

PERFORMANCE ANALYSIS AND COMPARISON OF LINEAR PRECODING  
TECHNIQUES IN MASSIVE MIMO UNDER DIFFERENT FADING CHANNEL

MSc. Thesis

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TECHNIQUES IN MASSIVE MIMO UNDER DIFFERENT FADING CHANNEL

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A THESIS SUBMITTED TO THE HAWASSA UNIVERSITY INSTITUTE OF  
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ADVISORS' APPROVAL SHEET  
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## Declaration

I declare that this thesis is my work and that all source materials used for this thesis have been properly acknowledged. This thesis has been submitted in partial fulfillment of the requirements for MSc. degree in communication engineering and Networking at Hawassa University. On this thesis titled **Performance Analysis and Comparison of Linear Precoding Techniques in Massive MIMO under Different Fading Channel**. I earnestly declare that this thesis is not submitted to any other institution anywhere for the award of any academic degree, diploma or certificate.

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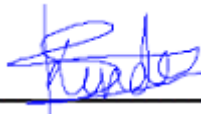
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## **Abstract**

Wireless communication and its featuring technology is continuously moving forward because of high demand in wireless communication services. Precoding techniques are very important in future telecommunication technology because in order to increase data rate the simplest method is increasing bandwidth. This method is difficult, because of scarcity in radio-frequency spectrum therefore, research over the last ten years has been focused towards improving spectral efficiency, so that higher data rates can be achieved with in a given band width for all above mentioned scenarios M-MIMO gives practical solution.

Massive MIMO technology is now attracting attention of both academic and industry. This motivates us to work on this topic. Most of the studies considered the uplink performance. Here, we study massive MU-MIMO for down-link system with linear precoding. We deliberate the system performance when the number of antennas and the number of users are large. Our area of interest in this work focus on performance analysis and comparison of transmitter precoding so, it is very much essential to know all the aspects of precoding techniques.

The main objective of this thesis is to analyze and compare the performance of linear precoding schemes in massive MIMO under Rayleigh fading channel and Rician fading channel. The performance Analysis and comparison issues include number of base station antenna, number of users, signal to noise ratio and Spectral efficiency under both complete and incomplete channel state information at the transmitter (CSIT) using MATLAB software.

Finally from this thesis work it was observed that as the number of users increases the mathematical complexity of SINR and spectral efficiency increases for all MF, ZF and RZF linear precoding schemes. Generally, in thesis work it is observed that ZF gives better performances at all transmission power and in an increasing number of BS antenna over RZF and MF in both perfect and imperfect channel state information under Rayleigh and Rician fading channel.

**Key word:** *Massive MIMO, precoding, Rayleigh, Rician and CSIT.*

## Table of Content

Declaration.....	i
Acknowledgement .....	ii
Abstract .....	iii
Table of Content .....	iv
List of Figures .....	vii
List of Tables .....	viii
List of Abbreviation .....	ix
List of Symbols.....	x
Chapter One.....	1
1. Introduction .....	1
1.1. Statement of Problem.....	4
1.2. Objectives.....	5
1.2.1. General Objective .....	5
1.2.2. Specific Objective.....	5
1.3. Review of Related Work .....	6
1.4. Methodology.....	8
1.5. Thesis Contributions .....	9
1.6. Thesis Organization .....	9
1.7. Scope of the Thesis .....	10
Chapter Two .....	11
2. Massive MIMO system.....	11
2.1. Benefits of massive MIMO systems .....	11
2.2. Downlink operation of a Massive MIMO link.....	12
2.3. Channel fading in wireless communications .....	13

2.3.1.	What is fading.....	13
2.3.2.	Channel Environments .....	14
2.4.	Channel state information (CSI).....	21
2.4.1.	Types of CSI.....	21
2.4.2.	Estimation Channel of CSI.....	22
2.4.3.	Downlink Data Transmission fading channel with perfect CSI at the transmitter ..	23
2.4.4.	Downlink Data Transmission fading channel with imperfect CSI at transmitter....	23
Chapter Three .....		25
3.	Downlink Precoding Schemes.....	25
3.1.	Linear Precoding algorithm.....	27
3.1.1.	Matched filter (MF) .....	31
3.1.2.	Zero forcing (ZF).....	33
3.1.3.	Regularized zero forcing (RZF).....	36
Chapter 4 .....		39
4.	Simulation Results and Analysis .....	39
4.1.	Spectral efficiency .....	39
4.2.	Massive MIMO Downlink System with perfect CSIT .....	41
4.2.1.	Change in capacity with the increasing the number of Base station antenna .....	41
4.2.2.	Spectral Efficiency against number of BS antennas over Rayleigh Fading.....	43
4.2.3.	Spectral Efficiency against the number of users over Rayleigh Fading .....	44
4.3.	Massive MIMO Downlink System with imperfect CSIT .....	45
4.3.1.	Change in capacity with the increase of the number of Base station antenna.....	45
4.3.2.	Spectral Efficiency against number of BS antennas .....	47
4.3.3.	Spectral Efficiency against the number of users .....	48
4.4.	M-MIMO Downlink System over Rician Fading channel at the transmitter.....	48

4.4.1.	Change in capacity with the increase the number of BS antenna in Rician.....	48
4.4.2.	Spectral Efficiency against number of BS antennas under Rician Channel .....	51
4.4.3.	SE against the number of users under Rician Fading Channel .....	53
Chapter 5	.....	56
5.	Conclusion and Recommendations for Future Work.....	56
5.1.	Conclusion.....	56
5.2.	Recommendations for Future Work.....	57
Reference	.....	58

## List of Figures

Figure 2-1 Downlink operation of a Massive MIMO link [1][2] .....	12
Figure 3-1 Functional block diagram for the single-cell downlink with MF processing.....	33
Figure 3-2 Functional block diagram for the single-cell downlink with ZF processing.....	36
Figure 3-3 Functional block diagram for the single-cell downlink with RZF processing .....	38
Figure 4-1 Block diagram of System design [1] .....	40
Figure 4-2 Spectral Efficiency versus SINR for ZF, RZF and MF. $M = 30$ and $K = 15$ .....	41
Figure 4-3 Spectral Efficiency v SINR for ZF, RZF and MF. $M = 60$ and $K = 15$ .....	42
Figure 4-4 SE v No BS antennas over Rayleigh fading for ZF, RZF and MF. ....	43
Figure 4-5 SE vs number of users over Rayleigh fading for ZF, RZF and MF. ....	44
Figure 4-6 SE v SINR for ZF, RZF and MF. $M= 30$ and $K = 15$ under imperfect CSI.....	45
Figure 4-7 SE v SINR for ZF, RZF and MF. $M= 60$ , $K = 15$ under imperfect CSI.....	45
Figure 4-8 SE v No BS antennas for ZF, RZF and MF. under imperfect CSI.....	47
Figure 4-9 SE v number of Users for ZF, RZF and MF. under imperfect CSI .....	48
Figure 4-10 SE vs SINR for ZF, RZF and MF (R factor $K=0.2$ ). $M = 30$ , $K = 15$ .....	49
Figure 4-11 SE vs SINR for ZF, RZF and MF, (R factor $K=10$ ). ....	49
Figure 4-12 SE v BS antennas ZF, RZF and MF (R factor $K=0.2$ ) $K = 15$ , SNR = 10dB. ....	51
Figure 4-13 SE v BS antennas ZF, RZF and MF (R factor $K=10$ ). $K = 15$ , SNR = 10dB. ....	51
Figure 4-14 SE v number of users ZF, RZF and MF (factor $K=0.2$ ). $M = 60$ , SNR = 10dB.....	53
Figure 4-15 SE v number of users ZF, RZF and MF (R factor $K=10$ ). $M = 60$ , SNR = 10dB .....	53

## List of Tables

Table 4-1 Value of different parameters .....	41
Table 4-2 For Rayleigh fading channel Spectral Efficiency versus SINR.....	42
Table 4-3 For Rayleigh fading channel Spectral Efficiency versus BS Antennas .....	43
Table 4-4 For Rayleigh fading channel Spectral Efficiency versus Number of Users .....	44
Table 4-5 For Rayleigh fading channel Spectral Efficiency versus SINR imperfect CSI .....	46
Table 4-6 For Rayleigh fading channel SE v BS Antennas imperfect CSI.....	47
Table 4-7 For Rayleigh fading channel SE versus Number of Users imperfect CSI .....	48
Table 4-8 For Rician fading channel SE versus SINR for Rician Factor $\mathbf{K}=0.2$ and $\mathbf{K}=10$ .....	50
Table 4-9 For Rician fading channel Spectral Efficiency versus BS Antennas .....	52
Table 4-10 For Rician fading channel Spectral Efficiency versus Number of User .....	54
Table 4-11 Summary of Rayleigh and Rician fading Spectral Efficiency v Number of SINR.....	55

## List of Abbreviation

AWGN	Additive White Gaussian Noise
BS	Base Station
BPSK	Binary phase Shift Key
CSI	Channel State information
CSIT	Channel state information at transmitter
DPC	Dirty paper coding
FDD	Frequency-division duplex
LOS	Line of Sight
M-MIMO	Massive Multiple Input Multiple Output
MF	Matched Filter
MIMO	Multiple Input Multiple Output
M-PSK	phase Shift Key
M-QAM	Quadrature Amplitude Modulation
MRT	Maximum Ratio Transmission
MU-MIMO	Multiuser Multiple-in and multiple-out
NLOS	Non line of Sigh
PDF	Probability DencityFuncution
QoF	Quality of Service
QPSK	Quadrature phase Shift Key
REE	Radiated energy efficiency
RF	Radio Frequency
RZF	Regularize Zero Forcing
SINR	Signal to Interference plus Noise Ratio
SNR	Signal to Noise Ratio
SP	Spectral Efficiency
TDD	Time division duplex
VP	Vector Perturbation
ZF	Zero Forcing
5G	Fifth Generation

## List of Symbols

$\mathbf{h}$	Vector $h$
$H$	Matrix $H$
$K$	Number of users
$M$	Number of Base
$\mathbf{K}$	Rician factor
$(\cdot)^*$	Complex conjugate
$(\cdot)^T$	Transpose of matrix
$\text{tr}(\cdot)$	Trace of square matrix
$(\cdot)^{-1}$	Inverse of matrix
$I_N$	Identity matrix with size $N$
$E[\cdot]$	Mean of the random variable
$\ \cdot\ $	Norm of vector
$ \cdot $	Absolute value
$\mathbf{K}$	Rician $K$ factor
$h(k)$	channel matrix
$r(k)$	Receive signals
$x(k)$	Transmit signal information
$n(k)$	Noise at AWGN
$p_t$	transmitter power
$p_r$	Receive powers
$H$	Channel matrix
$R$	Receive signal
$P$	Coefficient for the average transmit power
$Q$	Data signal vector transmitted to users
$\mu$	means of the distributions
$\sigma^2$	variance of the distribution
$\wp \mathcal{N}(0, \delta^2)$	Circularly symmetric complex Gaussian random variable with zero mean and variance $\delta^2$
$\mathcal{N}(0, \delta^2)$	Gaussian random variable with zero mean and variance $\delta^2$

# Chapter One

## 1. Introduction

Massive multiple input multiple outputs (MIMO), well viewed as one of the important and has attracted techniques for 5G wireless [1]. The main idea of massive MIMO is to move a large-scale number of antennas and employment some simple precoding processing or detection technologies at the base station, which significantly reduces the impact of noise and user interference and accordingly spectral efficiency (SE), energy efficiency, transmission reliability, and quality of service (QoS) can be significantly to make better [2][3]. However, with the greatly increasing number of BS antennas, may be up to hundreds, the design of Massive MIMO system brings up the new challenges. Massive MIMO technology scales up the number of transmits ( $T_x$ ) and receive ( $R_x$ ) antennas in comparison to conventional MU-MIMO systems by at least an order of magnitude. In ideal situations, massive MIMO offers large improvements in data rates, link reliability, energy efficiency and multiplexing gains while simultaneously reducing inter-user interference through linear precoding techniques. MIMO system is consequent classified into three categories:

- ❖ Point-to-Point MIMO
- ❖ Multiuser MIMO and
- ❖ Massive MIMO

This thesis is about Massive MIMO, which possibly will be the ultimate embodiment of MIMO technology. Either in uplink or in downlink transmissions, all terminals occupy the full time frequency resources concurrently. On uplink, the base station has to recover the individual signals transmitted by the terminals. On the downlink, the base station has to ensure that each terminal receives only the signal planned for it. The base station's multiplexing and demultiplexing signal processing is made possible by utilizing a large number of antennas and by its possession of Channel state information (CSI). Precoding is one of the ways to improve the transmission using this channel state information at the transmitter (CSIT). This channel state information at the transmitter (CSIT) can be exploited to enhance the performance of massive MIMO system using various precoding techniques. The pre-coders adapt the transmission to the channel using CSIT to improve the performance of the system. Precoding also helps to reduce the inter-user interference by focusing the energy on the desired user. When multiple users are simultaneously served, precoding of the data at the transmitter can help increase the received

SINR at each user . There are two types of precoding techniques. Those are:- Linear precoding and Nonlinear precoding. Linear precoding techniques are sub optimal transmission which provides simple signal, reasonable performance low implementation, low complexity and used to simplify the MIMO receivers. Some well-known linear precoding techniques are, Minimum Mean Square Error (MMSE) precoding, Zero Forcing (ZF) precoding, Maximum Ratio Transmission (MRT) precoding, Matched filter (MF), Wiener Filtering (WF) Regularize Zero Forcing (RZF) and Eigen Beam Forming power allocation (EBF-PA). The Nonlinear precoding techniques assure high implementation high complexity. The different types of nonlinear precoding techniques are Dirty paper coding (DPC) and Vector Perturbation (VP).

In this thesis, analyze the performance of linear precoding techniques that means Zero Forcing (ZF), Matched filter (MF) and Regularize Zero Forcing (RZF) precoding in a massive MIMO system considering perfect and imperfect CSIT conditions and we consider a general Rayleigh and Rician fading channel model. The major challenges in wireless communication are includes spectral efficiency, link reliability, coverage and energy efficiency. This is due to the limited available bandwidth, the fading nature of the propagation channel and the mobility and autonomy of the wireless nodes. The rapid growth in demand for high speed and high quality multimedia services with diverse QoS requirements is also creating the opportunities and challenges for the system designers. However the wireless channel is affected by time-dispersive effects, doppler and multipath fading, as a result the spectral efficiency of the wireless system is degraded. In arrangement to increase the data rates attempt a simple approach is to increase the allocated bandwidth. This approach is difficult, since the radio-frequency spectrum is scarce resource. Therefore, research over the last years has been focused towards improving spectral efficiency, so that higher data rates can be achieved within a given bandwidth. To improve the spectral efficiency, the conventional scheme and the new massive MIMO cooperative scheme are used adaptively according to channel condition between source and destination [1][4][5].

### **Channel Models**

Consider the compact massive MIMO system as exhibited in which the BS is sends  $M$  transmitter antennas and  $K$  users being randomly distributed in the circular-shape cell. In this scenario, the  $M$  transmitter serves to  $K$  single-antenna users at the same time frequency resources and we consider that the number of antennas is far greater than the number of users sate the limited condition  $M \gg K$ . Before all users received the transmitted date, the BS shall use some

simple linear pre-coding techniques pre-process, which realizes the signal term maximization and interfere term minimization as much as possible. There are three different MIMO channel modeling methodologies are Rayleigh fading, Rician fading and AWGN. These channel models are assumed to be for a particular frequency at a particular instance in time and are thus appropriate for modeling the channel of a single frequency resource [3][6].

