dZ_2}({Z_1}^*, 0)$ is Metzler matrix (the diagonal element of A are non-negative) and Ω is a region where the model makes biological sense.

Consider matrix $A = \frac{\partial G}{\partial(C,Q)}({Z_1}^*, 0) = \begin{bmatrix} \beta S^* + \left(1 + \frac{\omega}{m+\omega}\right) - (\alpha+\mu+\rho) & 0 \\ (1-p)\alpha & -(\mu+\gamma) \end{bmatrix}$

Hence A is a Metzler matrix (off diagonal element are non-negative)

Hence $\tilde{G}(Z_1, Z_2) = AZ_2 - G(Z_1, Z_2)$

$$\tilde{G}(Z_1, Z_2) = \begin{bmatrix} \beta S_0 + \left(1 + \dfrac{\omega}{m+\omega}\right) - (\alpha+\mu+\rho) & 0 \\ (1-p)\alpha & -(\mu+\gamma) \end{bmatrix} \begin{bmatrix} C \\ Q \end{bmatrix} - \begin{bmatrix} \beta SC + \left(1 - \dfrac{\omega}{m+}\right) AC - (\alpha+\mu+\rho)C \\ (1-p)\alpha C - (\mu+\gamma)Q \end{bmatrix}$$

$$= \begin{bmatrix} \beta S_0 C + \left(1 + \dfrac{\omega}{m+\omega}\right) C - (\alpha+\mu+\rho)C \\ (1-p)\alpha C - (\mu+\gamma)Q \end{bmatrix} - \begin{bmatrix} \beta SC + \left(1 - \dfrac{\omega}{m+}\right) AC - (\alpha+\mu+\rho)C \\ (1-p)\alpha C - (\mu+\gamma)Q \end{bmatrix}$$

$$\tilde{G}(Z_1, Z_2) = \begin{bmatrix} \beta S_0 C - \beta SC \\ 0 \end{bmatrix} = \begin{bmatrix} \beta C(S_0 - S) \\ 0 \end{bmatrix} = (S_0 - S)\begin{bmatrix} \beta C \\ 0 \end{bmatrix} \geq 0$$

$(S_0 - S) \geq 0$ or $\begin{bmatrix} \beta C \\ 0 \end{bmatrix} \geq 0 \Rightarrow S_0 \geq S$

Which implies that $\tilde{G}(Z_1, Z_2) \geq 0$ for all$(Z_1, Z_2)\epsilon\Omega$, thus, the two condition of Castillo-Chaves theorem is satisfied. Therefore, the crime free equilibrium E_0 of the system (4.4-4.8) is globally asymptotically stable $R_0 < 1$.

4.5.3 Local stability of crime persistent equilibrium point

In this section we discuss the local stability of the crime persistent equilibrium point by stating in the form of a theorem.

Theorem 4.6: If $R_0 > 1$ then the persistent equilibrium point is locally asymptotically stable.

Proof: The stability of persistent equilibrium point is then determined based on the sign of the eigenvalues of the Jacobian matrix which is computed at the persistent equilibrium,

$$E^* = (S^*, A^*, C^*, Q^*, R^*)$$

Now consider system (4.4-4.8). The Jacobian matrix of the system at E^* is given by:

$$J(\mathrm{E}^*) = \begin{pmatrix} \frac{\partial f_1}{\partial S} & \frac{\partial f_1}{\partial A} & \frac{\partial f_1}{\partial C} & \frac{\partial f_1}{\partial Q} & \frac{\partial f_1}{\partial R} \\ \frac{\partial f_2}{\partial S} & \frac{\partial f_2}{\partial A} & \frac{\partial f_2}{\partial C} & \frac{\partial f_2}{\partial Q} & \frac{\partial f_2}{\partial R} \\ \frac{\partial f_3}{\partial S} & \frac{\partial f_3}{\partial A} & \frac{\partial f_3}{\partial C} & \frac{\partial f_3}{\partial Q} & \frac{\partial f_3}{\partial R} \\ \frac{\partial f_4}{\partial S} & \frac{\partial f_4}{\partial A} & \frac{\partial f_4}{\partial C} & \frac{\partial f_4}{\partial Q} & \frac{\partial f_4}{\partial R} \\ \frac{\partial f_5}{\partial S} & \frac{\partial f_5}{\partial A} & \frac{\partial f_5}{\partial C} & \frac{\partial f_5}{\partial Q} & \frac{\partial f_5}{\partial R} \end{pmatrix}$$

$$J(E^*) = \begin{bmatrix} -\beta C - \mu - \eta & 0 & -\beta S & 0 & 0 \\ \eta & -\mu - \left(1 - \frac{\omega}{m+\omega}\right)C & -\left(1 - \frac{\omega}{m+\omega}\right)A & 0 & \theta \\ \beta C & \left(1 - \frac{\omega}{m+\omega}\right)C & \beta S + \left(1 - \frac{\omega}{m+\omega}\right)A - (\alpha + \mu + \rho) & 0 & 0 \\ 0 & 0 & (1-p)\alpha & -\mu - \gamma & 0 \\ 0 & 0 & \alpha p & \gamma & -\mu - \theta \end{bmatrix}$$

$$=\begin{bmatrix} -(\beta C+\mu+\eta) & 0 & -\beta S & 0 & 0 \\ \eta & -\mu-\left(1-\frac{\omega}{m+\omega}\right)C & -\left(1-\frac{\omega}{m+\omega}\right)A & 0 & \theta \\ \beta C & \left(1-\frac{\omega}{m+\omega}\right)C & \beta S+\left(1-\frac{\omega}{m+\omega}\right)A-(\alpha+\mu+\rho) & 0 & 0 \\ 0 & 0 & (1-p)\alpha & -(\mu+\gamma) & 0 \\ 0 & 0 & \alpha p & \gamma & -(\mu+\theta) \end{bmatrix}$$

$$J(E^*) = \begin{bmatrix} -k & 0 & -\beta S^* & 0 & 0 \\ \eta & -h & -g & 0 & \theta \\ \beta C^* & j & f & 0 & 0 \\ 0 & 0 & (1-p)\alpha & -m & 0 \\ 0 & 0 & \alpha p & \gamma & -n \end{bmatrix}$$

Where $k = \beta C^* + \mu + \eta$, $h = \mu + \left(1 - \frac{\omega}{m+\omega}\right) C^*, f = \beta S^* + \left(1 - \frac{\omega}{m+\omega}\right) A^* - (\alpha + \mu + \rho), m = \mu + \gamma, n = \mu + \theta, g = \left(1 - \frac{\omega}{m+\omega}\right) A^*, j = (1 - \frac{\omega}{m+\omega})C^*$

The determinant polynomial equation of $J(E^*)$ is $|J(E^* - \lambda I)| = 0$ where I is 5×5 identity matrix and λ is eigenvalues of Jacobian matrix. Thus, we have

$$detJ(S^*, A^*, C^*, Q^*, R^*) = 0$$

$$\begin{vmatrix} -k-\lambda & 0 & -\beta S^* & 0 & 0 \\ \eta & h-\lambda & -g & 0 & \theta \\ \beta C^* & j & f-\lambda & 0 & 0 \\ 0 & 0 & (1-p)\alpha & -m-\lambda & 0 \\ 0 & 0 & \alpha p & \gamma & -n-\lambda \end{vmatrix} = 0$$

$DVVVNBNBNGFF$

$$\Rightarrow (-k-\lambda)\begin{vmatrix} h-\lambda & -g & 0 & \theta \\ j & f-\lambda & 0 & 0 \\ 0 & (1-p)\alpha & -m-\lambda & 0 \\ 0 & \alpha p & \gamma & -n-\lambda \end{vmatrix} - \beta S^* \begin{vmatrix} \eta & -h-\lambda & 0 & \theta \\ \beta C^* & j & 0 & 0 \\ 0 & 0 & -m-\lambda & 0 \\ 0 & 0 & \gamma & -n-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (-k-\lambda)(-h-\lambda)\begin{vmatrix} f-\lambda & 0 & 0 \\ (1-p)\alpha & -m-\lambda & 0 \\ \alpha p & \gamma & -m-\lambda \end{vmatrix}$$

$$-j(-k-\lambda)\begin{vmatrix} -g & 0 & \theta \\ (1-p)\alpha & -m-\lambda & 0 \\ \alpha p & \gamma & -n-\lambda \end{vmatrix} - \beta S^*\eta \begin{vmatrix} j & 0 & 0 \\ 0 & -m-\lambda & 0 \\ 0 & \gamma & -n-\lambda \end{vmatrix}$$

$$+\beta^2 S^* C^* \begin{vmatrix} -h-\lambda & 0 & \theta \\ 0 & -m-\lambda & 0 \\ 0 & \gamma & -n-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (-k-\lambda)(-h-\lambda)(f-\lambda)\begin{vmatrix} -m-\lambda & 0 \\ \gamma & -n-\lambda \end{vmatrix} + jg(-k-\lambda)\begin{vmatrix} -m-\lambda & 0 \\ \gamma & -n-\lambda \end{vmatrix}$$

$$-j\theta(-k-\lambda)\begin{vmatrix} (1-p)\alpha & -m-\lambda \\ \alpha p & \gamma \end{vmatrix} - \beta S^*\eta j \begin{vmatrix} -m-\lambda & 0 \\ \gamma & -n-\lambda \end{vmatrix}$$