## **1.1. Statement of Problem**

Due to limited available bandwidth, fading nature of the propagation channel and unexpected mobility of users wireless communication is highly sensitive for researching and study. However the wireless channel is affected by time-dispersive effects, doppler and multipath fading, as a result the spectral efficiency of the wireless system is degraded. It is known that the next wireless communication technology operates at high frequency. Whenever frequency of a signal increases wave length decreases and signal penetration power decreases. This are major challenges in next generation wireless communication. Due to the above mentioned problems designers and engineers in the perspective field of study are attracted and interested in the concepts of Massive MIMO. Massive MIMO attraction the spatial dimension by providing significant increases in all the advantages of conventional MIMO systems but on a much larger scale. In order to increase the data rates accessible, a simple approach is to increase the allocated bandwidth. This approach is difficult, since the radio-frequency spectrum is very scarce resource. While we are using Massive MIMO for high frequency communication system interference is a major problem. To solve this problem beam forming is an interesting technique and MIMO technology, is best approach for substantial improvement of spectral efficiency. Massive MIMO relies on the spatial multiplexing that in opportunity relies on the base station having sufficient CSIT. Precoding is one of the ways to develop the transmission using this channel state information at the transmitter. This channel state information at the transmitter (CSIT) can be exploited to improve the performance of massive MIMO system using several precoding techniques.

## **1.2. Objectives**

### **1.2.1. General Objective**

Analyzing and comparing the performance of linear precoding schemes in massive MIMO under Rayleigh fading channel and Rician fading channel.

### **1.2.2. Specific Objective**

- ✓ To analyze the effects of ZF, RZF and MF in a massive MIMO system under Rayleigh and Rician fading channel.
- ✓ To model mathematical expression of ZF, RZF and MF in massive MIMO system under Rayleigh and Rician fading channel.
- ✓ To compare performance of ZF, RZF and MF in massive MIMO under Rayleigh and Rician fading channel in terms of
  - Spectral Efficiency versus SINR
  - Spectral Efficiency versus number BS antennas and
  - Spectral Efficiency versus number of users.
- ✓ To Compare and analyze efficiency of linear precoding techniques and recommending best schemes.

### 1.3. Review of Related Work

There are different types of researches investigated for precoding techniques in massive MIMO system. Some of the researches guided on the area of linear precoding and related to this work are shortly reviewed below.

In [1] The performance analysis of a massive MIMO system utilizing different linear precoding techniques (MMSE, ZF and MRT) under Rayleigh fading channel with both complete and incomplete channel state information at the transmitter (CSIT) where proposed. In these research ZF gives better performances at high transmission power while MRT gives better performance at the low transmission power. However, MMSE gives the best performance at low and high transmission power.

In [3] proposes Analysis of downlink multi user precoding of massive multiple input multiple output (MIMO) system, where the base station (BS) has ideal channel state information (CSI) and adopts three types of different linear precoding schemes, those are maximum ratio transmission (MRT), zero-forcing (ZF) and minimum mean squared error (MMSE). Under a Rayleigh fading channel and the MRT precoding scheme is much inferior to the ZF and MMSE precoding schemes and tends to be at a fixed rate at the high SINR case. On the contrary, the MRT precoding scheme out performs ZF precoding schemes at the low SINR case.

In [6] Describes the error rate performance of precoding schemes over fading channels with the assumption of perfect channel state information at the transmitter. To estimate the bit error rate performance, different propagation environments namely, Rayleigh, Rician and Nakagami fading channels have been offered. This paper presents the error rate performance comparison of these fading channels based on precoding methods like channel inversion. The performance of precoding techniques was studied over Rayleigh, Rician and Nakagami fading channels in terms of BER versus SINR.

In [8] The performance of Massive MIMO downlink including linear precoding is evaluated. The achievable rate and the downlink transmit power of ZF and MRT precoding techniques are derived, analyzed and compared under the same conditions and assumptions. Several scenarios are considered to investigate these performance parameters. It is found that when the ratio number of BS antennas and number of users is large, ZF is better than MRT while when the ratio is quite small it makes MRT better than ZF for the same conditions.

In [20] A comprehensive overview of state-of-the-art research on the topic were proposed. It begin with an information theoretic analysis to illustrate the conjectured advantages of massive MIMO and then it was addressed that implementation issues related to channel estimation, detection and precoding schemes particularly focuses on the potential impact of pilot contamination caused by the use of non-orthogonal pilot sequences by users in adjacent cells. In this to make the benefits of massive MIMO a reality, significant additional research is needed on a number of issues, including channel correlation, hard ware implementations and impairments, interference management and modulation.

In [26] Precoding technology in 5G system analyzes the various methods of linear precoding, the maximum ratio transmission (MRT) precoding algorithm, zero forcing (ZF) precoding algorithm, minimum mean square error (MMSE) precoding algorithm based on maximum signal to leakage and noise ratio (SLNR). Precoding algorithms are analyzed and summarized in detail. At the same time, there also do some research on nonlinear precoding methods, such as dirty paper precoding and THP precoding algorithm. The linear and nonlinear precoding, linear precoding concern with low complexity and can obtain better performance at the same time, also to be able to analyze nonlinear precoding technology is close to the ideal channel capacity on the greatest degree. Thus, it is concluded that the optimal choice for precoding in various situations. Generally this thesis work mainly focuss on Analyzing and Comparing the performance of different linear precoding techniques using Zero Forcing (ZF), Matched filter (MF) and Regularize Zero Forcing (RZF) under Rayleigh fading channel and Rican fading channel with perfect and imperfect channel state information.

## 1.4. Methodology

The following methods was employed to achieve the objective of this thesis:

- Literature review: this includes the survey and reading of books, articles, journals and Simulation tools and all forms of other resources which are related to MIMO and precoding techniques
- System model: model mathematical expression of ZF, RZF and MF in massive MIMO system under Rayleigh and Rician fading channel using MATLAB software.
- Simulation: Is the approximation of the modeled massive MIMO system under Rayleigh and Rician fading channel with MATLAB software to investigate and compare performance of precoding techniques using Zero Forcing (ZF), Matched filter (MF) and Regularize Zero Forcing (RZF).
- Performance comparison: this involves Zero Forcing (ZF), Matched filter (MF) and Regularize Zero Forcing (RZF) of massive MIMO system under Rayleigh and Rician fading channel with MATLAB software.
- Analysis and interpretation of the results: It deals with the performance analysis and interpretation which is based on the results and findings obtained from the simulated results.

## **1.5. Thesis Contributions**

The problem of development impurity has received significant interest from the ongoing investigation on Massive MIMO. Theoretical evaluation has recognized pilot contamination as an imperative restriction on the throughput of massive MIMO systems.

The contribution of this thesis:-

A Massive-MIMO system using linear precoding system is ongoing. The system throughput expression and the achievable rate of a user over Rayleigh and Rician channel fadings are derived with different linear precoding scheme, the Massive MIMO achieves better performance. In this work, If the CSIT is not perfect, the missing information is obtained using statistical properties and there is noise in the channel. Since the pilot signals known at both ends of the link and the base station to make an estimate the channels propagate through the uplink channel, so in order to get channel state information at the downlink, the base station performs a de-spreading operation by correlating the received signals with each of pilot sequences.

In these case linear precoding at downlink MF, ZF and RZF are considered in the number of base station antenna is typically much larger than number of users in Massive MIMO. The spectral efficiency for Massive MIMO with given target of SINR were compared in Rayleigh and Rician fading channel. The complexities analyze of Massive MIMO with perfect and imperfect CSIT condition linear precoding are considered.

## **1.6. Thesis Organization**

Chapter 2 Reviews the Massive MIMO concept, including the advantages and their benefits, channel fading in wireless communication, Estimation channel of CSI, short description of perfect and imperfect CSIT and shows the equation for the achievable rate of a user in Massive MIMO system.

Chapter 3, Describes the downlink linear precoding techniques used in this work.

Chapter 4, Provide the simulation results and discussion of Massive MIMO for system including the linear precoding.

Chapter 5, Conclusion and suggestions for future work were presented.

## **1.7. Scope of the Thesis**

This thesis is focused on performance analysis of linear precoding techniques in Massive MIMO were studied with Zero Forcing (ZF), Matched filter (MF) and Regularize Zero Forcing (RZF) under Rayleigh fading channel and Rician fading channel in terms of Spectral Efficiency versus SINR, Spectral Efficiency versus number of BS antennas and Spectral Efficiency versus number of users.

## Chapter Two

### 2. Massive MIMO system

Massive MIMO systems use limited hundred antennas at the same time to give large number of wireless broad band terminals. The fifth generations of mobile networks (5G) main requirements are to achieve high data transmission rates, massive user capacity, low power consumption, high communication reliability and low quality. Massive MIMO systems, those that use large antenna arrangements at the base station to transmit multiple users to the same time-frequency resource, have been proposed to address these demands. MIMO systems have received significant concentration out standing to the growing number of served users and the increasing demand for big amounts of data [1][2].

#### 2.1. Benefits of massive MIMO systems

##### 1) Spectral efficiency (SE)

Massive MIMO can increase

- ✓ The spectral efficiency per cell as it can multiplex multiple users to obtain a multiplexing gain, which is the pre-log factor in the sum capacity formula.
- ✓ Can simultaneously create multiple very sharp beams.
- ✓ Many high-rate streams can be multiplexed over the same transmission resources.

##### 2) Radiated energy efficiency (REE)

Massive MIMO offers a large array gain that is equal to the number of antennas at the BS. The array gain is the factor of increase in terms of the received signal power. Thus to achieve the same data rate as in traditional systems, the transmit power in both uplink and downlink can be greatly reduced and there by increase the energy efficiency.

##### 3) Simpler processing

Channels between users become more and more orthogonal to each other. If the number of antenna is much greater than the number of users is simple linear processing that treats interference as noise is shown to perform close to the optimal capacity achieving schemes.

##### 4) Scalable

When operating in TDD mode, the number of antennas can theoretically be scaled indefinitely because the channel estimation over head is only limited by the number of users being served.

5) Near-optimal performance with simple transceivers:

Massive MIMO displays allow achieving good performance even with a simple precoding technology such as conjugate beam forming, which ignores multiuser interference in designing user beams.

6) Reduced inter user interference

When the number of antennas in the array increases the corresponding channel vectors of the terminals asymptotical become mutually orthogonal. The BS can purposely avoid transmitting into directions where signal speeding interface would be harm full [9]–[12][13][14].

### 2.2. Downlink operation of a Massive MIMO link.

Downlink operation, entails transmitting a different data stream to each user. Contemporary systems typically perform this multiplexing by some combination of sending the various data streams at different time-division multiplexing and over different frequency-division multiplexing [1][2][7]. Each base station is equipped with a large number of antennas  $M$  and serves a cell with a large number of terminals  $K$  through Rayleigh and Rician fading channels and that the channels will stay constant during a coherence interval of  $T$  symbols. The terminals typically throughout this thesis have a single antenna each.

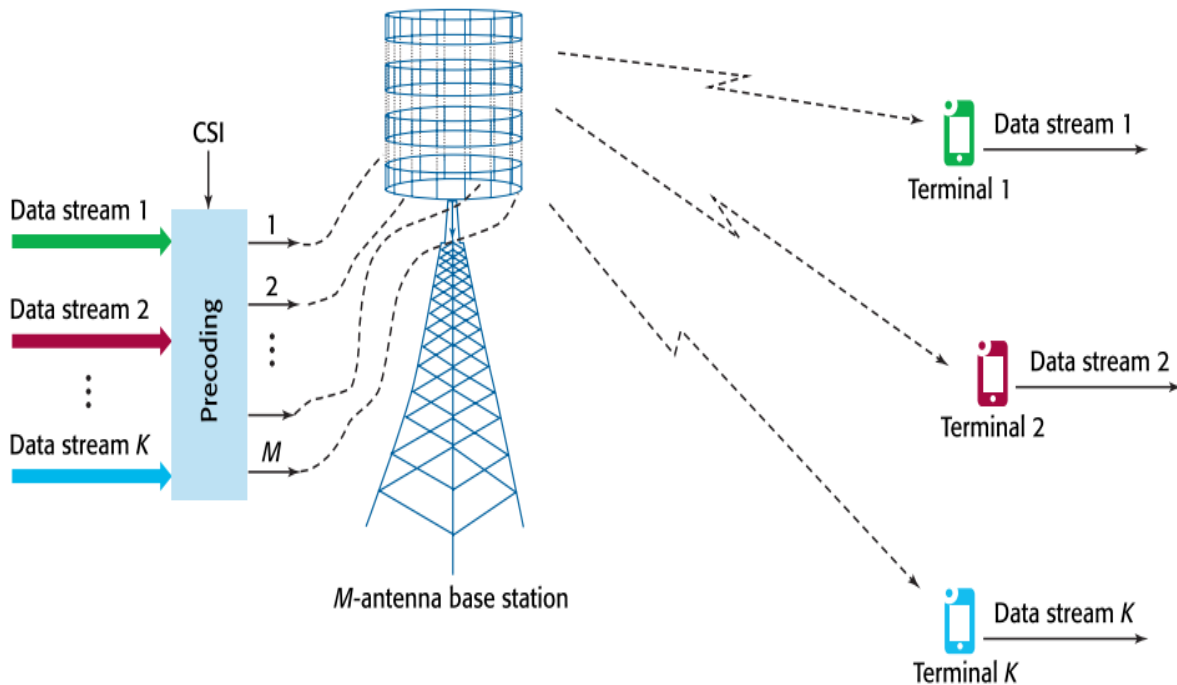


Figure 2-1 Downlink operation of a Massive MIMO link [1][2]

In large scale fading to happen when an obstacle comes in between transmitter and receiver. This interference type is reason significant amount of signal strength reduction. This is because electro magnetic wave is blocked by the obstacle. Small scale fading is concerned with rapid fluctuations of received signal strength over small distance and short time period. These multipath fading types is to be determined on propagation environment.

Let  $h_k^m$  be the channel gain between the  $k^{\text{th}}$  terminal and the  $m^{\text{th}}$  base station antenna.

$$h_k^m = \sqrt{\beta_k} g_k^m \dots \dots \dots k=1, \dots, K, m=1, \dots, M \quad (2.1)$$

Where  $\beta_k$  is a large scale fading coefficient that depends on  $k$  but not on  $m$  and

$g_k^m$  is represent the effect of small-scale fading

$h_k^m$  be the channel gain between the  $k^{\text{th}}$  terminal and the  $m^{\text{th}}$  base station antenna.

Let  $H$  be a matrix that comprises the channel gains between all terminals and all base station antennas [7].

$$H = \begin{bmatrix} h_1^1 & h_2^1 & \dots & h_K^1 \\ h_1^2 & \cdot & h_2^2 & h_K^2 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ h_1^M & h_2^M & \dots & h_K^M \end{bmatrix} \quad (2.2)$$

Throughout all performance analyses, we will assume that the small-scale fading is Rayleigh and Rician independent between the antennas and the terminals, so that  $h_k^m$  i.i.d.  $\mathcal{CN}(0,1)$  is random variables.