$$+\beta^2 S^* C^*(-h-\lambda)\begin{vmatrix} -m-\lambda & 0 \\ \gamma & -n-\lambda \end{vmatrix} = 0$$

$$\begin{aligned}\Rightarrow & (-k-\lambda)(-h-\lambda)(f-\lambda)(-m-\lambda)(-n-\lambda) + jg(-k-\lambda)(-m-\lambda)(-n-\lambda) \\ & - j\theta(-k-\lambda)(1-p)\alpha\gamma + (-k-\lambda)j\theta\alpha p(-m-\lambda) \\ & - \beta S^*\eta j(-m-\lambda)(-n-\lambda) + \beta^2 S^* C^*(-h-\lambda)(-m-\lambda)(-n-\lambda) = 0\end{aligned}$$

$$\begin{aligned}\Rightarrow & (kh + k\lambda + h\lambda + \lambda^2)(mnf + mf\lambda + nf\lambda + f\lambda^2 - mn\lambda - m\lambda^2 - n\lambda^2 - \lambda^3) - kmnjg \\ & - kmjg\lambda - knjg\lambda - kjg\lambda^2 - mnjg\lambda - mjg\lambda^2 - njg\lambda^2 - jg\lambda^3 + j\theta k\alpha\gamma \\ & - j\theta k\alpha p\gamma + j\theta\alpha\gamma\lambda - j\theta\alpha p\gamma\lambda + j\theta\alpha pkm + j\theta\alpha pk\lambda + j\theta\alpha pm\lambda + j\theta\alpha p\lambda^2 \\ & - \beta S^*\eta jmn - \beta S^*\eta jm\lambda - \beta S^*\eta jn\lambda - \beta S^*\eta j\lambda^2 - \beta^2 S^* C^* hmn - \beta^2 S^* C^* hm\lambda \\ & - \beta^2 S^* C^* hn\lambda - \beta^2 S^* C^* h\lambda^2 - \beta^2 S^* C^* mn\lambda - \beta^2 S^* C^* m\lambda^2 - \beta^2 S^* C^* n\lambda^2 \\ & - \beta^2 S^* C^* \lambda^3 = 0\end{aligned}$$

$$\Rightarrow khmnf + khmf\lambda + khnf\lambda + khf\lambda^2 - khmn\lambda - khm\lambda^2 - khn\lambda^2 - kh\lambda^3 + kmnf\lambda + kmf\lambda^2 + knf\lambda^2 + kf\lambda^3 - kmn\lambda^2 - km\lambda^3 - kn\lambda^3 - k\lambda^4 + kmnf\lambda + hmf\lambda^2 + hnf\lambda^2 + hf\lambda^3 - hmn\lambda^2 - hm\lambda^3 - hn\lambda^3 - h\lambda^4 + mnf\lambda^2 + mf\lambda^3 + nf\lambda^3 + f\lambda^4 - mn\lambda^3 - m\lambda^4 - n\lambda^4 - \lambda^5 - kmnjg - kmjg\lambda - knjg\lambda - kjg\lambda^2 - mnjg\lambda - mjg\lambda^2 - njg\lambda^2 - jg\lambda^3 + j\theta k\alpha\gamma - j\theta k\alpha p\gamma + j\theta\alpha\gamma\lambda - j\theta\alpha p\gamma\lambda + j\theta\alpha pkm + j\theta\alpha pk\lambda + j\theta\alpha pm\lambda + j\theta\alpha p\lambda^2 - \beta S^*\eta jmn - \beta S^*\eta jm\lambda - \beta S^*\eta jn\lambda - \beta S^*\eta j\lambda^2 - \beta^2 S^*C^*hmn - \beta^2 S^*C^*hm\lambda - \beta^2 S^*C^*hn\lambda - \beta^2 S^*C^*h\lambda^2 - \beta^2 S^*C^*mn\lambda - \beta^2 S^*C^*m\lambda^2 - \beta^2 S^*C^*n\lambda^2 - \beta^2 S^*C^*\lambda^3 = 0$$

$$\Rightarrow -\lambda^5 + (-k - h + f - m - n)\lambda^4 + (kf + hf + mf + nf - kh - km - kn - hm - hn - mn - jg - \beta^2 S^*C^*)\lambda^3 + (khf + kmf + knf + hmf + hnf + mnf + j\theta\alpha p - khm - khn - kmn - hmn - kjg - mjg - njg - \beta S^*\eta j - \beta^2 S^*C^*h - \beta^2 S^*C^*m - \beta^2 S^*C^*n)\lambda^2 + (khmf + khnf + kmnf + hmnf + j\theta\alpha\gamma + j\theta\alpha pk + j\theta\alpha pm - khmn - kmjg - knjg - mnjg - j\theta\alpha p\gamma - \beta S^*\eta jm - \beta S^*\eta jn - \beta^2 S^*C^*hm - \beta^2 S^*C^*hn - \beta^2 S^*C^*mn)\lambda + khmnf + j\theta k\gamma\alpha + j\theta\alpha pkm - kmnjg - j\theta k\alpha p\gamma - \beta S^*\eta jmn - \beta^2 S^*C^*hmn = 0$$

Multiplying both sides by negative one then we have the polynomial of degree 5 of the form

$$P(\lambda) = a_5\lambda^5 + a_4\lambda^4 + a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 \qquad 4.25$$

Where,

$a_5 = 1,$

$a_4 = k + h + m + n - f,$

$a_3 = kh + km + kn + hm + hn + mn + jg + \beta^2 S^*C^* - kf - hf - mf - nf,$

$a_2 = khm + khn + kmn + hmn + kjg + mjg + njg + \beta S^*\eta j^* + \beta^2 S^*C^*h + \beta^2 S^*C^*m + \beta^2 S^*C^*n - khf - kmf - knf - hmf - hnf - mnf - j\theta\alpha p,$

$$a_1 = khmn + kmjg + knjg + mnjg + j\theta\alpha p\gamma + \beta S^*\eta jm + \beta S^*\eta jn + \beta^2 S^* C^* hm + \beta^2 S^* C^* hn + \beta^2 S^* C^* mn - khmf - khnf - kmnf - hmnf - j\theta\alpha\gamma - j\theta\alpha pk - j\theta\alpha pm,$$

$$a_0 = kmnjg + j\theta k\alpha p\gamma + \beta S^*\eta jmn + \beta^2 S^* C^* hmn - khmnf - j\theta k\alpha\gamma - j\theta\alpha pkm$$

Solving the characteristic polynomial 4.25 for eigenvalues λ is very difficult, but for the stability we need to find the sign of the eigenvalues.We use the Routh-Hurwith criterion to determine whether all roots are negative or have negative real parts. The coefficients of characteristics polynomial equation $a_0, a_1, a_2, a_3, a_4, a_5$ are real positive. Thus, the necessary condition for stability of equilibrium point is satisfied. From the sufficient condition for stability of the system, the Hurwitz array for the characteristic polynomial is presented as follows:

$$\begin{array}{c|ccc}
\lambda^5 & a_5 & a_3 & a_1 \\
\lambda^4 & a_4 & a_2 & a_0 \\
\lambda^3 & b_1 & b_2 & b_3 \\
\lambda^2 & c_1 & c_2 & c_3 \\
\lambda^1 & d_1 & d_2 & d_3 \\
\lambda^0 & e_1 & e_2 & e_3
\end{array}$$

Where $a_0, a_1, a_2, a_3, a_4, a_5$ are characteristics polynomial coefficient and the elements in the rest of the array are computed using the following ways.

$$b_1 = \frac{-1}{a_4}\begin{vmatrix} a_5 & a_3 \\ a_4 & a_2 \end{vmatrix} = \frac{a_3 a_4 - a_2}{a_4} > 0 \text{ where } a_3 a_4 > a_2 \text{ and} a_4 > 0 \quad \text{since } a_5 = 1$$

$$b_2 = \frac{-1}{a_4}\begin{vmatrix} a_5 & a_1 \\ a_4 & a_0 \end{vmatrix} = \frac{a_1 a_4 - a_0}{a_4} = \frac{a_1 a_4 - a_0}{a_4} > 0, \text{ where } a_1 a_4 > a_0 \text{ and } a_0 > 0$$

$$b_3 = \frac{-1}{a_4}\begin{vmatrix} a_5 & 0 \\ a_4 & 0 \end{vmatrix} = 0,$$

$$c_1 = \frac{-1}{b_1}\begin{vmatrix} a_4 & a_2 \\ b_1 & b_2 \end{vmatrix} = \frac{b_1 a_2 - a_4 b_2}{b_1} > 0 \text{Where} b_1 a_2 > a_4 b_2$$

$$c_2 = \frac{-1}{b_1}\begin{vmatrix} a_4 & a_0 \\ b_1 & b_3 \end{vmatrix} = \frac{b_1 a_0 - a_4 b_3}{b_2} = a_0\ , \quad c_3 = \frac{-1}{b_1}\begin{vmatrix} a_4 & 0 \\ b_1 & 0 \end{vmatrix} = 0,$$

$\mathrm{d}_1 = \frac{-1}{\mathrm{c}_1}\begin{vmatrix} \mathrm{b}_1 & \mathrm{b}_2 \\ \mathrm{c}_1 & \mathrm{c}_2 \end{vmatrix} = \frac{\mathrm{c}_1\mathrm{b}_2 - \mathrm{b}_1\mathrm{c}_2}{\mathrm{c}_1} > 0$Where, $\mathrm{c}_1\mathrm{b}_2 > \mathrm{b}_1\mathrm{c}_2$,