### 2.3. Channel fading in wireless communications

#### 2.3.1. What is fading

Fading is the most important thing when designs any Radio frequency. The communication between the base station and the mobile station in mobile systems is basically non-LOS. The line of sight path between transmitter and receiver is affected by environment and obstructed by buildings and other objects. The mobile station is also changing place in different directions at different speeds. The Radio frequency signal from the transmitter is scattered by reflection and diffraction and reaches the receiver done many non-LOS paths. This non-line of sight path bases long-term and short term variations in the form of log-normal fading, Rayleigh and Rician fadings which degrades the performance of the Radio frequency channel. In wireless communication systems the fading effect is always a great challenges the receiver receives the

transmitted signal through multipath with distributed amplitudes and phases. The multipath signal will display various fading effects based on the density of the scatter signal.

### 2.3.2. Channel Environments

In telecommunications and computer networking, a communication channel, refers to a physical transmission medium and radio channel. A channel is used to take an information signal a digital bit stream from transmitters to receivers. Communicating data from one place to another requires some form of path way or medium. It uses two types of media: cable (twisted-pair wire, cable and fiber-optic cable) and broadcast (microwave, satellite, radio and infrared). Cable or wire line media use physical wires of cables to send data and information[15]. For our thesis we used

- Rayleigh fading channel and
- Rician fading channel

#### 2.3.2.1. Rayleigh Fading Channel

Rayleigh fading often occurs in multipath environments where there is no direct path between the transmitter and receiver. The Rayleigh fading model uses a statistical close to analyze the propagation and can be used in a number of medium. The Rayleigh fading model is normally viewed as a competent approach to take when analyzing and prediction radio wave propagation performance for areas such as cellular communications in a well made up urban environment where there are many reflections from buildings. Rayleigh fading is a model that can be used to draw the form of fading that occurs when multipath propagation exists. In any material environment a radio signal will travel via a number of different paths from the transmitter to the receiver. The most easy to see path is the direct or line of sight path. However there will be very many objects along the direct path [6][17][18]. These objects may slave to reflect, refract etc the signal.

The model assumption is that the summation of many statistically independent reflected and scattered paths with random amplitudes is an independent and identically distributed complex Gaussian random variable. The element of the channel matrix is a complex number

$$h_k^m = a + jb \quad (2.3)$$

The joint probability density function can be

$$f_{a,b}(a, b) = \frac{1}{2\pi\sigma^2} e^{\left(-\frac{a^2+b^2}{2\sigma^2}\right)} \quad (2.4)$$

Expressing  $h_k^m$  in polar form we obtain

$$x = a^2 + b^2 \quad (2.5)$$

$$\theta = \arctan\left(\frac{b}{a}\right) \quad (2.6)$$

From polar form we can write the joint PDF of R and  $\theta$

$$f_{R,\theta}(x, \theta) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2}{2\sigma^2}} \quad (2.7)$$

By integration with  $\theta$  we obtain the PDF of R as

$$P_R(x) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} \quad (2.8)$$

By Rayleigh fading, we mean that the magnitude of the channel response (i.e.  $|h(k)|$  is a Rayleigh random variable. As we will see, this fact follows naturally from the mathematical definition of  $h(k)$ . Recall that for flat fading,  $h(k) = c(t_k; f = 0)$

Where  $h(k)$  channel matrix

$r(k)$  Receive signals

$x(k)$  Transmit signal information

$n(k)$  Noise at AWGN

Impulse response of channels

$$C(t, \tau) = \sum \alpha_i(t) e^{-j2\pi f_c \tau_i(t)} \delta(\tau - \tau_i)(t) \quad (2.9)$$

Transfer functions of channels

$$C(t, f) = \int_{-\infty}^{\infty} C(t, \tau) e^{-j2\pi f \tau} d\tau \quad (2.10)$$

$t_k$  Time at which the  $k^{\text{th}}$  symbols (at users)

$$r(k) = h(k)x(k) + z(k) \quad (2.11)$$

From equation (2.9)  $t_k$  time at which the  $k^{\text{th}}$  symbols (at users)

$$C(t_k, \tau) = \sum \alpha_i(t_k) e^{-j2\pi f_c \tau_i(t_k)} \delta(\tau - \tau_i)(t_k) \quad (2.12)$$

$$C(t, f = 0) = \int_{-\infty}^{\infty} C(t, \tau) d\tau \quad (2.13)$$

$$h(k) = C(t_k; f = 0)$$

$$h(k) = \int_{-\infty}^{\infty} C(t, \tau) d\tau \quad (2.14)$$

$$= \int_{-\infty}^{\infty} \sum \alpha_i(t_k) e^{-j2\pi f_c \tau_i(t_k)} \delta(\tau - \tau_i)(t_k) d\tau \quad (2.15)$$

$$= \sum \alpha_i(t_k) e^{-j2\pi f_c \tau_i(t_k)} \\ = \left[ \sum \alpha_i(t_k) \cos(2\pi f_c \tau_i(t_k)) \right] - j \left[ \sum \alpha_i(t_k) \sin(2\pi f_c \tau_i(t_k)) \right] \quad (2.16)$$

In equation (2.16) replace the first bracketed term by **A** and the second by **B**, we can write  $h(k)$

$$|h(k)| = \sqrt{A^2 + B^2} \quad (2.17)$$

If we can show that **A** and **B** are IID zero-mean Gaussian random variables with equal variances,  $\sigma^2$ , then, by definition,  $h(k)$  is a Rayleigh random variable with a probability density function,  $f_{|h(k)|}(x)$  given by

$$f_{|h(k)|}(x) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} \quad (2.18)$$

To show that **A** and **B** are independent and identically distributed zero-mean Gaussian random variables. Further more, the circular symmetry implies that the real and imaginary components of  $h(k)$  each have zero mean. Since  $h(k)$  is a complex Gaussian random variable, its real and imaginary components are jointly Gaussian. The final step in the argument is to appeal to the fact that uncorrelated jointly Gaussian random variables are independent. Therefore, if we can show that the two terms in the rectangular brackets above are uncorrelated, it follows that  $h(k)$  is the square root of the sum of the squares of two independent zero mean Gaussian random variables, which is the definition of Rayleigh random variables. The correlation between two random variables **a** and **b** is defined as

$$\text{Corr}(a, b) = \frac{E\{(a - \mu_a)(b - \mu_b)^*\}}{\delta_a \delta_b} \quad (2.19)$$

Where  $\mu_a$  and  $\mu_b$  are the mean values of **a** and **b**

\* complex conjugate

We conclude that  $h(k)$  is Rayleigh. It is possible to express  $f_{|h(k)|}(x)$  in terms of the transmission gain (or attenuation) of the channel. To do this, we start by noting that

$$|h(k)|^2 = a^2 + b^2. \quad (2.20)$$

Where a and b are zero mean, and

The variance of  $h(k)$ , which we denote by  $\sigma_h^2$ , can be expressed as

$$\sigma_h^2 = E\{|h(k)|^2\} \quad (2.21)$$

$$\sigma_h^2 = E\{a^2\} + E\{b^2\}$$

$$\sigma_h^2 = 2\sigma^2 \quad (2.22)$$

It is sometimes incorrectly stated without qualification that uncorrelated Gaussian random variables are independent. It is only true if the Gaussian random variables are jointly Gaussian, meaning that they are individually Gaussian and they have a joint PDF that is Gaussian.

Since we are considering flat fading, the absence of thermal noise, it follows that

$$|r(k)|^2 = |h(k)|^2|x(k)|^2 \quad (2.23)$$

That means time to be discrete where  $t_k$  denotes the time which the  $k^{\text{th}}$  symbols  $s(k) \triangleq \mu(t_k)$  is transmitted and the channel response

$$h(k) \triangleq (ct_k, 0)$$

$$r(k) = h(k)x(k) \quad (2.24)$$

Where  $r(k) \triangleq r(t_k)$  received signal.

We can, therefore, write the received signal power

$$P_r \triangleq E\{|r(k)|^2\} \quad (2.25)$$

Interims of transmitted power

$$P_t \triangleq E\{|x(k)|^2\}$$

From equation (2.25) and (2.24)

$$P_r = E\{|h(k)|^2 E|x(k)|^2\}$$

$$P_r = 2\sigma^2 E\{|x(k)|^2\}$$

$$P_r = 2\sigma^2 P_t \quad (2.26)$$

Where we have used the fact that  $h(k)$  and  $r(k)$  are independent of each other the PDF of  $|h(k)|$  in Rayleigh fading channel is

$$f_{|h(k)|}(x) = \frac{x}{\sigma^2} e^{\left(-\frac{x^2}{2\sigma^2}\right)} \quad (2.27)$$

$$f_{|h(k)|}(x) = \frac{2x}{P_r/P_t} e^{\left(-\frac{x^2}{P_r/P_t}\right)} \quad (2.28)$$

Where

$$P_r = 2\sigma^2 P_t \quad (2.29)$$

$$f_{|h(k)|}(x) = \frac{2xP_t}{P_r} e^{-\frac{x^2 P_t}{P_r}}$$

Where  $P_t$  transmitter power and  $P_r$  received power

### 2.3.2.2. Rician Fading Channel

Rician fading occurs when the received signal consists of two components a random portion that fluctuates in time and a non-fluctuating part. One way in which this might occur is if some of the transmitted signal energy propagates directly to the receiver and some of it gets scattered off objects along the path between the transmitter and receiver. The Rician fading propagation channel can be utilized to model wireless propagation when receiver can receive transmitted signal via a dominant line that means signals travelling in a direct path. The achievable spectral efficiency (SE) of Massive MIMO systems with imperfect channel state information (CSI) have been thoroughly characterized and enhanced for fading channels modeled by either spatially uncorrelated or spatially correlated Rayleigh fading. However, in practical scenario, channels can be a combination of deterministic LOS path combined with small-scale fading as a result of multipath propagation, which can be modeled by the Rician fading model which is a frequency operate alternative model for wireless channel where there are large line of sight components with big independent paths [18][19].

- ❖ Due to multipath propagation of the signal.
- ❖ There is a partially scattered field.
- ❖ One dominant signal.
- ❖ Others are weaker.

Rician  $\mathbf{K}$ -factor parameter is used to describe the statistical model of a Rician distribution and it is the ratio between the LOS and NLOS power signals from the massive MIMO antennas and hence the K-factors divide the signal gain into scattered and direct components. The dominant LOS component in a Rician fading channel can be modeled by combining two Gaussian random variables those are zero mean and non-zero mean. If we consider two Gaussian random variables  $a \sim N(m_I, \sigma^2)$  and  $b \sim N(m_Q, \sigma^2)$ . Where  $m_I$  and  $m_Q$  represents the means of the distributions and  $\sigma^2$  represents variance of the distribution [18]. The probability density function or the magnitude of the received complex envelop at time  $\mathbf{t}$  has a Rician distribution as shown in equation below

$$f_{h(k)}(x) = \frac{x}{\sigma^2} e^{\left(-\frac{x^2+p^2}{2\sigma^2}\right)} \quad (2.30)$$

Where the constant  $p$  is the strength of the direct component or the power of the LOS component  $x$  is given by

$$x = \sqrt{[m_I^2 + m_Q^2]}$$

is the non-centrality parameter.

If  $p > 0$  that implies  $k > 0$  the Rice model process becomes a Rayleigh process of which the amplitudes of the received signal are being presented by a Rayleigh distribution. Assuming squares of  $k$  independent normal random variables with non-zero mean (mean  $\mu=0$ ,  $\sigma=1$ ) are summed, it produces a non-central Chi-squared distribution. Non-centrality parameter is defined as the summation of means of each independent underlying normal random variable. A basic Rician model makes the presumption that the mean  $m_I$  and  $m_Q$  representing the in phase and Quadrature components of the received signal are invariable with time i.e.  $m_{I(t)} = m$  and  $m_{I(Q)} = m$  and the in-phase and Quadrature component do not change with time as well. The means of the in-phase  $m_I$  and mean of Quadrature  $m_Q$  are given by

$$m_{I(t)} = r \cdot \cos(2\pi f_m \cos \theta_0 t + \phi_0)$$

$$m_{Q(t)} = r \cdot \sin(2\pi f_m \sin \theta_0 t + \phi_0)$$

Where  $f_m \cos \theta_0$  Doppler shift and

$\phi_0$  Random phase off set of the signal with the LOS component

The Rician  $K$ -factor is one of the major features of a wireless channel and is defined as the ratio of the signal power of the line-of-sight (LOS) component to the signal power in the NLOS component

$$\mathbf{K} = \frac{\text{Power in direct component}}{\text{Power in scattered component}}$$

$$\mathbf{K} = \frac{P_{LOS}}{P_{NLOS}}$$

the Rice factor,  $\mathbf{K}$ , is defined as the ratio of the Power in direct component  $p^2$  to scattered power  $2\sigma^2$

$$\mathbf{K} = \frac{p^2}{2\sigma^2} \quad (2.31)$$

A Rician statistical channel can be mathematically described using the  $\Omega$  and  $\mathbf{K}$  parameter where

$$\Omega = p^2 + 2\sigma^2 \quad (2.32)$$

When  $\mathbf{K} = 0$  the channel is transformed into a Rayleigh fading

$\mathbf{K} = \infty$  there is no fading in the channel. The envelope distribution can be rewritten in terms of the Rice factor and the average envelope power [19]. Therefore the theoretical PDF of the signal at the receiver channel can hence be rewritten as shown in equation.

$$f_{h(k)}(x) = \frac{2x(k+1)}{\Omega} e^{-\left[\frac{(k+1)x^2}{\Omega} - k\right]} \quad \text{For } x \geq 0, k \geq 0, \Omega \geq 0 \quad (2.33)$$

Insert equation (2.32) into equation (2.33)

$$f_{h(k)}(x) = \frac{2x(k+1)}{p^2 + 2\sigma^2} e^{-\left[\frac{(k+1)x^2}{p^2 + 2\sigma^2} - k\right]} \quad (2.34)$$

Insert equation (2.31) into equation (2.34)

$$\begin{aligned} f_{h(k)}(x) &= \frac{2x(k+1)}{2\sigma^2 k + 2\sigma^2} e^{-\left[\frac{(k+1)x^2}{2\sigma^2 k + 2\sigma^2} - \frac{p^2}{2\sigma^2}\right]} \\ f_{h(k)}(x) &= \frac{2x(k+1)}{2\sigma^2(k+1)} e^{-\left[\frac{(k+1)x^2}{2\sigma^2(k+1)} - \frac{p^2}{2\sigma^2}\right]} \\ f_{h(k)}(x) &= \frac{2x(k+1)}{2\sigma^2(k+1)} e^{-\left[\frac{(k+1)x^2}{2\sigma^2(k+1)} - \frac{p^2}{2\sigma^2}\right]} \\ f_{h(k)}(x) &= \frac{x}{\sigma^2} e^{-\left[\frac{x^2}{2\sigma^2} - \frac{p^2}{2\sigma^2}\right]} \\ f_{h(k)}(x) &= \frac{x}{\sigma^2} e^{-\left[-\frac{x^2 + p^2}{2\sigma^2}\right]} \end{aligned} \quad (2.35)$$

From equation (2.29) and equation (2.35)

$$f_{h(k)}(x) = \frac{x}{p_r/2p_t} e^{-\left[-\frac{x^2 + p^2}{p_r/p_t}\right]} \quad (2.36)$$

Where  $p_r$  is received power and  $p_t$  is transmitted power  $p_r = 2\sigma^2 p_t$

$$f_{h(k)}(x) = \frac{2xp_t}{p_r} e^{-p_t \left[-\frac{x^2 + p^2}{p_r}\right]} \quad (2.37)$$

## 2.4. Channel state information (CSI)

In wireless communications, channel state information refers to called channel properties of a communication link. Channel estimation is the information to make how a signal propagates from the transmitter to the receiver and represents the combined effect of scattering, fading and power decay with the distance method. The channel state information produce adjust transmissions to current channel conditions, which is crucial for achieving reliable communication with high data rates in multi antenna systems [20][21].