$\mathrm{d}_2 = \frac{-1}{\mathrm{c}_1}\begin{vmatrix} \mathrm{b}_1 & \mathrm{b}_3 \\ \mathrm{c}_1 & \mathrm{c}_3 \end{vmatrix} = \frac{\mathrm{c}_1\mathrm{b}_3 - \mathrm{b}_1\mathrm{c}_3}{\mathrm{c}_1} = 0$ Where, $\mathrm{c}_1\mathrm{b}_3 > \mathrm{b}_1\mathrm{c}_3$,

$$\mathrm{d}_3 = \frac{-1}{\mathrm{c}_1}\begin{vmatrix} \mathrm{b}_1 & 0 \\ \mathrm{c}_1 & 0 \end{vmatrix} = 0, \mathrm{e}_1 = \frac{-1}{\mathrm{d}_1}\begin{vmatrix} \mathrm{c}_1 & \mathrm{c}_2 \\ \mathrm{d}_1 & \mathrm{d}_2 \end{vmatrix} = \mathrm{a}_0 > 0,$$

$$\mathrm{e}_2 = \frac{-1}{\mathrm{d}_1}\begin{vmatrix} \mathrm{c}_1 & \mathrm{c}_3 \\ \mathrm{d}_1 & \mathrm{d}_3 \end{vmatrix} = 0, \mathrm{e}_3 = \frac{-1}{\mathrm{d}_1}\begin{vmatrix} \mathrm{c}_1 & 0 \\ \mathrm{d}_1 & 0 \end{vmatrix} = 0$$

Thus, the Routh-Hurwitz array is

$$\begin{array}{c|ccc} \lambda^5 & 1 & a_3 & a_1 \\ \lambda^4 & a_4 & a_2 & a_0 \\ \lambda^3 & b_1 & b_2 & 0 \\ \lambda^2 & c_1 & a_0 & 0 \\ \lambda^1 & d_1 & 0 & 0 \\ \lambda^0 & a_0 & 0 & 0 \end{array}$$

From the above, all the first column of the Routh-Hurwitz array are positive. That is in the first column there is no sign change. Therefore, the persistent equilibrium point of system$(4.4 - 4.8)$ is locally asymptotically stable.

4.6 Sensitivity analysis of $\mathbf{R_0}$ with respect to the model parameters

Sensitivity analysis is used to determine the relative importance of model parameters to crime transmission and its prevalence. Changing the value of the most sensitivity parameter will be the most sensitive strategy in changing the result of the model. Sensitivity indices allow us to measure the relative change in a state variable when a parameter that have a high impact on R_0 and should be targeted by intervention strategies. We perform the analysis by calculating the sensitivity indices of the basic reproduction number, R_0 (computing the partial derivative of R_0 with respect to each parameter at any time) [9]. Sensitivity analysis helps in developing efficient and effective intervention strategies in the control of crime in order to have a crime free community. We derive an analytical expression for its sensitivity to each parameter using the normalized forward sensitivity index MI that depends differentiably on a parameter S, as [32] is defined as

$$M_I = \frac{\partial R_0}{\partial S} \times \frac{S}{R_0} forSrepresentallthebasicparameters \quad (4.26)$$

Definition 4.1 The normalized forward sensitivity index of a variable to a parameter is the ratio of the relative change in the variable to the relative change in the parameter. The larger the magnitude of the sensitivity index leads to more sensitivity R_0 with respect to that parameters. We can easily derive an analytical expression for the sensitivity of R_0 with respect to each parameter that comprisesR_0. The basic reproduction number is given by:

$$R_0 = \frac{\mu\beta\delta\Lambda(m+\omega)+m\Lambda(\mu+\eta-\delta\mu)}{(\mu+\eta)(m+\omega)(\alpha+\mu+\rho)\mu}$$

The magnitude of the basic reproduction number is depending on the parameters that are associated with the crime. If the result is negative, then the relationship between the parameters and R_0 is inversely proportional. On the other hand, a positive sensitivity index means an increase in the value of the parameter [32].

Lets us check this by computing the partial derivative of basic reproduction number with respect to each parameter.

$$M_\Lambda^{R_0} = \frac{\partial R_0}{\partial \Lambda} \times \frac{\Lambda}{R_0} = 1 > 0$$

$$M_\beta^{R_0} = \frac{\partial R_0}{\partial \beta} \times \frac{\beta}{R_0} = \frac{\beta\mu\delta(m+\omega)}{\beta\mu\delta(m+\omega)+m(\mu+\eta-\delta\mu)} > 0$$

$$M_\delta^{R_0} = \frac{\partial R_0}{\partial \delta} \times \frac{\delta}{R_0} = \frac{\delta\mu\beta(m+\omega)-\delta m\mu}{\mu\beta\delta(m+\omega)+m(\mu+\eta-\delta\mu)} > 0$$

$$M_\eta^{R_0} = \frac{\partial R_0}{\partial \eta} \times \frac{\eta}{R_0} = -\frac{\mu\eta\delta\Lambda(\beta m+\beta\omega)+m))}{(\mu+\eta)[\mu\beta\Lambda(m+\omega)+m\Lambda(\mu+\eta-\delta\mu)]} > 0$$

$$M_\alpha^{R_0} = \frac{\partial R_0}{\partial \alpha} \times \frac{\alpha}{R_0} = -\frac{\alpha}{\alpha+\mu+\rho} < 0$$

$$M_{\rho}^{R_0} = \frac{\partial R_0}{\partial \rho} \times \frac{\rho}{R_0} = -\frac{\rho}{\alpha + \mu + \rho} < 0$$

$$M_{m}^{R_0} = \frac{\partial R_0}{\partial m} \times \frac{m}{R_0} = \frac{m\omega\Lambda(\alpha + \eta - \mu)}{(m + \omega)[(\mu\beta\delta\Lambda)(m + \omega) + m\Lambda(\mu + \eta - \delta\mu)]} > 0$$

$$M_{\omega}^{R_0} = \frac{\partial R_0}{\partial \omega} \times \frac{\omega}{R_0} = -\frac{\omega m\Lambda(\mu + \eta - \delta\mu)}{(m + \omega)[\mu\beta\delta\Lambda(m + \omega) + m\Lambda(\mu + \eta - \delta\mu)]} > 0$$

$$M_{\mu}^{R_0} = \frac{\partial R_0}{\partial \mu} \times \frac{\mu}{R_0}$$

$$= \frac{m\mu(\mu + \eta)(\alpha + \rho + \mu\delta - \eta) - [\beta\delta\mu^2(m + \omega)(\alpha + 2\mu + \rho + \eta) + m\mu(\alpha + \mu + \rho)(\mu + \eta + \delta\eta) + m(\mu + r}{(\mu + \eta)(\alpha + \mu + \rho)[\beta\delta\mu(m + \omega) + m\Lambda(\mu + \eta - \mu\delta)]}$$

$$< 0$$

The basic reproduction number of the system depends on nine parameters namely, the recruitment rate Λ, the contact rate between susceptible and criminal individuals(β, ω, m), the rate of crime unaware susceptible become crime aware susceptible η, death rate (μ, ρ),the rate proportion of crime unaware susceptible (δ) and incarceration rate (α).

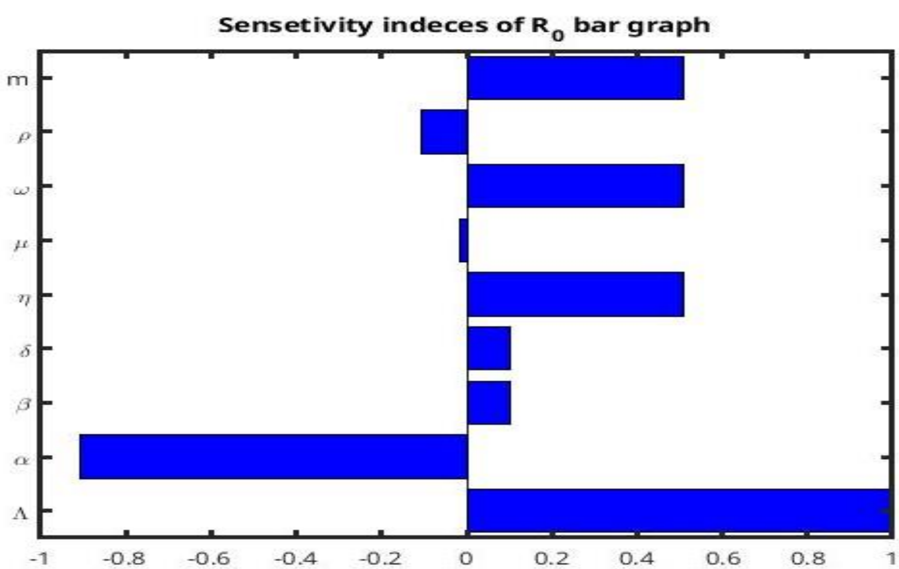


Figure 4.3 the sensitivity indices of $R_0$with respect to the parameters

Figure 4.3 shows that the parameters $\Lambda, \beta, \delta, \eta, \omega\ and\ m$ have a positive correlation of R_0. This indicates that the spread of crime decreases with decrease of these parameters. The parameters

α *and* ρ have a negative correlation of R_0. This implies that the spread of the crime decreases with an increase of these parameters.

CHAPTER FIVE

5. NUMERICAL SIMULATION

5.1 Numerical simulation and discussion

In this section, we use the numerical simulation to show the dynamic behavior of our model. The numerical simulation of the model system (4.4-4.8) were carried out to graphical illustrate with help of ode45 Mat lab tools. The sources of these parameters are mainly from assumptions and the aim of the simulation is to investigate the response of model parameters for crime. Table 5.1 below shows the set of parameter values and the state variable which were used in order to support the analytical results. We consider the following initial conditions.

$$S(0) = 100, A(0) = 150, C(0) = 300, Q(0) = 150, R(0) = 80$$

Parameter symbol	Values per year	Sources
Λ	300	Assumed
β	0.00005	Assumed
δ	0.523	Assumed
γ	0.05	Assumed
θ	0.005	Assumed
η	0.00001	Assumed
μ	0.39	Assumed
ρ	0.9	Assumed
ω	0.49	Assumed
α	0.05	Assumed
m	0.00026	Assumed
p	0.005	Assumed

TABLE 5.1 THE PARAMETER VALUE OF THE MODEL

In the model equation (4.4-4.8) simulation study is conducted and the population graphs are given in figure below. According to figure (5.1) we observe that the susceptible population (both aware and unaware) increase with time starting from initial until disease (crime) dies out and become constant at equilibrium point and while criminal, quarantined(jailed) and recovered population decrease and converges to zero. Crime free equilibrium point is locally asymptotically stable for the value $R_0 < 1$. This is due to that all curves of state variables with different initial conditions move in to their components of crime free equilibrium point.

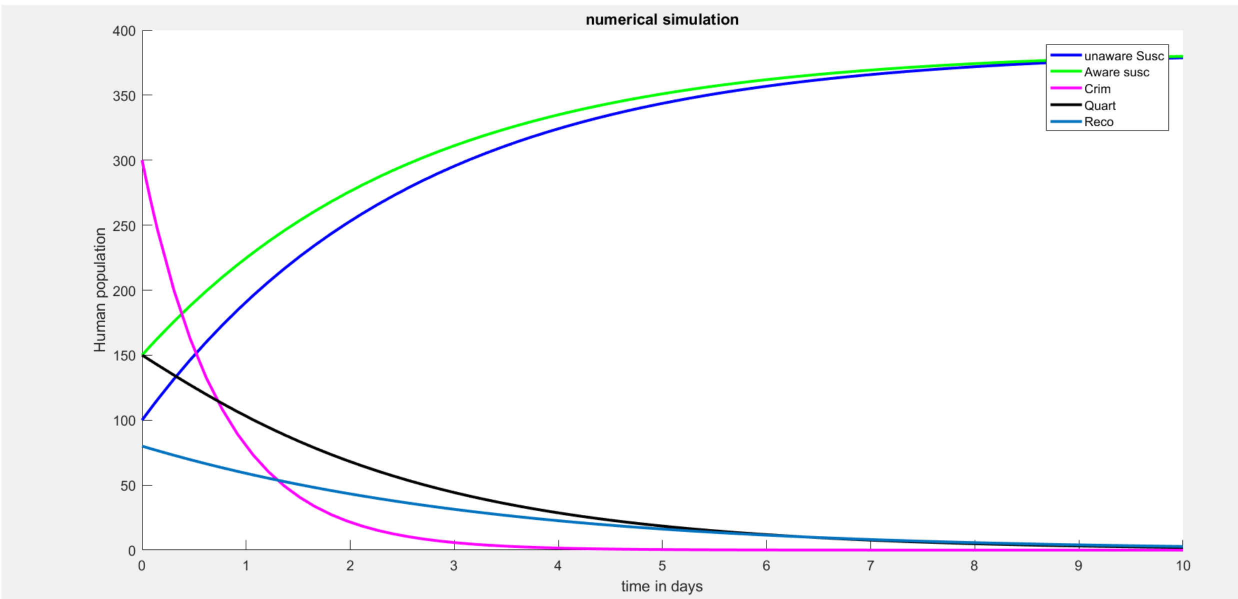


Figure5.1 The graph of dynamics of the state variable of model with $R_0 = 0.0296$.

Figure (5.2) below shows that the modified simulations at endemic equilibrium point.The crime persists in the population and spread in the population. We also observe that for $R_0 > 1$, all criminal solution of curve move away from crime free equilibrium point. This indicates that the crime free equilibrium point is unstable for the value$R_0 > 1$, and the solution will move in to the endemic equilibrium point. As a result, the crime overrun in the population. Also in the figure the endemic equilibrium point is locally asymptotically stable for the values of $R_0 > 1$. This is due to that all curves of state variable with different initial conditions move in to their components of the endemic equilibrium point.

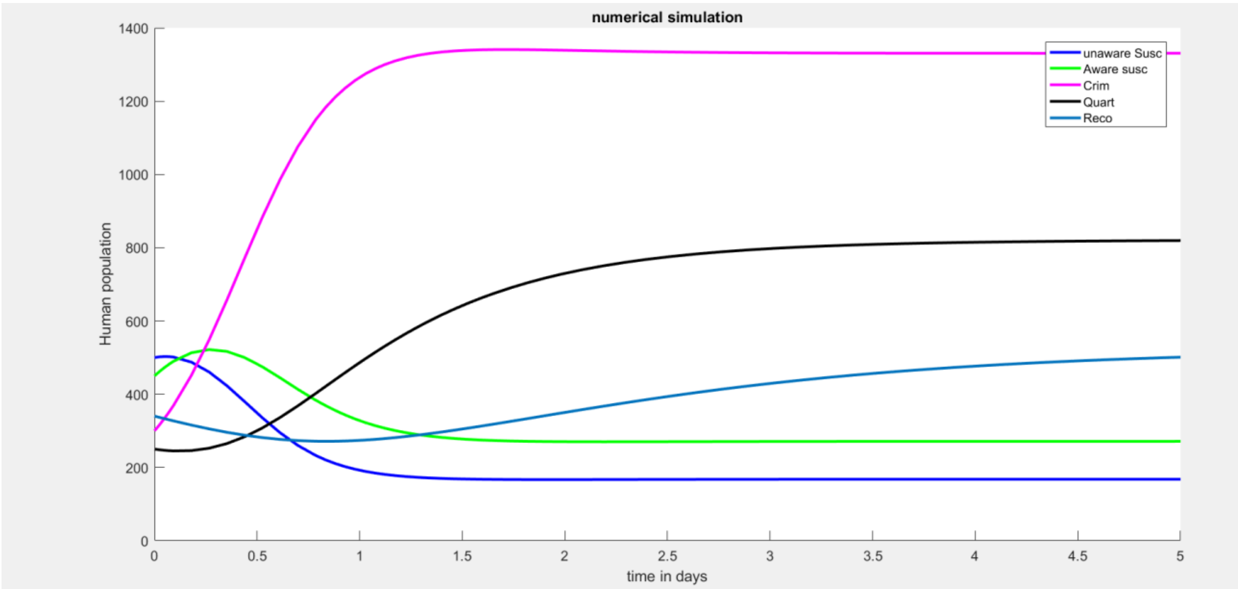
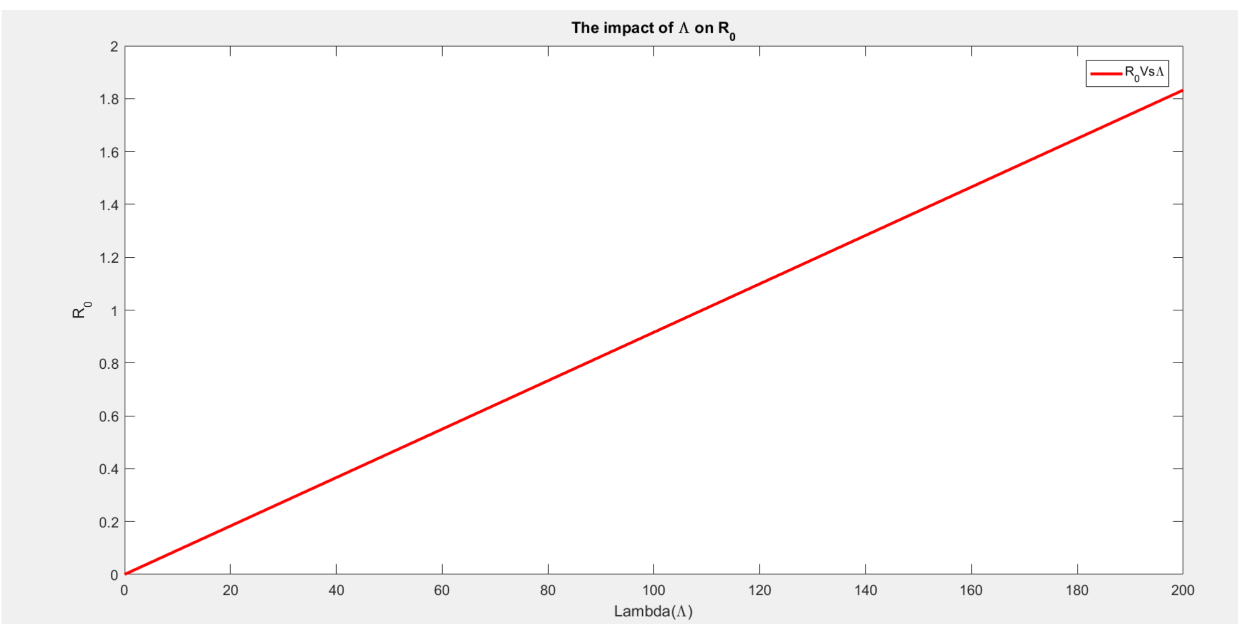
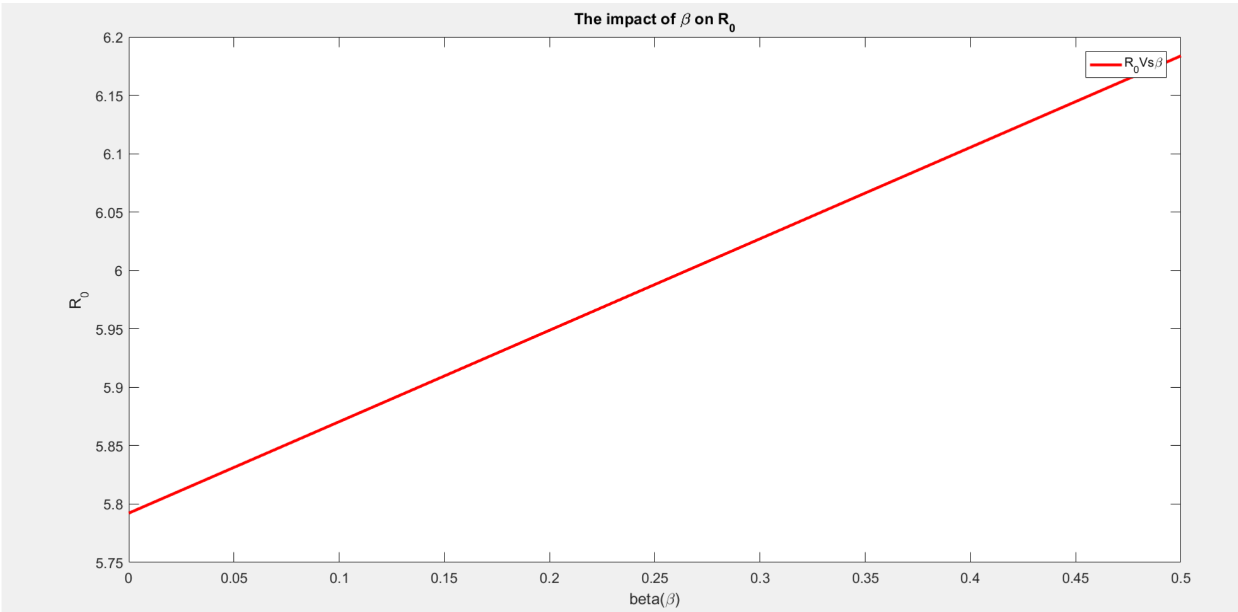