### Advantages of Channel State Information

- ❖ CSI do not have to be transmitted which submit low latency and high capacity.
- ❖ More power can be allocated to the sub channels with higher channel gain.
- ❖ Can be obtained at the transmitter exploiting reciprocity in TDD.
- ❖ CSI at transmitters of wireless link permits the application of adaptively techniques in order to either boot transitions rate and reliability or decreases transmit power required to achieve certain performance.

#### 2.4.1. Types of CSI

There are basically two levels of channel state information

**Instantaneous CSI (or short-term CSI)-:** the current channel conditions are known, which can be viewed as knowing the impulse response of a digital filter. It gives a chance to adapt the transmitted signal to the impulse response and there by optimize the received signal for spatial multiplexing or to achieve low bit error rates.

**Statistical CSI (or long-term CSI)-:** a statistical characterization of the channel is known. This description can include the type of fading distribution, the average channel gain, the line-of-sight component and the spatial correlation. In a narrow band flat-fading channel with multiple transmit and receive antennas MIMO, the system is modeled as

$$r = Hx + n \quad (2.38)$$

Where  $r$  and  $x$  is received and transmit signal  $H$  and  $n$  are the channel matrix and the noise vector, respectively. The noise is often modeled as circular symmetric complex normal with  $c = CN(0, \delta)$  where the mean value is zero and the noise covariance matrix  $\delta$  is known.

### 2.4.2. Estimation Channel of CSI

The content of channel hardening is that effects of small-scale fading and frequency dependence disappear when  $M$  is large. Further more, channel hardening to cause channel estimation at the terminals and the associated transmission of downlink pilots. Typically, the channel estimation is done on the downlink. More precisely, each user estimates the channel using pilot sequences transmitted from the base station and it feedbacks this CSI to the base station. This channel estimation over head will be proportional to the number of base station antennas. Therefore, with M-MIMO system, this is very inefficient. Thus, here we assume that the channel is estimated at the base station via uplink pilots, assuming channel reciprocity. The downlink and uplink channels cannot be perfectly reciprocal in practice because of hardware chains mismatch. But with proper calibration, this non reciprocity can be removed. In this work, it is assumed that the hardware chain calibration is perfect. A popular path is training sequence, known signal is transmitted and the channel matrix  $H$  is estimated using the combined knowledge of the transmitted and received signal. Let the training sequence be denoted  $p_1 p_2 \dots p_N$ , where the vector  $x_k$  is transmitted over the channel as

$$r_i = Hp_i + n_i \quad (2.39)$$

By combining the received training signals  $r_i$  for  $i = 1 \dots N$ , the total training signaling becomes

$$r_i = Hp_i + n_i \quad (2.40)$$

$$r = HP + N$$

With the training matrix  $P = [p_1 p_2 \dots p_N]$  and the noise matrix  $N = [n_1 n_2 \dots n_N]$  with this notation, channel estimation means that  $H$  should be recovered from the knowledge of  $r$  and  $p$ .

#### Least-square estimation

If the channel and noise distributions are unknown, then the least-square estimator [7]

$$H_{LS,estimation} = rP^H(PP^H)^{-1} \quad (2.41)$$

Where  $(\cdot^H)$  denotes the Hermitian channel matrix.

### 2.4.3. Downlink Data Transmission fading channel with perfect CSI at the transmitter

The BS has to ensure that each terminal receives only the signal voluntary for it. The base station's multiplexing and de-multiplexing signal processing is made possible by utilizing a large number of antennas and by its possession of channel state information to process the signals before transmitting them to K users. The base station uses the channel estimates obtained from channel estimation phase to process the signals before transmitting them to K users and the base station uses linear precoding techniques [22]. Downlink data transmission entails a linear precoding operation that combines the message bearing symbols with the downlink channel estimates to create the actual signals that the array transmits. Using linear precoding the received signal  $r \in \mathbb{C}^{K \times 1}$  will be

$$r = \sqrt{p}qWH + n \quad (2.42)$$

$$x = \sqrt{p}qw$$

p Coefficient for the average transmit power

q Data signal vector transmitted to users

Perfect channel state information at transmitters means that at the transmitter knows all the components of H instantaneously and without errors. In an ideal scenario like this, the performance of the system can be increased very much using the channel state information at transmitters.

### 2.4.4. Downlink Data Transmission fading channel with imperfect CSI at transmitter

If the channel state information at the transmitter is imperfect

- ❖ Perform any precoding and using old transmission using other techniques rather than M-MIMO techniques.
- ❖ The missing values of the channel state information at the transmitter can be assumed. The missing information is obtained using statistical properties and there is noise in the channel. since the pilot signals (known at both ends of the link and from which the base station estimates the channels) propagate through the uplink channel, So in order to get CSI at downlink.
- ❖ The base station performs a de-spreading operation by correlating the received signals with each of pilot sequences. During de-spreading operation the noise is also spread.

The missing information can be obtained using statistical properties. We next extend the previous model to the case of an unknown channel gain. The signal x and the noise n are

uncorrelated, but not necessarily independent. Where  $X \in \mathbb{C}^{m \times 1}$  the transmitted signal from the base station,  $W \in \mathbb{C}^{M \times K}$  is a linear precoding matrix  $p$  a coefficient for the average transmit power  $q$  Data signal vector transmitted to users and  $n \in \mathbb{C}^{K \times 1}$  is the AWGN (Additive White Gaussian Noise) with zero-mean and variance. The received signal by  $k^{\text{th}}$  user using linear precoding technique is given by

$$r_k = \sqrt{p} h_k w_k q_k + \sqrt{p} \sum_{j=1, j \neq k}^k h_k w_j q_j + n_k \quad (2.43)$$

Where

$\sqrt{p} h_k w_k q_k$  is the desired signal

$\sqrt{p} \sum_{j=1, j \neq k}^k h_k w_j q_j$  the interference from the other users

$n_k$  the noise

Direct calculation shows that  $\sqrt{p} \sum_{j=1, j \neq k}^k h_k w_j q_j$  and  $n_k$  terms of (2.43) are mutually correlated and uncorrelated with  $x$ . Considering the equation  $\sqrt{p} \sum_{j=1, j \neq k}^k h_k w_j q_j$  and  $n_k$  as effective noise, the channel in (2.43) is, with appropriate normalization, equivalent to the model in deterministic channel with Additive Non-Gaussian Noise the received signal to interference plus noise ratio of the  $k^{\text{th}}$  user can be expressed as

$$\text{SINR}_k = \frac{p |w_k h_k|^2}{1 + p \sum_{j=1, j \neq k}^k |h_k w_j|^2} \quad (2.44)$$

## Chapter Three

### 3. Downlink Precoding Schemes

Precoding is a generalization of beam forming, here the second aim to adjust the weight of the channel, includes the amplitude and phase shift at each antenna to constructively add the signal at the receiving antenna. Precoding is the pre processing coding of the symbols that are ready to be transmitted to the receiver. In Massive MIMO systems, Space Division Multiple Access (SDMA) is implemented using precodes. Because of different data streams occupy the same frequencies and times. Precoding is an important signal processing procedure in down links, which utilizes the channel state information at the transmitter. With reliable CSI, precoding can be used to maximize link performance. CSI is estimated by linear channel estimators through uplink pilot transmissions, which leverages the channel reciprocity. In time division duplex (TDD) operation, the time required to acquire CSI is dependent on the number of users but independent of the number of BS antennas. Therefore, TDD is a more preferable operating mode for massive MIMO systems than the frequency-division duplex (FDD) mode, where the amount of time required by FDD to obtain CSI relies on the excessive number of BS antennas [20]. The precoding technique in complexity optimization of 5G system to make better the performance of capacity, reduce the error rate, simplify the receiver, can reduce energy consumption and better the speed and quality of mobile communication, it has a good application explore. At present, the mobile communication (5G) technology has the advantages of low energy consumption, higher security, faster speed and high efficiency. Massive MIMO and precoding technology is the core technology of MIMO can improve the capacity, minimize the bit error rate and simplify the complexity of the receiver, which plays a good capacity in the performance of the system. Data throughput is one of the most important performance indicators for communication systems. Multipath effect has long been considered as the main obstacle that prevents high through put transmission [23]. This can be simply explained in the time domain. Due to the multipath effect, we may need to insert guard interval between transmitted symbols to prevent inter-symbol interference (ISI), where the guard interval should be larger than the channel delay spread. The insertion of guard interval limits the transmission data throughput. The multipath effect in time domain leads to frequency selective fading in frequency domain. If the signal occupies the whole channel band width, the signal will experience the frequency selective fading and the performance degrade significantly in this case.

To avoid selective frequency fading, we may transmit signal with narrow band communications. In narrow band communication systems the signal band width is far less than channel coherent band width. Hence, although the channel has frequency selective fading, the narrow band signal will not experience a dramatic frequency-selective fading since the signal occupies only a small portion of the channel band width. However, transmit signal with narrow band width implies that the data throughput cannot be high [24]. Precoding is a pre processing technique that exploits channel-state information at the transmitter (CSIT) to match the transmission to the instantaneous channel. Linear precoding in particular provides a simple and efficient method to utilize CSIT. One of the most critical issues in communication systems is the multipath effect. Multipath effect will cause inter-symbol interference (ISI) and hence it limits the transmission speed. Communication channels are usually band width limited. In order to approach capacity, the transmission band must be expanded to the entire usable available band width of the channel. Precoding is one of the ways to improve the transmission using this channel state information at the transmitter (CSIT). This channel state information at the transmitter (CSIT) can be exploited to enhance the performance of massive MIMO system using various precoding techniques. The pre-coders adapt the transmission to the channel using CSIT to improve the performance of the system. Precoding also helps to reduce the inter-user interference by focusing the energy on the desired user. There are two types of precoding techniques. Those are:-

- ❖ Linear precoding and
- ❖ Non linear precoding.

Some well-known linear precoding techniques are, Minimum Mean Square Error (MMSE), Zero Forcing (ZF), Maximum Ratio Transmission (MRT), Matched filter (MF), Wiener Filtering (WF), Regularize Zero Forcing (RZF) and Eigen Beam Forming power allocation (EBF-PA).

The Non linear precoding techniques are,

- ❖ Tomlinson-Harashima (TH) precoding and
- ❖ Vector Perturbation (VP) etc.

In this paper, we have analyzed the performance of linear precoding techniques that means Zero Forcing (ZF), Matched filter (MF) and Regularize Zero Forcing (RZF). Precoding in a massive MIMO system considering perfect and imperfect CSIT condition and consider a general Rayleigh fading channel and Rician fading channel.

### 3.1. Linear Precoding algorithm

Linear precoding is divided into single antenna precoding and multi-antenna precoding algorithm. In the implementation of precoding technology, it can get better performance, while the complexity is also low and easy to realize. In a single-cell large-scale MIMO system, the base station precodes the signal and sends the signal vector to the user [6][21][8][25].

At present, the mobile communication (5G) technology has the better of low energy consumption, higher security, faster speed, high efficiency and more coverage. How to make better the capacity of the system and reduce the energy consumption is an important part of the research on the key technologies of 5G. In large scale MIMO and precoding technology is the core technology of MIMO can improve the capacity, reduce the bit error rate and simplify the complexity of the receiver, which plays an important role in the performance of the system. Base station to provide with  $M$  antennas serves and  $K$  users. The  $K$  user data signals are processed and transformed into an  $M \times 1$  signal vector  $q$  which is to be transmitted from the  $M$  base station antennas.  $q$  contains the information for all  $K$  users.

Define  $P_t$  as the average total transmit power, then we have

$$P_t = E[\|x\|^2] \quad (3.1)$$

The received signal vector is given by

$$r = Hx + n \quad (3.2)$$

$$x = \sqrt{p}Wq \quad (3.3)$$

$$r = \sqrt{p}WqH + n \quad (3.4)$$

Where  $p_t$  transmitter power

$p_r$  Receive powers

$H$  Channel matrix

$r$  Receive signal

$W$  Precoding

$P$  Coefficient for the average transmit power

$q$  Data signal vector transmitted to users

$$P_t = E[\|\sqrt{p}Wq\|^2] \quad (3.5)$$

$q$  Is normalized and independent

$$[qq^H] = I_k \quad (3.6)$$

From equation (3.5) the coefficient can be calculated

$$p = \frac{P_t}{E[\|Wq\|^2]} \quad (3.7)$$

From equation (3.2) and (3.3) the received signal vector  $r$  is given by

$$r = \sqrt{p}HWq + n \quad (3.8)$$

To explain the complications, consider the uplink of a single-cell Massive MIMO system with  $K$  single-antenna users and  $M$  antennas at the base station. The received dimensional signal is

$$r_k = \sqrt{p}h_k w_k q_k + n_k \quad (3.9)$$

Where  $h_k \in C^M$  the fading is channel from this user and  $n_k$  is unit-power additive Gaussian noise. In general, the base station will only have access to an imperfect estimate of  $h_k, k = 1, \dots, K$  suppose the base station uses  $h_1 \dots h_k$  to select receive combining vector user  $K$ .

$$r_k = \sqrt{p}h_k w_k q_k + \sqrt{p} \sum_{j=1, j \neq k}^k h_k w_j q_j + n_k \quad (3.10)$$

The performance of the transmission with interference, an important criterion is the signal-to-interference-plus-noise-ratio (SINR), defined as the desired signal power divided by the sum of the interference power and the noise variance. Then from equation (3.10) in our model, the SINR at User  $k$  can be calculated to be

$$\text{SINR}_k = \frac{p|w_k h_k^m|^2}{1 + p \sum_{j=1, j \neq k}^k |h_k^m w_j|^2} \quad (3.11)$$

The Capacity or spectral efficiency is given by

$$\frac{C}{B} = \text{SE} = \log_2(1 + \text{SINR}_k) \quad (3.12)$$

**But for Rayleigh channel fading** from downlink data transmission fading channel with perfect CSI at the transmitter find  $p$  and insert equation (3.11).