Figure5.2 The graph of dynamics of the state variable of the model with $R_0 = 7.6773$

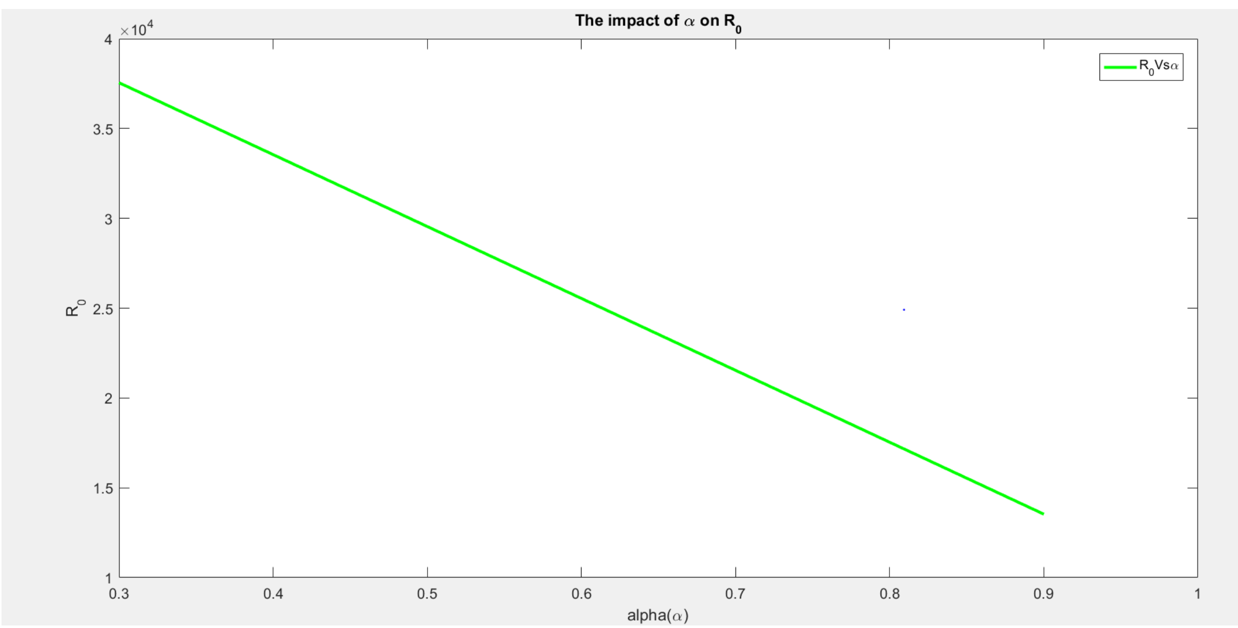
5.2.Some impact of parameters on R_0



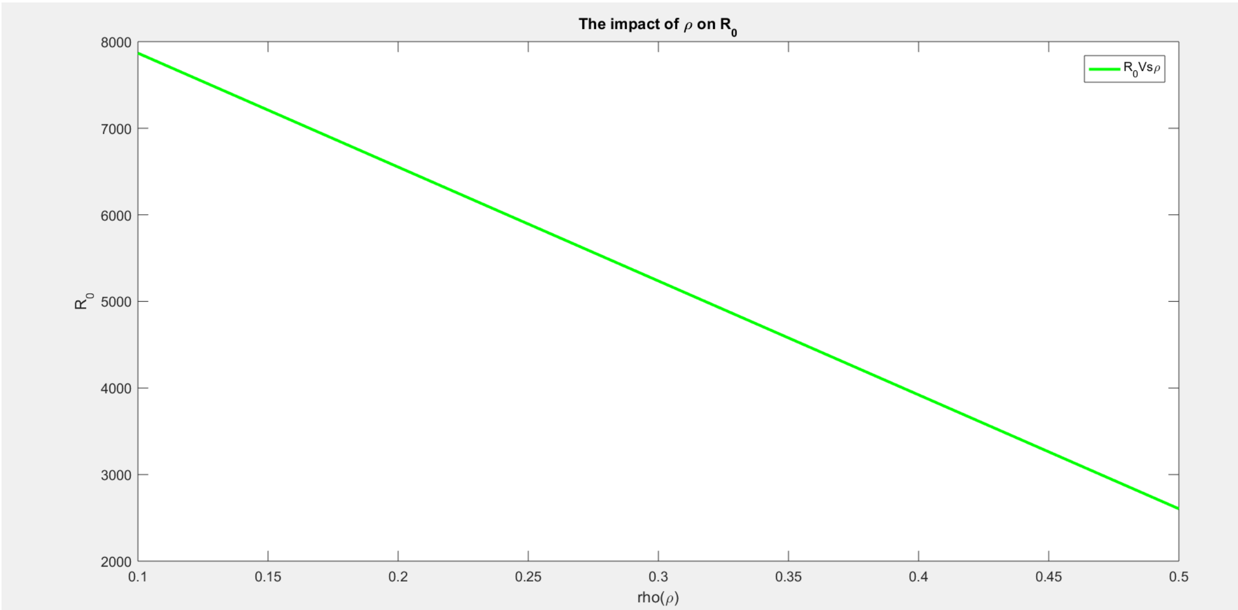
(a)


The impact of β on R_0
R_0 Vs β
6.2
6.15
6.1
6.05
6
5.95
5.9
5.85
5.8
5.75
R_0
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
beta(β)

(b)


The impact of α on R_0
$\times 10^4$
R_0 Vs α
4
3.5
3
2.5
2
1.5
1
R_0
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
alpha(α)

(c)



(d)

Figure5.3 Sensitivity analysis of reproduction number of parameter Λ, β, α and ρ

The reproduction number is increasing as parameter Λ and β are increasing with time, that is, Λ and β are directly proportional. And α and ω have negative impact which means increasing the value of those parameters decreases the number of criminals.

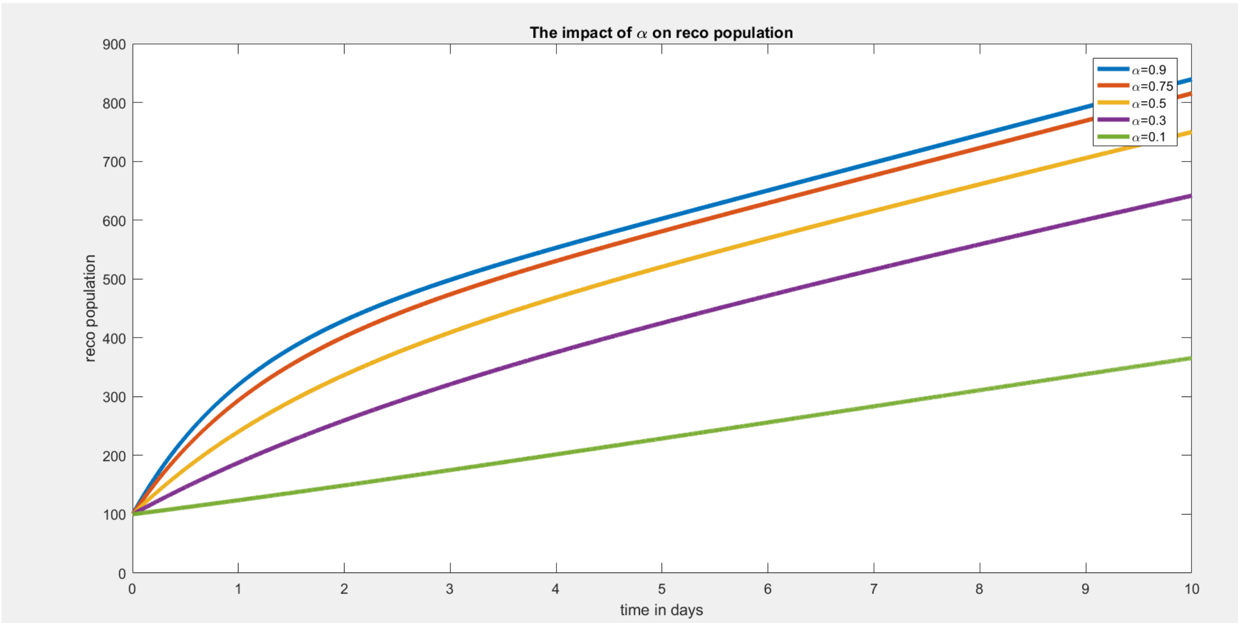


Figure5.4 The effect of varying transmission rate of α on recovered population

In the figure 5.4 above, we observe that, if the transmission or contact rate α is increases then the recovered population also increases which means that the crime in the population decreases due to moral or religious or media coverage.

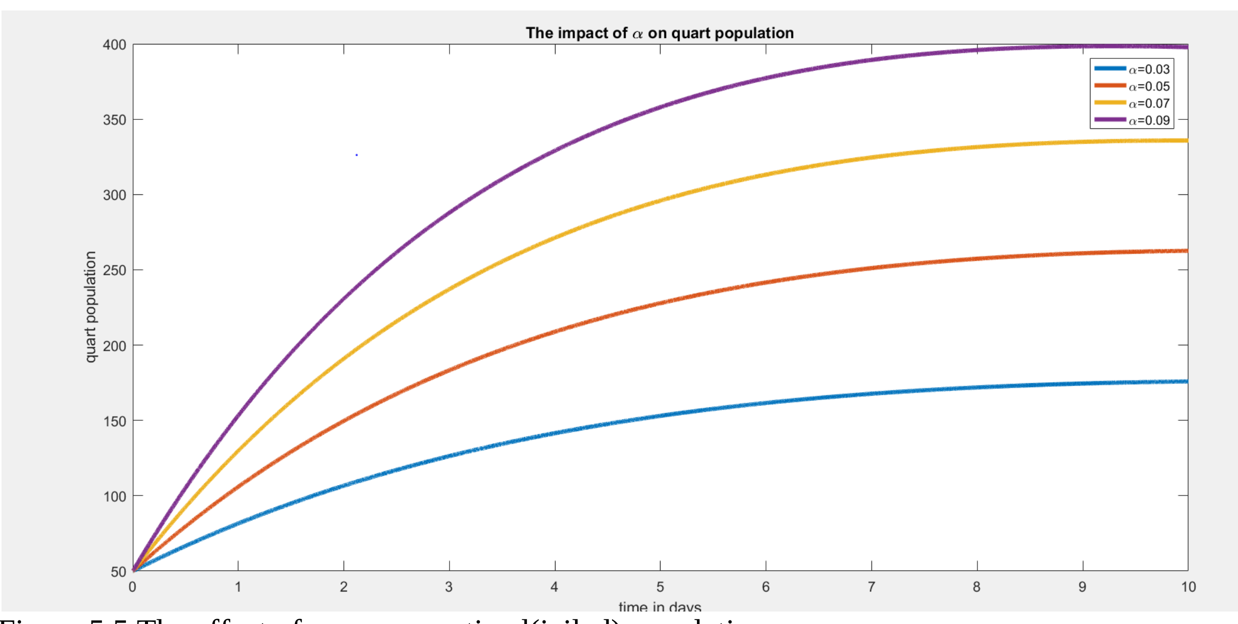


Figure 5.5 The effect of α on quarantined(jailed) population

From the figure 5.5 above one can observe that if the incarceration rate α is increasing then the jailed population or individuals are also increasing and viceversa.

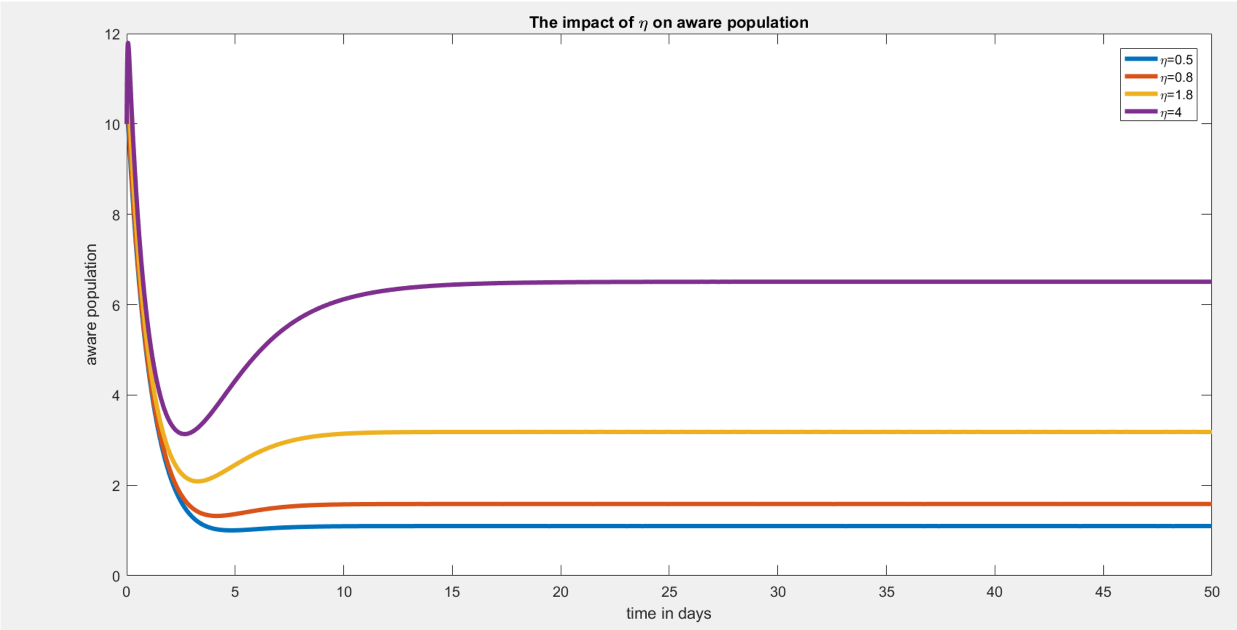


Figure 5.6 The impact of η on aware population

From figure 5.6above we observe that, if the contact rate η is increasing, the crime aware population is also increasing and this leads to the decrease of crime in the population. As much as possible giving aware about crime can reduce the spread of crime in the population.

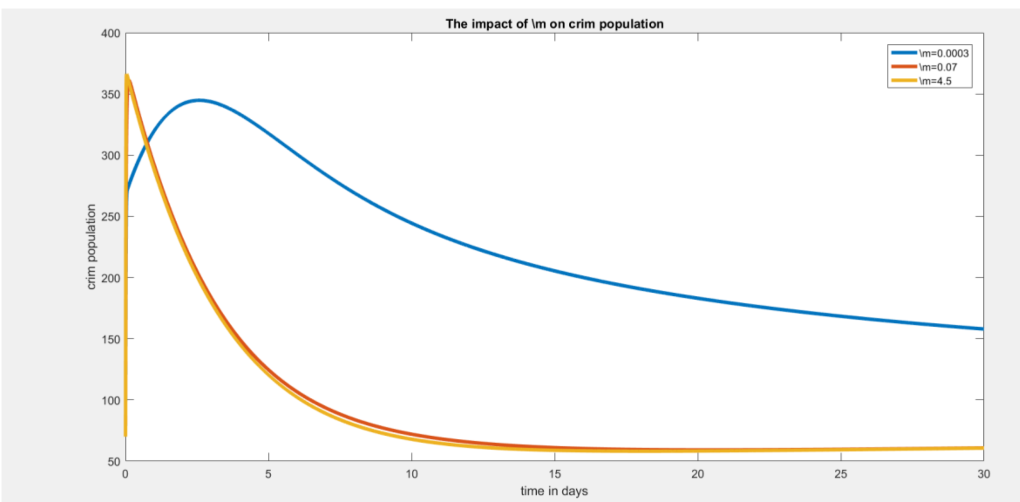


Figure 5.7 The impact of m on the criminal population

In figure 5.7 above we observe that ineffective orlack of media coverage $(m = 0.0003)$ increases the number of criminal individuals and effective use of media coverage $(m = 4.5)$ decreases the number of criminal individuals from the population. Thus to reduce the crime in the population effective use of media have a vital role.