That means

$$r = \sqrt{p}HX + n \quad (3.13)$$

Interims of users

$$\left(\frac{r_k - n_k}{x_k}\right) * h_k^{m-} = \sqrt{p}$$

$$P = \left(\left(\frac{r_k - n_k}{x_k}\right) * h_k^{m-}\right)^2 \quad (3.14)$$

Therefore SINR is calculated by insert equation (3.14) into equation (3.11)

$$\text{SINR}_k = \frac{\left(\left(\frac{r_k - n_k}{x_k}\right) * h_k^{m-}\right)^2 |w_k h_k|^2}{1 + \left(\left(\frac{r_k - n_k}{x_k}\right) * h_k^{m-}\right)^2 \sum_{j=1, j \neq k}^k |h_k w_j|^2}$$

Then multiply by  $\frac{1}{\left(\left(\frac{r_k - n_k}{x_k}\right) * h_k^{m-}\right)^2}$  nominator and denominator

$$\text{SINR}_k = \left( \frac{\left(\left(\frac{r_k - n_k}{x_k}\right) * h_k^{m-}\right)^2 |w_k h_k|^2}{1 + \left(\left(\frac{r_k - n_k}{x_k}\right) * h_k^{m-}\right)^2 \sum_{j=1, j \neq k}^k |h_k w_j|^2} \right) \frac{\frac{1}{\left(\left(\frac{r_k - n_k}{x_k}\right) * h_k^{m-}\right)^2}}{\frac{1}{\left(\left(\frac{r_k - n_k}{x_k}\right) * h_k^{m-}\right)^2}}$$

$$\text{SINR}_k = \frac{|w_k h_k^m|^2}{\left(\left(\frac{r_k - n_k}{x_k}\right) * h_k^{m-}\right)^2 + \sum_{j=1, j \neq k}^k |h_k^m w_j|^2} \quad (3.15)$$

The Capacity or spectral efficiency is given by

$$\frac{C}{B} = \text{SE} = \log_2(1 + \text{SINR}_k) \quad (3.16)$$

**But in Rician channel fading** from downlink data transmission fading channel with perfect CSI at the transmitter equation (2.31) find p and insert equation (3.11).

$$\mathbf{k} = \frac{p^2}{2\delta^2} \quad (3.17)$$

Where k is Rician factor

P is average transmitter power or power in direct component

$2\delta^2$  Scatter power

From equation (3.17)  $2\delta^2 = \frac{p^2}{k}$  but from equation (2.29)  $P_r = 2\delta^2 P_t$  Equate two equations (3.17)

and (2.29) then

$$\frac{p^2}{\mathbf{k}} = \frac{P_r}{P_t}$$

$$p = \sqrt{\frac{\mathbf{k}P_r}{P_t}} \quad (3.18)$$

Then for Rician channel fading from downlink data Transmission fading channel with perfect CSI at the transmitter equation equate equation (3.14) and equation (3.18). That means

$$\sqrt{\frac{\mathbf{K}P_r}{P_t}} = \left( \left( \frac{r_k - n_k}{x_k} \right) * h_k^{m^-} \right)^2$$

$$\frac{\mathbf{K}P_r}{P_t} = \left( \left( \frac{r_k - n_k}{x_k} \right) * h_k^{m^-} \right)^4$$

$$P_t = \frac{\mathbf{K}P_r}{\left( \left( \frac{r_k - n_k}{x_k} \right) * h_k^{m^-} \right)^4} \quad (3.19)$$

Equate two equations (3.19) and equation (3.5)

$$P_t = PE[||Wq||^2] = \frac{\mathbf{K}P_r}{\left( \left( \frac{r_k - n_k}{x_k} \right) * h_k^{m^-} \right)^4}$$

$$P = \frac{\mathbf{K}P_r}{E[|w_k q_k|] \left( \left( \frac{r_k - n_k}{x_k} \right) * h_k^{m^-} \right)^4} \quad (3.20)$$

Therefore SINR is calculated by insert equation (3.20) into equation (3.11)

$$\text{SINR}_k = \frac{\frac{\mathbf{K}P_r}{E[|w_k q_k|] \left( \left( \frac{r_k - n_k}{x_k} \right) * h_k^{m^-} \right)^4} |w_k h_k|^2}{1 + \frac{\mathbf{K}P_r}{E[|w_k q_k|] \left( \left( \frac{r_k - n_k}{x_k} \right) * h_k^{m^-} \right)^4} \sum_{j=1, j \neq k}^k |h_k w_j|^2}$$

Then multiply  $E[|w_k q_k|] \left( \left( \frac{r_k - n_k}{x_k} \right) * h_k^{m^-} \right)^4$  nominator and denominator

$$= \left( \frac{\frac{\mathbf{K}P_r}{E[|w_k q_k|] \left( \left( \frac{r_k - n_k}{x_k} \right) * h_k^{m^-} \right)^4} |w_k h_k|^2}{1 + \frac{\mathbf{K}P_r}{E[|w_k q_k|] \left( \left( \frac{r_k - n_k}{x_k} \right) * h_k^{m^-} \right)^4} \sum_{j=1, j \neq k}^k |h_k w_j|^2} \right) \frac{E[|w_k q_k|] \left( \left( \frac{r_k - n_k}{x_k} \right) * h_k^{m^-} \right)^4}{E[|w_k q_k|] \left( \left( \frac{r_k - n_k}{x_k} \right) * h_k^{m^-} \right)^4}$$

$$\text{SINR}_k = \frac{\mathbf{K}P_r |w_k h_k^m|^2}{E[|w_k q_k|] \left( \left( \frac{r_k - n_k}{x_k} \right) * h_k^{m-} \right)^4 + \mathbf{K}P_r \sum_{j=1, j \neq k}^k |h_k^m w_j|^2} \quad (3.21)$$

The Capacity or spectral efficiency is given by

$$\frac{C}{B} = \text{SE} = \log_2(1 + \text{SINR}_k) \quad (3.22)$$

### 3.1.1. Matched filter (MF)

MF precoding maximizes the desired signal power the interference elimination is not taken into consideration in the precoding design. Precoding algorithm is very simple, is close to the optimal algorithm of constrained system, each user can maximize the signal-to-noise ratio (SNR), but did not consider the interference between users [26][27]. MF precoding can maximize the signal-to-noise ratio of each user. In the case where the interference between users is negligible compared with noise, MF is a near-optimal algorithm in signal constrained systems and is very simple. But it does not take into account the interference between users; it does not apply to such as the interference between the larger user complex transmissions environments. Matched-filter (MF) precoding maximizes the sum of K users SINRs. Each column vector  $W_k$  ( $k = 1, 2 \dots K$ ) Of the MF precoding matrix  $W$  is derived by maximizing the following sum of the ratios shown as

$$\begin{aligned} (w_{1,\text{MF}} w_{2,\text{MF}} \dots w_{K,\text{MF}}) &= \text{argmax} \sum_{k=1}^K \frac{E \left[ \left| \sqrt{\rho_{\text{MF}}} h_k W_k q_k \right|^2 \right]}{E[|n_k|^2]} \\ \text{s. t. } E \left[ \left\| \sqrt{\rho_{\text{MF}}} W_k q_k \right\|^2 \right] &= P_t \end{aligned} \quad (3.23)$$

Where  $(.)^H$  denotes the Hermitian matrix. Since the noise power is normalized to 1, that means  $E[|h_k|^2 = 1]$  MF precoding equivalently maximizes the sum of each user's desired signal power under the transmit power constraint[7][28].

$$w_k^{\text{MF}} = h_k^H \quad (3.24)$$

$w_k$  is the  $k^{\text{th}}$  column of the precoding matrix  $W$  then MF precoding matrix  $W$  is given by

$$\begin{aligned} W^{\text{MF}} &= [w_1^{\text{MF}} w_2^{\text{MF}} \dots w_k^{\text{MF}}] \\ W^{\text{MF}} &= [h_1^H h_2^H \dots h_k^H] \\ W^{\text{MF}} &= H^H \end{aligned} \quad (3.25)$$

From (3.25), the received vector of  $k^{\text{th}}$  user with MF is given by

$$r_k^{\text{MF}} = \sqrt{p} h_k^m h_k^H q_k + \sqrt{p} \sum_{j=1, j \neq k}^k h_k^m h_j^H q_j + n_k \quad (3.26)$$

The SINR for User  $k$  is MF linear precoding under Rayleigh fading channel insert equation (3.24) into equation (3.15) then

$$\text{SINR}_k^{\text{MF}} = \frac{|h_k^H h_k^m|^2}{\left( \left( \frac{r_k - n_k}{x_k} \right) * h_k^{m-} \right)^2 + \sum_{j=1, j \neq k}^k |h_k^m h_j^H|^2} \quad (3.27)$$

The Spectral efficiency is given by

$$\text{SE}_k^{\text{MF}} = \log_2(1 + \text{SINR}_k^{\text{MF}}) \quad (3.28)$$

$$\text{SE}_k^{\text{MF}} = \log_2 \left( 1 + \frac{|h_k^H h_k^m|^2}{\left( \left( \frac{r_k - n_k}{x_k} \right) * h_k^{m-} \right)^2 + \sum_{j=1, j \neq k}^k |h_k^m h_j^H|^2} \right)$$

The SINR for User  $K$  is MF linear precoding under Rician fading channel insert equation (3.24) into equation (3.21) then

$$\text{SINR}_k^{\text{MF}} = \text{KP}_r \left( \frac{|h_k^H h_k^m|^2}{E[|h_k^H q_k|] \left( \left( \frac{r_k - n_k}{x_k} \right) * h_k^{m-} \right)^4 + \text{KP}_r \sum_{j=1, j \neq k}^k |h_k^m h_j^H|^2} \right) \quad (3.29)$$

The spectral efficiency is given by

$$\text{SE}_k^{\text{MF}} = \log_2(1 + \text{SINR}_k^{\text{MF}}) \quad (3.30)$$

$$\text{SE}_k^{\text{MF}} = \log_2 \left( 1 + \text{KP}_r \left( \frac{|h_k^H h_k^m|^2}{E[|h_k^H q_k|] \left( \left( \frac{r_k - n_k}{x_k} \right) * h_k^{m-} \right)^4 + \text{KP}_r \sum_{j=1, j \neq k}^k |h_k^m h_j^H|^2} \right) \right)$$

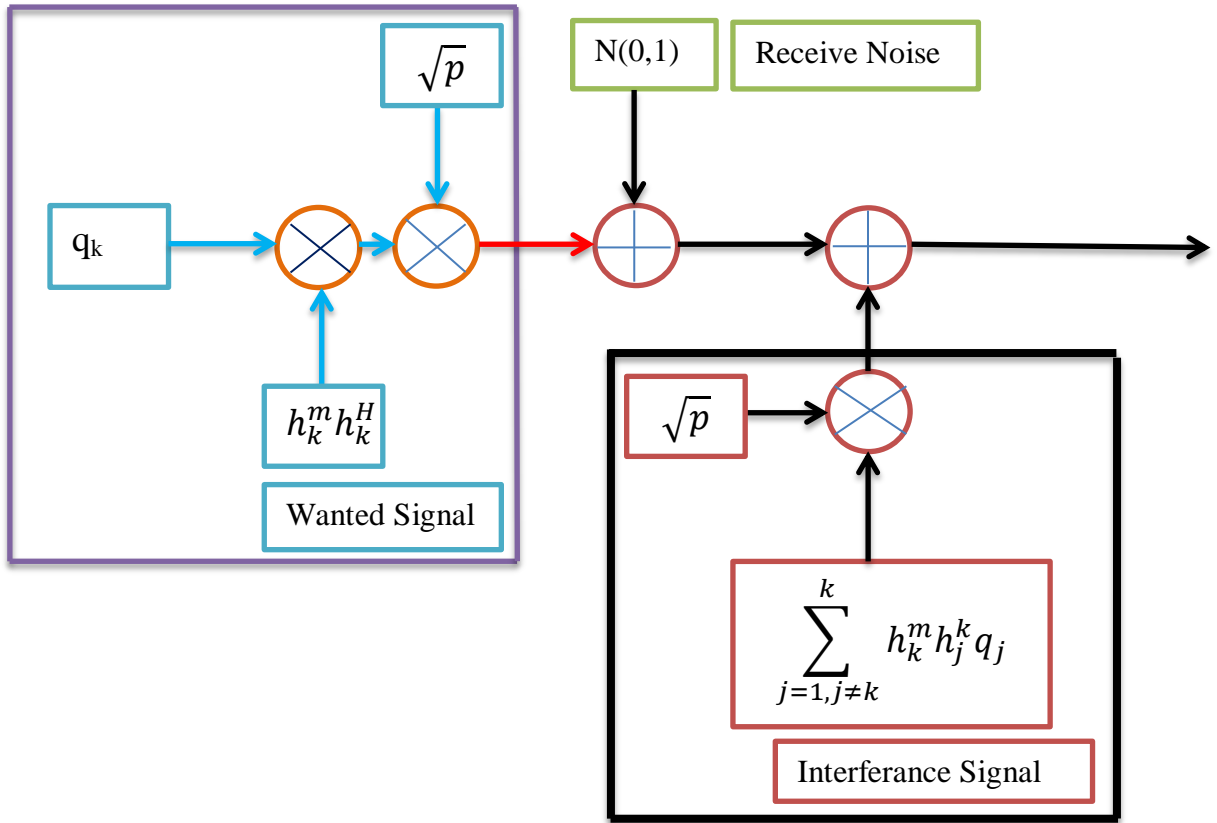


Figure 3-1 Functional block diagram for the single-cell downlink with MF processing

### 3.1.2. Zero forcing (ZF)

The zero forcing (ZF) precoding algorithms can completely remove the interference from other users in the cell, but it will lose some signal gain, without considering the influence of noise on the signal. According to the scheme, each antenna cannot be processed by using the ZF precoding algorithm [29][27]. It completely eliminates the interference between users by projecting the signals to be transmitted over the orthogonal complement of the components causing the interference between users. ZF precoding aims to completely remove the user interference. If the product of the channel matrix  $H$  and the precoding matrix  $W$  is an identity matrix, shown as

$$HW = I_k$$

The received signal vector  $r$  in can be rewritten as

$$r = \sqrt{\rho}qWH + n \quad (3.31)$$

the received signal at User k is known by

$$r_k = \sqrt{p}w_k q_k h_k + n_k \quad (3.32)$$

All entries of q are independent of each other and normalized to one that means

$$E[|q_k|^2] = I_k$$

Define q as the  $K \times 1$  data signal vector where the  $k^{\text{th}}$  entry  $q_k$  is the signal to be transmitted to user k. Each entry of q is normalized and independent we have

$$E[qq^H] = I_k$$

Where  $I_k$  is the  $k \times k$  identity matrix

It can be clearly observed that each user contains only the desired signal and the noise signal while the user-interference is completely removed. ZF precoding matrix was derived [30] as

$$W_{ZF} = H^H(HH^H)^{-1} \quad (3.33)$$

$$P_t = E\{\|\sqrt{p}Wq\|^2\}$$

$$p = \frac{P_t}{E\{\|Wq\|^2\}}$$

Where  $P_t$  transmitter power

$p$  is coefficient for the transmit power

Let  $r_t, q_k$  and  $n_k$  be the  $k^{\text{th}}$  elements of  $K \times 1$  vectors  $r$ ,  $q$  and  $n$  respectively and we define the  $k^{\text{th}}$  column of  $W^{ZF}$  as

$$W^{ZF} = H^H(HH^H)^{-1}$$

$$\text{Let } m_k = (HH^H)^{-1}$$

$$w_k^{ZF} = h_k^H m_k \quad (3.34)$$

The term  $H^H$  is Hermitian matrix and  $HH^H$  forms a Gram matrix whose diagonal elements denote power imbalance the channels while the off diagonal elements characterize mutual correction between the channels [31].