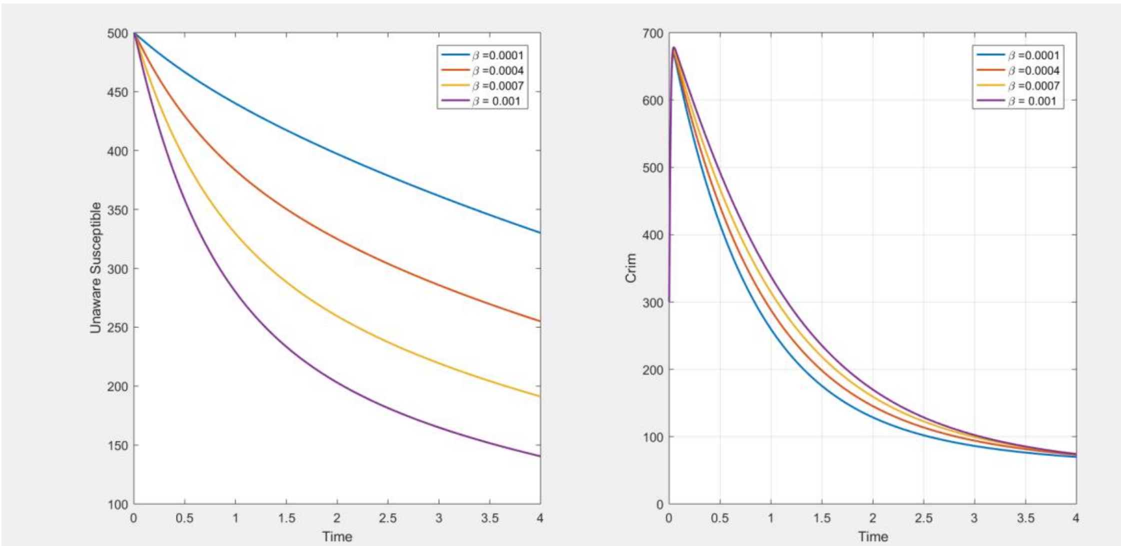


Figure 5.8The effect of β on Unaware and Criminal population

From figure 5.8 above we observe that the effect of contact rate β on unaware susceptible population S that the contact rate of β is increase the unaware susceptible population is decrease and the reverse is true as expected. And the effect of contact rate β on criminal population C that the contact rate β is increase the criminal population is also increase and viceversa.

CHAPTER SIX

6. CONCLUSION AND RECOMMENDATION

In this unit, we would put a summary of this research work, some the recommendation and the limitations that we had during the study of this thesis.

6.1 Conclusion

Crime affects everyone in one way or the other. It is observed that criminal behavior is contagious like epidemics and spreads through peer influence. Thus, the epidemic modeling approach can readily be applied to model the dynamics of crime in the society and its control. In view of this, we formulated and analyzed a deterministic mathematical model that includes media coverage. We have considered that criminals in the society increase by interaction among criminals and people prone to criminality. Vital mathematical features which include; the invariant region, existence and uniqueness, positivity of the solution are presented. In this case, two equilibria namely; crime-free equilibrium (CFE) and crime-persistent equilibrium (CPE) are obtained. The existence and stability criteria of the crime-free and the persistent equilibrium have been derived in terms of the basic reproduction number, R_0. From the theoretical point of view, we have shown that the basic reproduction ratio, R_0 is the distinguishing threshold parameter of the extinction or the persistence of the crime. When is R_0less than unity, crime cannot persist in the society, whereas for R_0 greater than unity, the crime remains persistent. This implies that whenever the threshold parameter, $R_0$is less than unity, thus the crime cannot progress.

The persistent equilibria are determined and shown to be locally asymptotically stable when the threshold parameter values are greater than unity. In this study, the effect of media coverage on the crime dynamics is considered. Unlike some of other previous model, we have taken into account the impact of media on crime and we added additional compartment. Media are widely acknowledged as a key tool for influencing people behavior towards the crime to devise proper policies for controlling the epidemic. Awareness programs through media make people aware about the crime and instruct them on how to take various precautions (e.g. social distancing from criminals), to reduce their chances of being criminal. Awareness among the human population thus may profoundly influence the pattern of crime spread and more importantly it helps in reducing the rate of to be criminals. To perform the numerical simulation of the model, we have applied the Matlab Ode45.

The results show that media coveragehas substantial influence on the dynamics of crime and can greatly influence the spread of the crime, thus, it is crucial to educate people by media coverage to quit crime. Our study suggests that if we increase the rate of implementation of awareness program through the media the number of criminal individuals decline and the system remains stable. On comparing the results obtained from the sensitivity analysis, decreasing the transmission rate decrease the prevalence of the crime due to effective use of media coverage.

6.2 Recommendations

Based on the results from this study, we recommend the following points.

1. The government should pay more attention on efficacy of media coverage for effective control of crime.

2. Effort should be made to minimize the interaction of susceptible and criminals, since the model shows that the spread of the crime largely depend on the interaction (contact) rate with criminals.

3. More research should be conducted on the prevalence dynamics of crime. Institutions which are dealing with tropical neglected crime should be expanded.

4. Educational campaigns like seminars and trainings should be conducted to create awareness on the depressing and spread of crime and control measures. Because educated person can be commit a crime with lower rate than those not educated.

5. This study can be extended by applying optimal control theory for a better transmission control and cost effectiveness of different crime intervention strategies.

6.3 Limitation of the study

Throughout the study we focused on the analysis and numerical simulation of the modified model. So estimating the parameters in this model was challenging, due to the scarcity of real data required to quantify the parameter values.

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Appendix

Appendix I: MatLab Codes for SACQR Model

```
function dy=abduro11(~,y)
global p m Lambda beta omega delta alpha gamma rho theta mu eta
dy=zeros(5,1);
%t=[0,100];
% S=y(1);
% A=y(2);
% C=y(3);
% Q=y(4);
% R=y(5);
%N=y(1)+y(2)+y(3)+y(4)+y(5);
dy(1)=delta*Lambda-beta*y(1)*y(3)-(mu+eta)*y(1);
dy(2)=(1-delta)*Lambda+eta*y(1)+theta*y(5)-mu*y(2)-(1-omega/(m+omega))*y(2)*y(3);
dy(3)=beta*y(1)*y(3)+(1-omega/(m+omega))*y(2)*y(3)-(mu+alpha+rho)*y(3);
dy(4)=(1-p)*alpha*y(3)-(mu+gamma)*y(4);
dy(5)=alpha*p*y(3)+gamma*y(4)-(theta+mu)*y(5);
end
```

Appendix II MatLab Codes for figure 5.1

```
global Lambda beta omega delta alpha gamma rho theta mu eta p m
Lambda=300; beta=0.00005; omega=0.49; delta=0.5; alpha=0.05; gamma=0.05; rho=0.9;
theta=0.005; mu=0.39; eta=0.000001; p=0.005; m=0.000026;
N=[100 150 300 150 80];
t_span=[0 10];
[t,y]=ode45('abduro11',[0 10],N);
R_0=(mu*beta*delta*Lambda*(m+omega)+m*Lambda*(mu+eta-
delta*mu))./((mu+eta)*(m+omega)*(alpha+mu+rho)*mu);
hold on
plot(t,y(:,1),'-b',t,y(:,2),'-g',t,y(:,3),'-m',t,y(:,4),'-k',t,y(:,5),'LineWidth',2)
```

```
legend('unawareSusc','Aware susc','Crim','Quart','Reco')
xlabel('time in days')
ylabel('Human population')
title('numerical simulation')
```

Appendix III MatLab Codes for figure 5.2

```
global Lambda beta omega delta alpha gamma rho theta mu eta p m
Lambda=2500; beta=0.005; omega=0.09; delta=0.5; alpha=0.8; gamma=0.5; rho=0.03;
theta=0.005; mu=0.79; eta=0.008; p=0.005; m=0.00026;
N=[500 450 300 250 340];
t_span=[0 5];
[t,y]=ode45('abduro11',[0 5],N);
R_0=(mu*beta*delta*Lambda*(m+omega)+m*Lambda*(mu+eta-
delta*mu))./((mu+eta)*(m+omega)*(alpha+mu+rho)*mu)
hold on
plot(t,y(:,1),'-b',t,y(:,2),'-g',t,y(:,3),'-m',t,y(:,4),'-k',t,y(:,5),'LineWidth',2)
legend('unawareSusc','Aware susc','Crim','Quart','Reco')
xlabel('time in days')
ylabel('Human population')
title('numerical simulation')
```