From (3.34), the received vector of  $k^{\text{th}}$  user with ZF is given by

$$r_k^{ZF} = \sqrt{p}h_k^m h_k^H m_k q_k + \sqrt{p} \sum_{j=1, j \neq k}^k h_k^m h_j^H m_j q_j + n_k \quad (3.35)$$

The SINR for User k is ZF linear precoding under Rayleigh fading channel insert equation (3.33) into equation (3.15) then

$$\text{SINR}_k^{\text{ZF}} = \frac{|h_k^H m_k h_k^m|^2}{\left(\left(\frac{r_k - n_k}{x_k}\right) * h_k^{m-}\right)^2 + \sum_{j=1, j \neq k}^k |h_k^m h_j^H m_j|^2} \quad (3.36)$$

The spectral efficiency is given by

$$\text{SE}_k^{\text{ZF}} = \log_2(1 + \text{SINR}_k^{\text{ZF}})$$

$$\text{SE}_k^{\text{ZF}} = \log_2 \left( 1 + \frac{|h_k^H m_k h_k^m|^2}{\left(\left(\frac{r_k - n_k}{x_k}\right) * h_k^{m-}\right)^2 + \sum_{j=1, j \neq k}^k |h_k^m h_j^H m_j|^2} \right) \quad (3.37)$$

The SINR for User k is ZF linear precoding under Rician fading channel insert equation (3.34) into equation (3.21) then

$$\text{SINR}_k^{\text{ZF}} = \mathbf{K}P_r \left( \frac{|h_k^H m_k h_k^m|^2}{E[|h_k^H m_k q_k|] \left(\left(\frac{r_k - n_k}{x_k}\right) * h_k^{m-}\right)^4 + \mathbf{K}P_r \sum_{j=1, j \neq k}^k |h_k^m h_j^H m_j|^2} \right) \quad (3.38)$$

The spectral efficiency is given by

$$\text{SE}_k^{\text{ZF}} = \log_2(1 + \text{SINR}_k^{\text{ZF}})$$

$$\text{SE}_k^{\text{ZF}} = \log_2 \left( 1 + \mathbf{K}P_r \left( \frac{|h_k^H m_k h_k^m|^2}{E[|h_k^H m_k q_k|] \left(\left(\frac{r_k - n_k}{x_k}\right) * h_k^{m-}\right)^4 + \mathbf{K}P_r \sum_{j=1, j \neq k}^k |h_k^m h_j^H m_j|^2} \right) \right) \quad (3.39)$$

Block diagram below shows the corresponding functional block. The numerators of the SINRs have identical form. For the downlink case, the effective noise depends only the channel estimation error for the  $k^{\text{th}}$  terminal weighted by the total transmitted power, because the  $k^{\text{th}}$  terminal receives power only through its own channel.

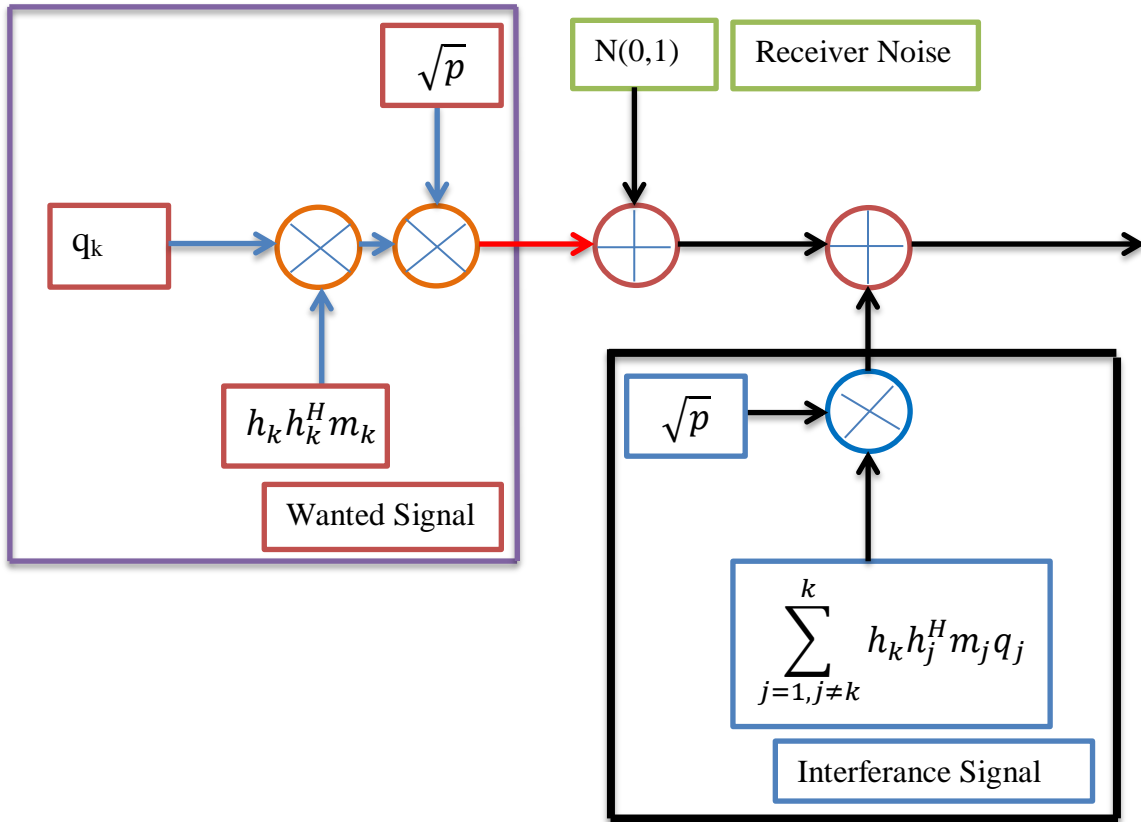


Figure 3-2 Functional block diagram for the single-cell downlink with ZF processing

### 3.1.3. Regularized zero forcing (RZF)

Precoding algorithms can completely remove the interference from other users in the cell, but it will lose some signal gain, without considering the influence of noise on the signal. According to the scheme, each antenna cannot be processed by using the ZF precoding algorithm [21][32][33]. The received signal vector  $r$  in can be rewritten as

$$r = \sqrt{\rho}qWH + n$$

the received signal at User  $k$  is

$$r_k = \sqrt{\rho}w_k q_k h_k + n_k$$

All entries of  $q$  are independent of each other and normalized to one that means

$$E[|q_k|^2] = I_k$$

Define  $q$  as the  $K \times 1$  data signal vector where the  $k^{\text{th}}$  entry  $q_k$  is the signal to be transmitted to user  $k$ . Each entry of  $q$  is normalized and independent we have

$$E[qq^H] = I_k$$

Where  $I_k$  is the  $k \times k$  identity matrix

RZF precoding, as a generalization of ZF precoding, is proposed to RZF precoding matrix.

It is given by

$$W^{RZF} = H^H(HH^H + cI_k)^{-1} \quad (3.40)$$

Where  $c > 0$  is a constant and the regularization factor,

$$w_k^{RZF} = h_k^H(HH^H + cI_k)^{-1}$$

Let

$$\begin{aligned} N_c &= (HH^H + cI_k)^{-1} \\ w_k^{RZF} &= h_k^H N_c \end{aligned} \quad (3.41)$$

From (3.41), the received vector of  $k^{\text{th}}$  user with ZF is given by

$$r_k^{RZF} = \sqrt{p} h_k^m h_k^H N_c q_k + \sqrt{p} \sum_{j=1, j \neq k}^k h_k^m h_j^H N_c q_j + n_k \quad (3.42)$$

The SINR for User  $k$  is RZF linear precoding under Rayleigh fading channel insert equation (3.41) into equation (3.15) then

$$\text{SINR}_k^{RZF} = \frac{|h_k^H N_c h_k^m|^2}{\left(\left(\frac{r_k - n_k}{x_k}\right) * h_k^{m-}\right)^2 + \sum_{j=1, j \neq k}^k |h_k^m h_j^H N_c|^2} \quad (3.43)$$

The spectral efficiency is given by

$$\begin{aligned} \text{SE}_k^{RZF} &= \log_2(1 + \text{SINR}_k^{RZF}) \\ \text{SE}_k^{RZF} &= \log_2 \left( 1 + \frac{|h_k^H N_c h_k^m|^2}{\left(\left(\frac{r_k - n_k}{x_k}\right) * h_k^{m-}\right)^2 + \sum_{j=1, j \neq k}^k |h_k^m h_j^H N_c|^2} \right) \end{aligned} \quad (3.44)$$

The SINR for User  $k$  is ZF linear precoding under Rician fading channel insert equation (3.41) into equation (3.21) then

$$\text{SINR}_k^{RZF} = \text{KP}_r \left( \frac{|h_k^m h_k^H N_c|^2}{E[|h_k^H N_c q_k|] \left(\left(\frac{r_k - n_k}{x_k}\right) * h_k^{m-}\right)^4 + \text{KP}_r \sum_{j=1, j \neq k}^k |h_k^m h_j^H N_c|^2} \right) \quad (3.45)$$

The spectral efficiency is given by

$$SE_k^{RZF} = \log_2(1 + \text{SINR}_k^{RZF}) \quad (3.46)$$

$$SE_k^{RZF} = \log_2 \left( 1 + \text{KPr} \left( \frac{|h_k^m h_k^H N_c|^2}{E[|h_k^H N_c q_k|] \left( \left( \frac{r_k - n_k}{x_k} \right) * h_k^{m-} \right)^4 + \text{KPr} \sum_{j=1, j \neq k}^k |h_k^m h_j^H N_c|^2} \right) \right) \right)$$

The RZF precoder becomes the ZF precoder as  $c \rightarrow 0$ , and becomes the MF precoder as  $c \rightarrow \infty$ . The sum-rate of RZF precoding approaches the sum capacity for low SINR with the assumption of Rayleigh fading channel and  $K = M$ . For high SINR, the sum-rate of RZF precoding approaches the sum-rate of ZF precoding. The performance of RZF precoding for a single-cell massive MIMO system is when the number of antennas at the BS,  $M$  is much larger than the number of users  $K$  and grows to infinity, that means  $M \gg K$  and  $M \rightarrow \infty$  [34].

Block diagram below shows the corresponding functional block.

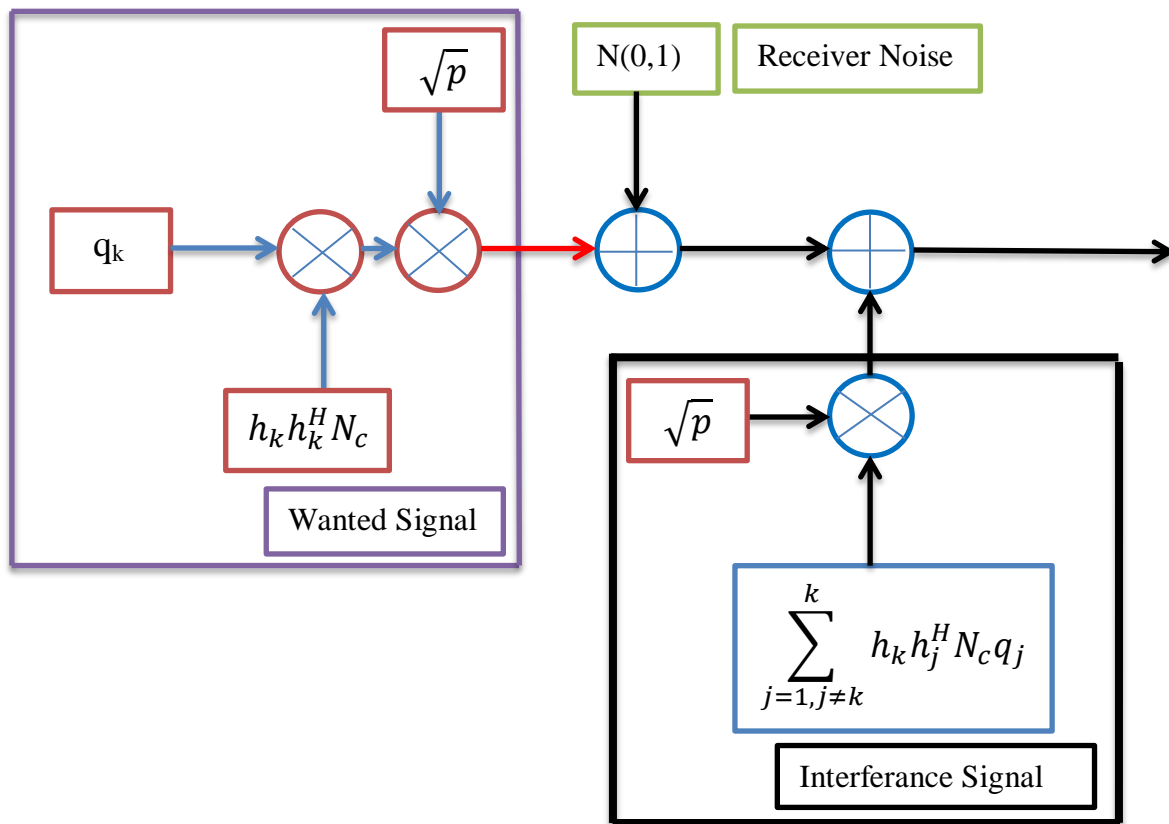


Figure 3-3 Functional block diagram for the single-cell downlink with RZF processing

## Chapter 4

### 4. Simulation Results and Analysis

The limitations of models are that the number of users  $K$  is not greater than the number of antennas  $M$  ( $K \ll M$ ). We considered a single cell Massive MIMO system, over Rayleigh fading channel and Rician Fading Channel with ZF, RZF and MF optimal linear pre-coding schemes.

All results are shown in terms of;

- ✓ The Spectral efficiency versus Signal-to-Interference Ratio (SINR)
- ✓ The Spectral efficiency versus the number of antenna arrays
- ✓ The Spectral efficiency versus the number of users under both fading channel

Main steps of calculating the capacity of the system are generating channel, defining precoding matrices with their power constraints, calculating SINR for each user, calculating the achievable rate of each user, calculating total capacity of all users hence total capacity of the system and finally taking an average of all realization to find the final system capacity.

#### 4.1. Spectral efficiency

The spectral efficiency of an encoding/decoding scheme is the average number of bits information per complex valued sample that it can reliably transmit over the channel under consideration. The spectral efficiency can be viewed as the average number of bit/s/Hz over the fading realizations [27][35]. In a single cell Massive MIMO system the spectral efficiency is given in.

$$SE = \sum_{k=1}^K A_k \quad (\text{bits/s/Hz})$$

Where  $S_e$  is the spectral efficiency in bits/s/Hz and  $A_k$  is the achievable rate of user  $k$ ,

The simulation analysis is done in below block diagram steps.

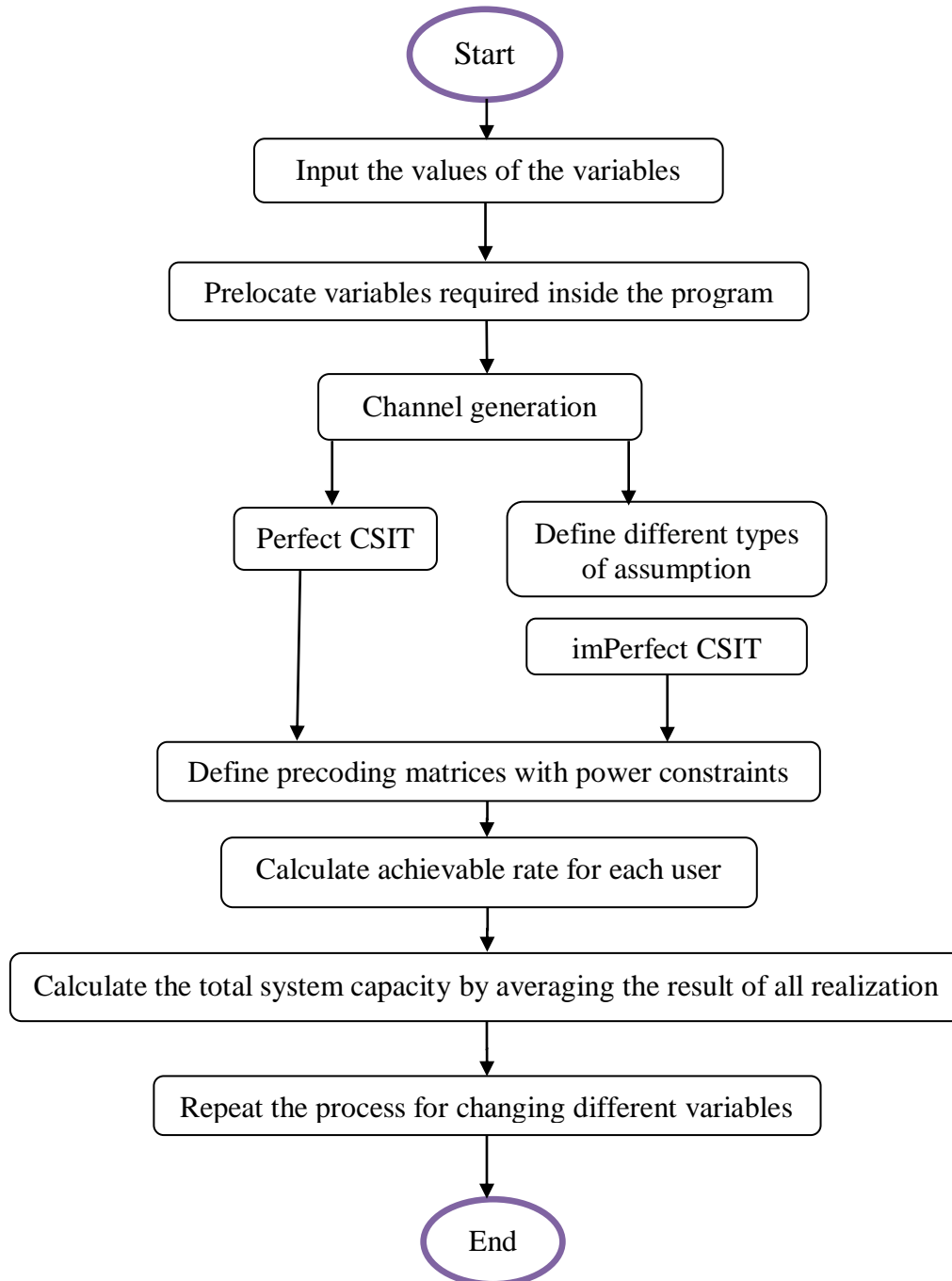


Figure 4-1 Block diagram of System design [1]

## The Massive MIMO Downlink System over Rayleigh Fading channel at the transmitter

Table 4-1 Value of different parameters

Parameter	Values	Remarks
Number of BS Antenna (Fixed)	M=30 and 60	Fixed
Range of BS antenna (When varied)	20 to 110	Variable
Number of users (Fixed)	K=15	Fixed
Number of users (Varied)	25 to 70	Variable
Rician model ( <b>K</b> ) factor	<b>K</b> =0.2 and 10	Variable

### 4.2. Massive MIMO Downlink System with perfect CSIT

#### 4.2.1. Change in capacity with the increasing the number of Base station antenna

Fifteen number of users were selected and the number of antennas changed from 20 to 110. Figure 4.1 shows the spectral efficiency of users across the entire SINR range. This system consists of the number of antennas  $M = 30$  and the number of users  $K = 15$ .

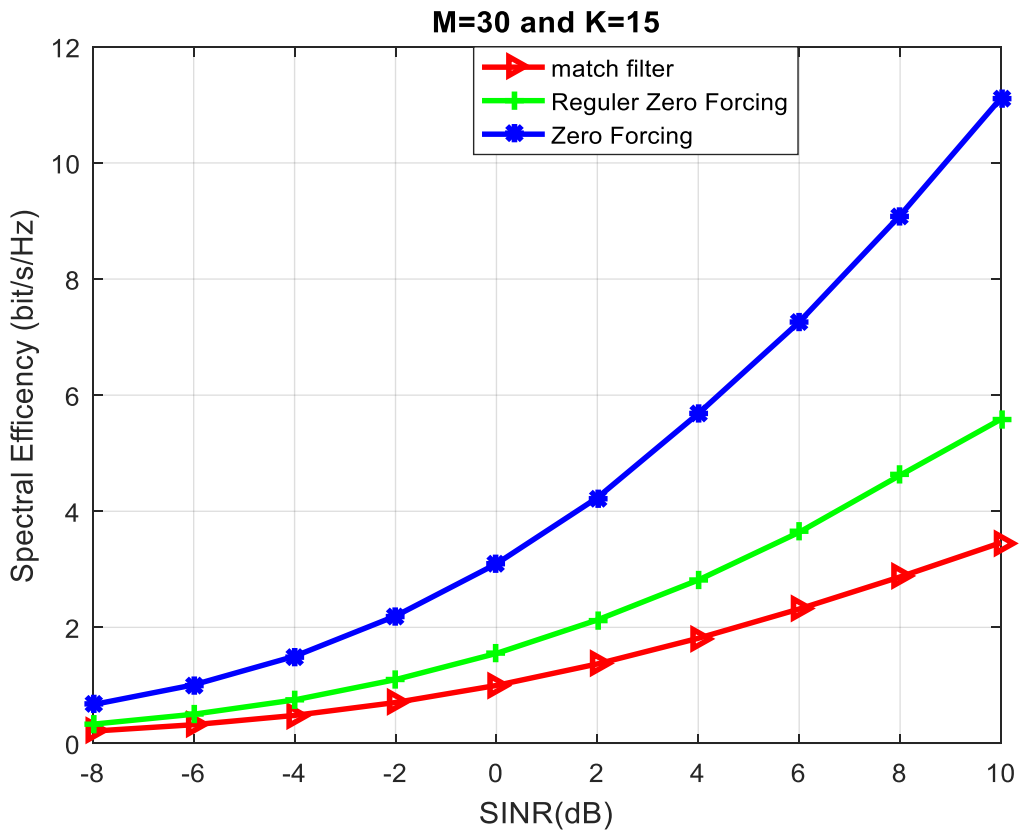


Figure 4-2 Spectral Efficiency versus SINR for ZF, RZF and MF.  $M = 30$  and  $K = 15$

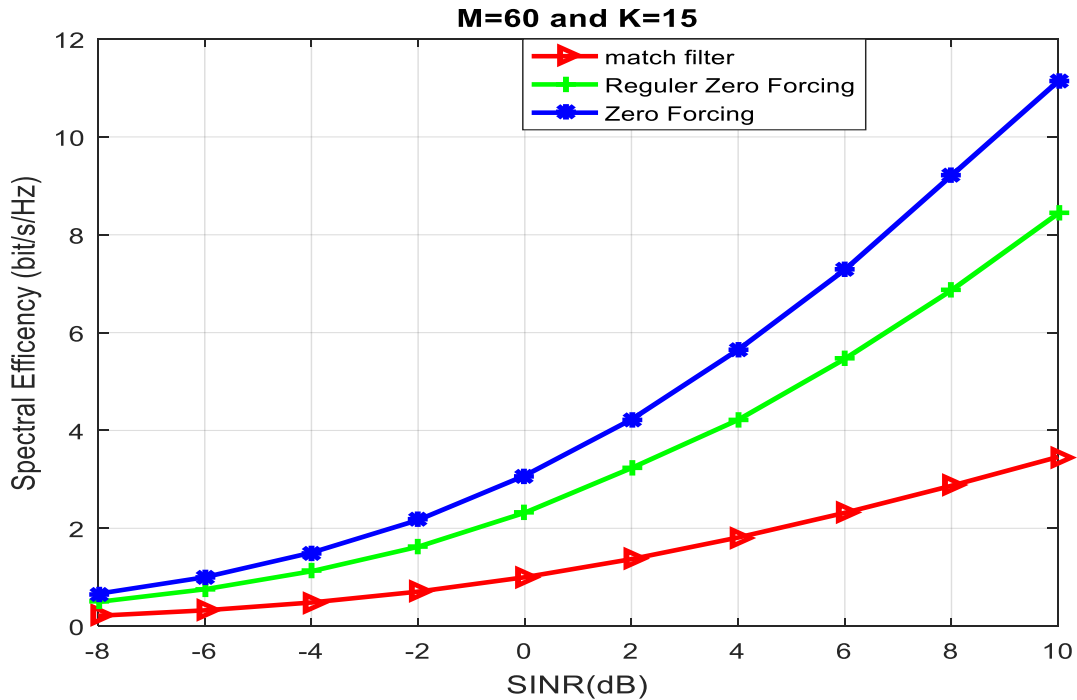


Figure 4-3 Spectral Efficiency v SINR for ZF, RZF and MF. M = 60 and K = 15

Table 4-2 For Rayleigh fading channel Spectral Efficiency versus SINR

SINR(dB)	No of BS antennas	Spectral Efficiency (bits/s/Hz)		
		MF	RZF	ZF
-6	M=30	0.321	0.51	1.01
	M=60	0.4	0.7536	1.3
2	M=30	1.36	2.128	4.231
	M=60	1.368	3.232	4.235
8	M=30	2.87	4.62	9.095
	M=60	2.95	6.865	9.999

As illustrated on figure 4.2 and 4.3 and table 4.2 all results shows that the system performance is improved. The spectral efficiency of users increase when the number of antennas from 30 to 60. The spectral efficiency of ZF, RZF and MF are increased by increasing the number of antennas. When ZF, RZF and MF are compared ZF linear pre-coding performs better than RZF and MF linear precoders.

#### 4.2.2. Spectral Efficiency against number of BS antennas over Rayleigh Fading

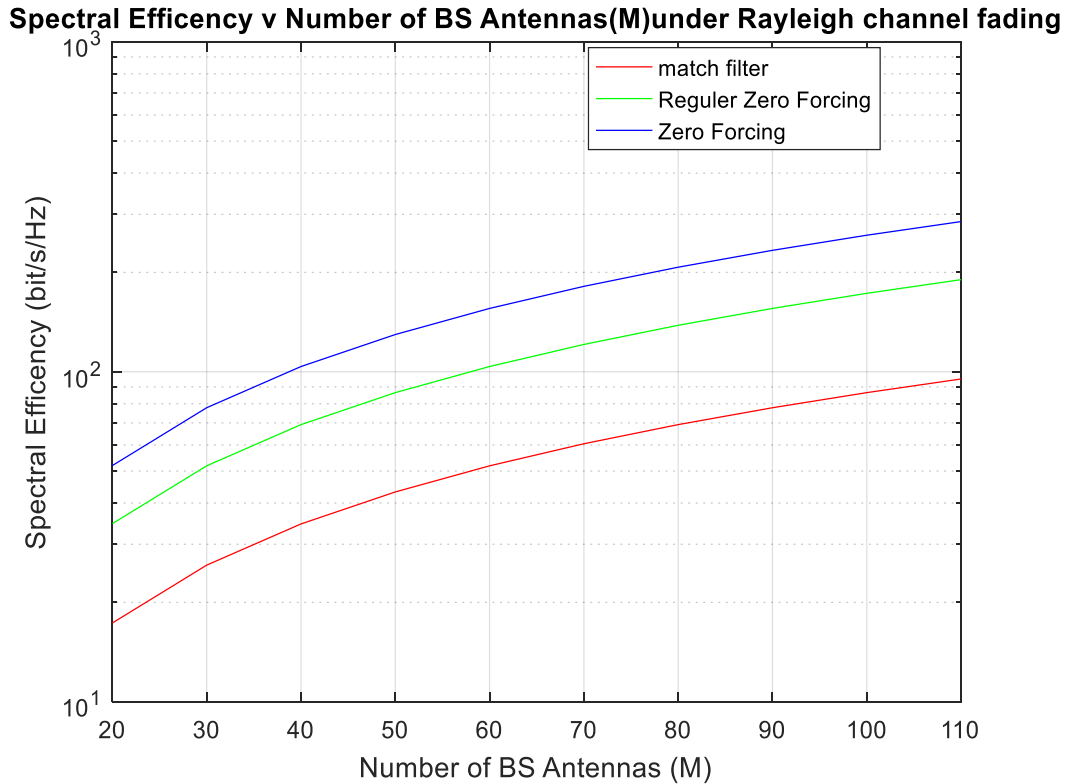


Figure 4-4 SE v No BS antennas over Rayleigh fading for ZF, RZF and MF.

Table 4-3 For Rayleigh fading channel Spectral Efficiency versus BS Antennas

No BS Antennas	Spectral Efficiency (bits/s/Hz)		
	MF	RZF	ZF
30	25.96	51.87	77.8
60	51.8	103	155
90	77	154	230

From figure 4.4 and table 4.3 increasing the number of base station antennas the spectral efficiency of all system precoding increases. As spectral efficiency increased to 10bits/s/Hz when the antennas were increased from 20 to 110, the number of users is  $K=15$  and the input SINR is 10dB. Therefore, it can be noticed that the total spectral efficiency increases when the number of BS antennas grows, which implies that the large-scale antenna dramatically benefits the spectral efficiency. Further more it can be observed that both the ZF and the RZF has a higher spectral efficiency with increasing number of antennas as compared to the MF. The

results show that ZF gives better performance at RZF and MF. All spectral efficiencies with linear precoding increased very importantly by increasing the number of BS antennas.

#### 4.2.3. Spectral Efficiency against the number of users over Rayleigh Fading

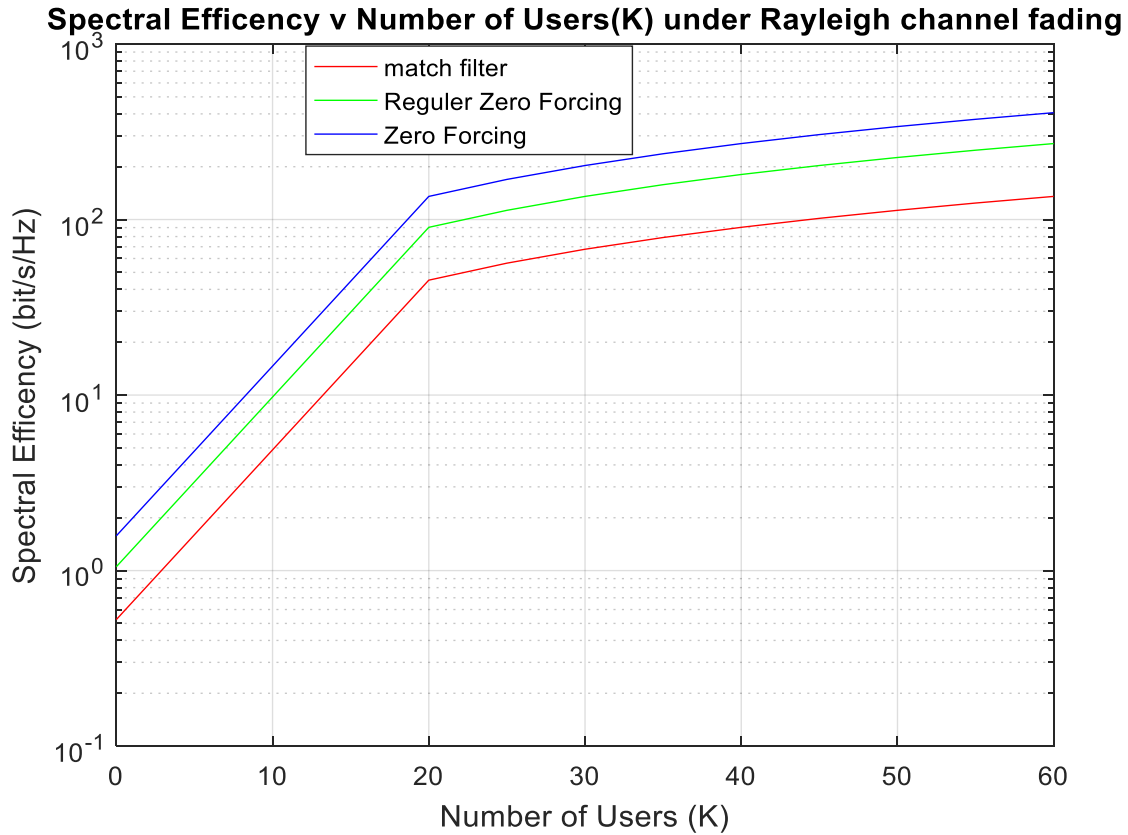


Figure 4-5 SE vs number of users over Rayleigh fading for ZF, RZF and MF.