Appendix IV MatLab Codes for figure 5.3(a)

```
global Lambda beta omega delta alpha gamma rho theta mu eta p m
Lambda=0:50:200; beta=0.75; omega=0.05; delta=0.4; alpha=0.35; gamma=0.045;
rho=0.03;theta=0.025; mu=0.578; eta=0.6541; p=0.25; m=0.0025;
R_0=mu*beta*delta*Lambda*(m+omega)+m*Lambda*(mu+eta-
delta*mu)/(mu+eta)*(m+omega)*(alpha+mu+rho)*mu;
plot(Lambda,R_0,'r','LineWidth',2)
legend('R_{0}Vs\Lambda');
title('The impact of \Lambda on R_{0}');
xlabel('Lambda(\Lambda)');
```

```
ylabel('R_{0}');
hold on
```

Appendix V MatLab Codes for figure 5.3(b)

```
global Lambda beta omega delta alpha gamma rho theta mu eta p m
Lambda=0.6; beta=0:0.5:0.7; omega=0.05; delta=0.234; alpha=0.33; gamma=0.045;
rho=0.003;theta=0.025; mu=0.08; eta=0.3541; p=0.025; m=0.025;
R_0=(mu*beta*delta*Lambda*(m+omega)+m*Lambda*(mu+eta-
delta*mu))./((mu+eta)*(m+omega)*(alpha+mu+rho)*mu);
plot(beta,R_0,'r','LineWidth',2)
legend('R_{0}Vs\beta');
 title('The impact of \beta on R_{0}');
xlabel('beta(\beta)');
ylabel('R_{0}');
hold on
```

Appendix VI MatLab Codes for figure 5.3(c)

```
global Lambda beta omega delta alpha gamma rho theta mu eta p m
Lambda=2000; beta=0.7; omega=0.05; delta=0.4; alpha=0.3:0.6:0.9; gamma=0.045;
rho=0.03;theta=0.025; mu=0.008; eta=0.6541; p=0.25; m=0.0025;
R_0=(mu*beta*delta*Lambda*(m+omega)+m*Lambda*(mu+eta-
delta*mu))./((mu+eta)*(m+omega)*(alpha+mu+rho)*mu);
plot(alpha,R_0,'g','LineWidth',2)
legend('R_{0}Vs\alpha');
title('The impact of \alpha on R_{0}');
xlabel('alpha(\alpha)');
ylabel('R_{0}');
hold on
```

Appendix VII MatLab Codes for figure 5.3(d)

```
global Lambda beta omega delta alpha gamma rho
theta mu eta p m
```

```
Lambda=200; beta=0.75; omega=0.005; delta=0.64;
rho=0.1:0.4:0.7; gamma=0.45;
alpha=0.053;theta=0.025; mu=0.045; eta=0.6541;
p=0.25; m=0.0025;
R_0=(mu*beta*delta*Lambda*(m+omega)+m*Lambda*(mu+et
a-
delta*mu))./((mu+eta)*(m+omega)*(alpha+mu+rho)*mu);
plot(rho,R_0,'g','LineWidth',2)
legend('R_{0}Vs\rho');
title('The impact of \rho on R_{0}');
xlabel('rho(\rho)');
ylabel('R_{0}');
hold on
```

Appendix VIII MatLab Codes for figure 5.4

```
C=['-b','-m','-g','-r'];
global Lambda beta omega delta gamma rho theta mu eta p m
Lambda=300; beta=0.5; omega=0.05; delta=0.97; gamma=0.045; rho=0.03;
theta=0.25; mu=0.005; eta=0.35;p=0.25; m=0.025;
N_0=[600 500 400 250 100];
t_span=[0 10];
for alpha=[0.9 0.75 0.5 0.3 0.1];
%for i=(i:length(alpha(i)):4);
[t,y]=ode45('abduro11',t_span,N_0);
plot(t,y(:,5),'LineWidth',3)
hold on
%legend('\S','A','C','Q','R')
legend('\alpha=0.9','\alpha=0.75','\alpha=0.5','\alpha=0.3','\alpha=0.1')
xlabel('time in days')
ylabel('reco population')
title('The impact of \alpha on reco population')
```

end

Appendix IX**MatLab Codes for figure 5.5**

```
C=['-b','-m','-g','-r'];
global Lambda beta omega delta gamma rho theta mu
eta p m
Lambda=100; beta=0.95; omega=0.055; delta=0.4;
gamma=0.045; rho=0.03;
theta=0.025; mu=0.08; eta=0.0541;p=0.25; m=0.025;
N_0=[800 600 450 50 150];
t_span=[0 10];
for alpha=[0.03 0.05 0.07 0.09];
%for i=(i:length(alpha(i)):4);
[t,y]=ode45('abduro11',t_span,N_0);
plot(t,y(:,4),'LineWidth',3)
hold on
%legend('\S','A','C','Q','R')
legend('\alpha=0.03','\alpha=0.05','\alpha=0.07','\
alpha=0.09')
xlabel('time in days')
ylabel('quart population')
title('The impact of \alpha on quart population')
end
```

Appendix X MatLab Codes for figure 5.6

C=['-b','-m','-g','-r'];

global Lambda beta omega delta gamma rho theta mu alpha p m

```
Lambda=10; beta=0.55; omega=0.55; delta=0.97; gamma=0.0065; rho=0.123;
theta=0.625; mu=0.85; alpha=0.035;p=0.0025; m=0.0175;
N_0=[20 10 40 55 30];
t_span=[0 50];
for eta=[0.5 0.8 1.8 4];
%for i=(i:length(eta(iA)):4);
[t,y]=ode45('abduro11',t_span,N_0);
plot(t,y(:,2),'LineWidth',3)
hold on
%legend('\S','A','C','Q','R')
legend('\eta=0.5','\eta=0.8','\eta=1.8','\eta=4')
xlabel('time in days')
ylabel('aware population')
title('The impact of \eta on aware population')
end
```

Appendix XI MatLab Codes for figure 5.7

```
C=['-r','-g','-m'];
global Lambda beta omega delta gamma rho theta mu alpha p eta
Lambda=15; beta=0.5; omega=0.65; delta=10; gamma=0.0065; rho=0.0002;
theta=0.65; mu=0.0005; alpha=0.345;p=0.05; eta=0.75;
N_0=[200 100 70 55 30];
t_span=[0 30];
for m=[0.0003 0.07 4.5];
%for i=(i:length(m(i)):4);
[t,y]=ode45('abduro11',t_span,N_0);
plot(t,y(:,3),'LineWidth',3)
hold on
%legend('\S','A','C','Q','R')
legend('\m=0.0003','\m=0.07','\m=4.5')
xlabel('time in days')
```

ylabel('crim population')

title('The impact of \m on crim population')

end

Appendix XII Mat Lab Codes for figure 5.8

```
function dy=murad(~,y,beta)
%Initialize the derivative vector
dy=zeros(5,1);
%parameters
Lambda=30;     omega=0.95;   delta=0.08;   alpha=0.4;
rho=0.9;
mu=0.02;                                          eta=0.07;
m=0.2;p=0.25;theta=0.5;gamma=0.04;

%Compartment values
 S=y(1);%Unaware sus
 A=y(2);%Aware sus
 C=y(3);%Crim
 Q=y(4);%Quart
R=y(5);%Reco
%Differential equations
dy(1)=delta*Lambda-beta*y(1)*y(3)-
(mu+eta)*y(1);%Unaware sus change
dy(2)=(1-delta)*Lambda+eta*y(1)+theta*y(5)-mu*y(2)-
(1-omega/(m+omega))*y(2)*y(3);%Aware sus change
dy(3)=beta*y(1)*y(3)+(1-omega/(m+omega))*y(2)*y(3)-
(mu+alpha+rho)*y(3); %Crim change
dy(4)=(1-p)*alpha*y(3)-(mu+gamma)*y(4);%Quart
change
dy(5)=alpha*p*y(3)+gamma*y(4)-(theta+mu)*y(5);%Reco
change
dy=[dy(1);dy(2);dy(3);dy(4);dy(5)];
endbeta= linspace(0.005, 0.006,3);
y0=[500 400 300 200 100]; % Initial populations
tspan = [0 30]; % Time range
figure; % Create a new figurefor i = 1:length(beta)
```

```
    [t, y] = ode45(@(t, y) murad(t, y, beta(i)), tspan, y0); % Pass beta as a parameter
    % Plot each compartment
   subplot(1, 2, 1);
     plot(t, y(:, 1), 'LineWidth', 1.5);
     ylabel('Unaware Susceptible');
    xlabel('Time');
    legend(strcat('\beta =',num2str(beta')))
%     hold on
%     subplot(3, 2, 2);
%     plot(t, y(:, 2), 'LineWidth', 1.5);
%     ylabel('Aware Susceptible');
%     xlabel('Time');
%     legend(strcat('\beta=',num2str(beta')))
    hold on
   subplot(1, 2, 2);
     plot(t, y(:, 3), 'LineWidth', 1.5);
     ylabel('Crim');
     xlabel('Time');
    legend(strcat('\beta =',num2str(beta')))
%     hold on
%       subplot(3, 2, 4);
%      plot(t, y(:, 4), 'LineWidth', 1.5);
%      ylabel('Quart');
%     xlabel('Time');
%      legend(strcat('\beta=',num2str(beta')))
%      hold on
%       subplot(3, 2, 5);
%      plot(t, y(:, 5), 'LineWidth', 1.5);
%       ylabel('Recovered');
%      xlabel('Time');
%      legend(strcat('\beta =',num2str(beta')))
      hold ongrid onend

%legend('Unaware Susceptible', 'Aware Susceptible','Crim','Quart','Reco')
% title('The impact of \beta on  population');
```