Table 4-4 For Rayleigh fading channel Spectral Efficiency versus Number of Users

No of Users	Spectral Efficiency (bits/s/Hz)		
	MF	RZF	ZF
10	0.6	1.5	1.8
20	45	90.3	135
50	112	220	338

From figure 4.5 and table 4.4 the spectral efficiency versus the number of users MF increases slowly as compared to ZF and RZF, although the number of users K increases. Additionally, the spectral efficiency with MF is less than ZF and RZF. Ironically, the results show that ZF gives better performance than RZF and MF.

### 4.3. Massive MIMO Downlink System with imperfect CSIT

#### 4.3.1. Change in capacity with the increase of the number of Base station antenna

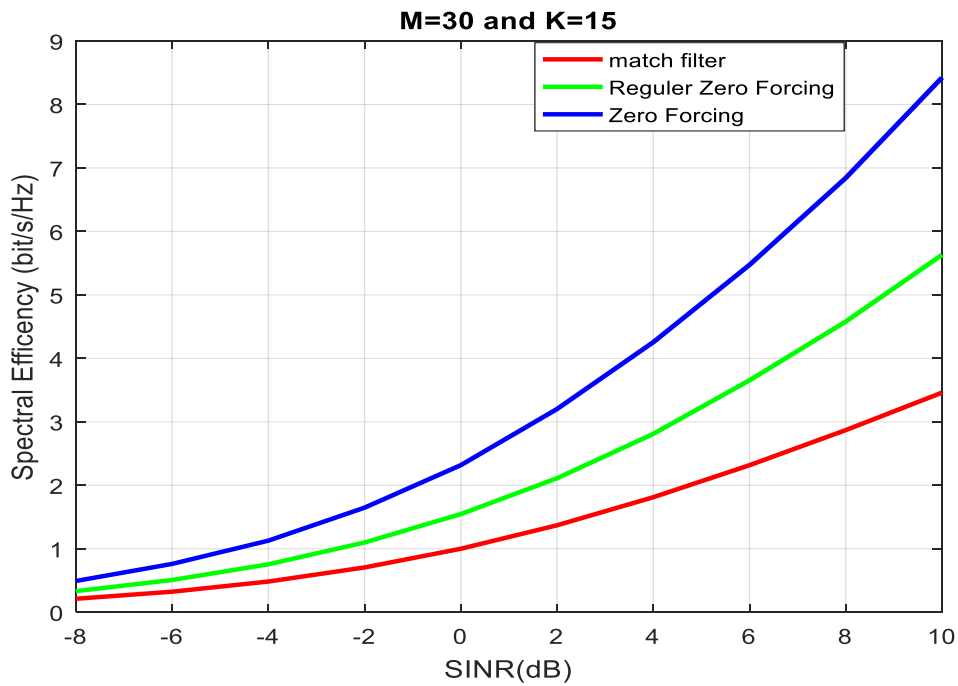


Figure 4-6 SE v SINR for ZF, RZF and MF. M= 30 and K = 15 under imperfect CSI

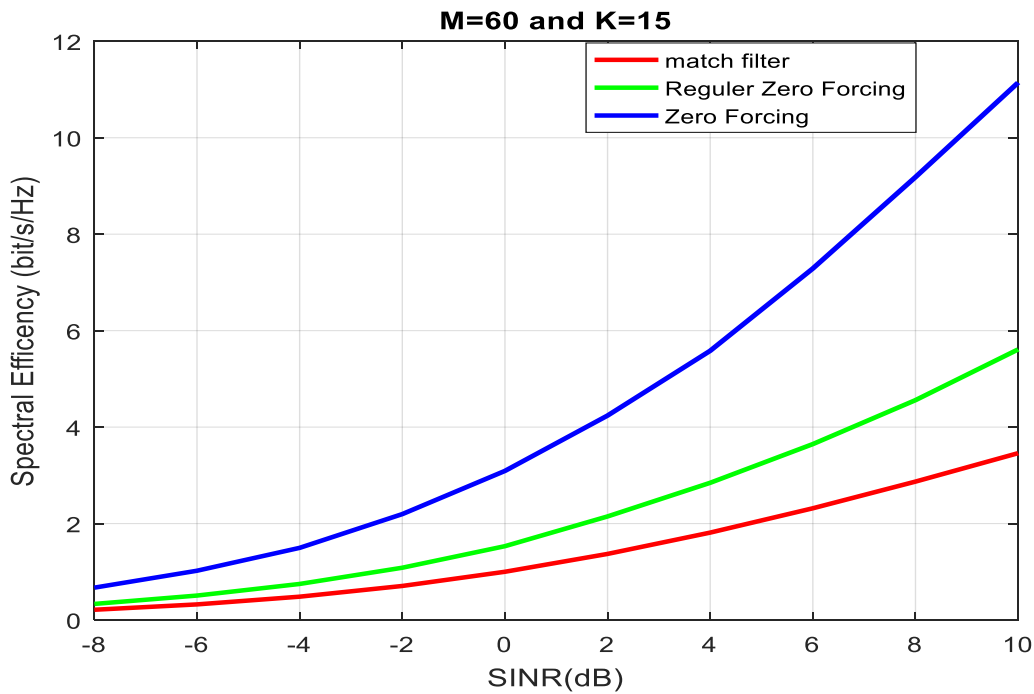


Figure 4-7 SE v SINR for ZF, RZF and MF. M= 60, K = 15 under imperfect CSI

Table 4-5 For Rayleigh fading channel Spectral Efficiency versus SINR imperfect CSI

SINR(dB)	No of BS antennas	Spectral Efficiency (bits/s/Hz)		
		MF	RZF	ZF
-6	M=30	0.3223	0.5017	0.7613
	M=60	0.3334	0.599	1.1
2	M=30	0.7057	1.1	1.648
	M=60	0.757	1.3	2.196
8	M=30	2.6	4.15	6.843
	M=60	2.7	4.9	9.184

From table 4.5 Regarding the spectral efficiency versus the number of SINR MF increases slowly related to ZF and RZF, although the number of Base station antenna increases. The Spectral efficiency of ZF, RZF and MF are increased by increasing the number of antennas. Comparing ZF, RZF and MF, ZF gives the better performance SINR. Increasing the number of antennas makes ZF able to be used at high SINR. Due to imperfect channel state information at the transmissions, the capacity of all precoder go down from high place or position than the capacity with perfect channel state information at the transmissions.

### 4.3.2. Spectral Efficiency against number of BS antennas

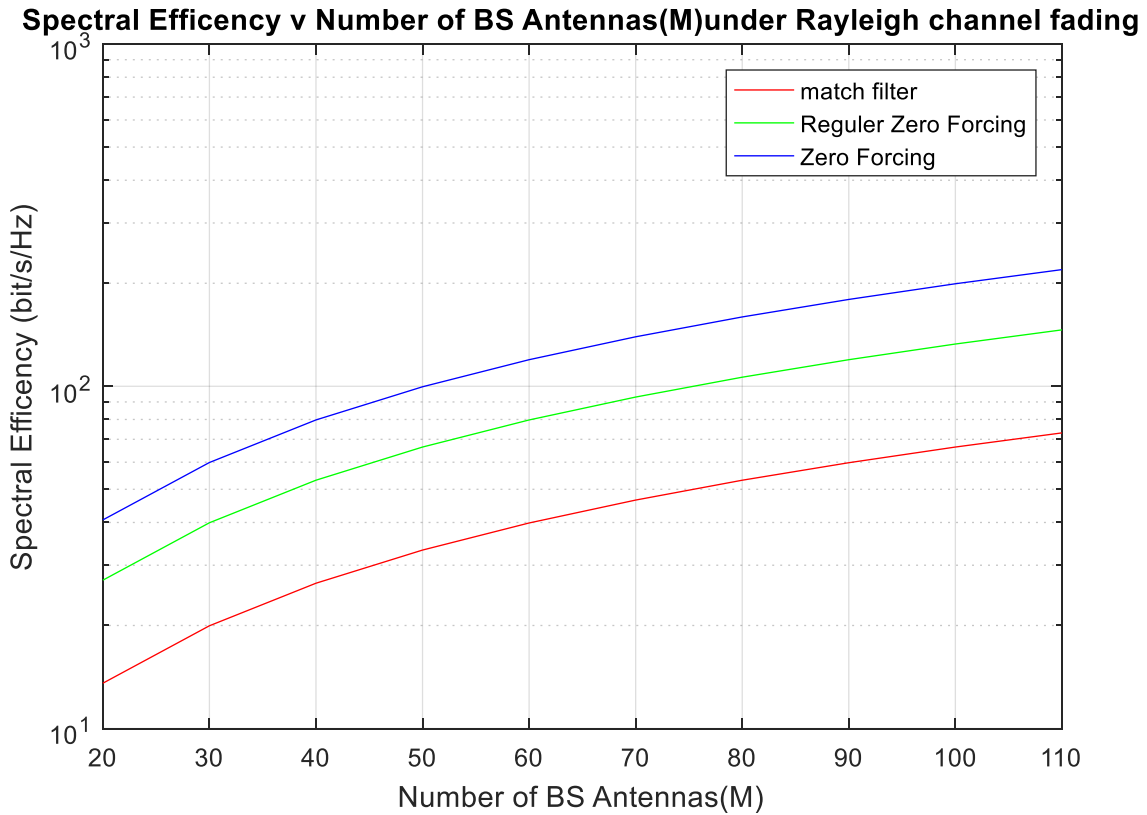


Figure 4-8 SE v No BS antennas for ZF, RZF and MF. under imperfect CSI

**Table 4-6 For Rayleigh fading channel SE v BS Antennas imperfect CSI**

No BS Antennas	Spectral Efficiency (bits/s/Hz)		
	MF	RZF	ZF
30	19.96	39.8	59.8
60	39.8	79.7	119
90	59	119	179

From figure 4.8 and table 4.6 as increase the number of base station antennas the spectral efficiency of all system precoding increases. The spectral efficiency and the number of base station antennas so we can observe that the spectral efficiency increased to 10bits/s/Hz when the antennas were increased from 20 to 110, the number of users is  $K=15$  and the input SINR is 10dB.

### 4.3.3. Spectral Efficiency against the number of users

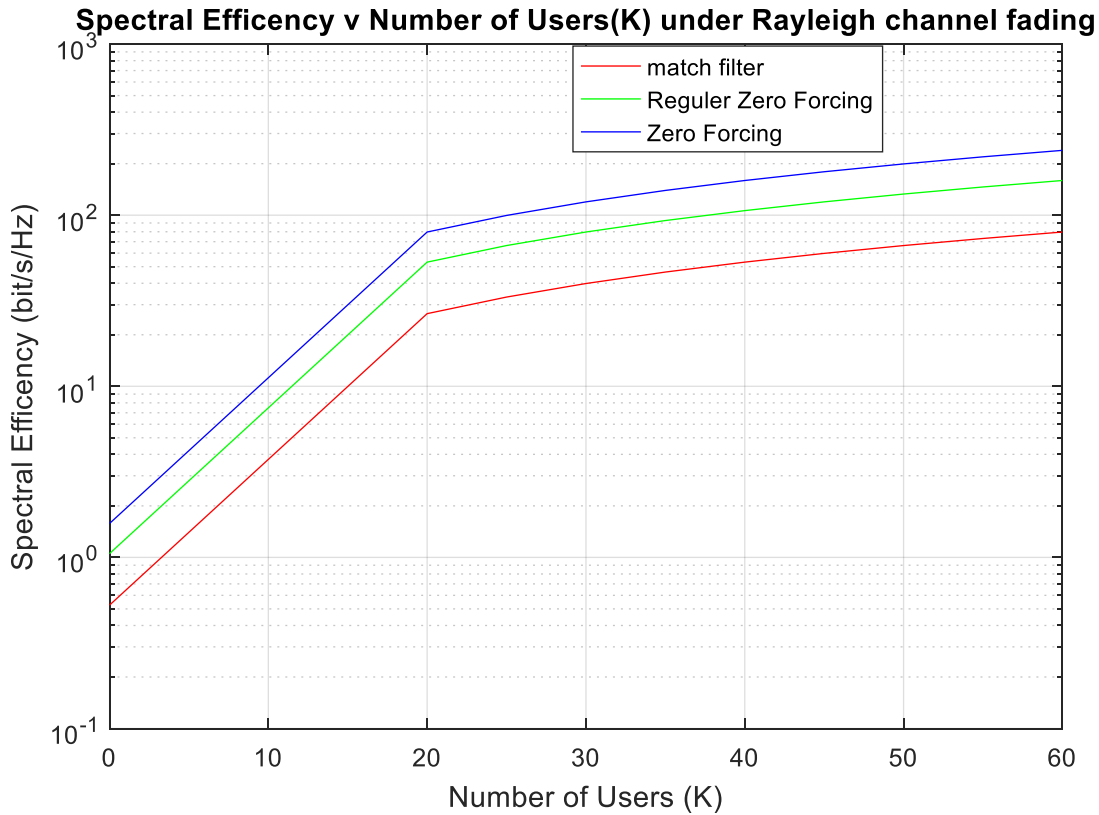


Figure 4-9 SE v number of Users for ZF, RZF and MF. under imperfect CSI

Table 4-7 For Rayleigh fading channel SE versus Number of Users imperfect CSI

No of Users	Spectral Efficiency (bits/s/Hz)		
	MF	RZF	ZF
10	0.526	1.05	1.58
20	26.5	53.1	79.5
50	66	132	199.3

From figure 4.9 and table 4.7 the spectral efficiency versus the number of users with MF increases slowly relate to ZF and RZF although the number of users K increases. In this case, the performance of MF is less affected by arbitrary assumption than ZF and RZF.

## 4.4. M-MIMO Downlink System over Rician Fading channel at the transmitter

### 4.4.1. Change in capacity with the increase the number of BS antenna in Rician

The number of Base station antenna  $M=30$  and users  $K = 15$  was choosen. The number of antennas was changes from 20 to 110. Figure 4.10 and figure 4.11 shows the spectral efficiency

of users across the entire SINR range. This system consists of the number of antennas  $M = 30$ , the number of users  $K = 15$  and Rician  $\mathbf{K}$  factor ( $\mathbf{K}=0.2$  and  $\mathbf{K}=10$ ).

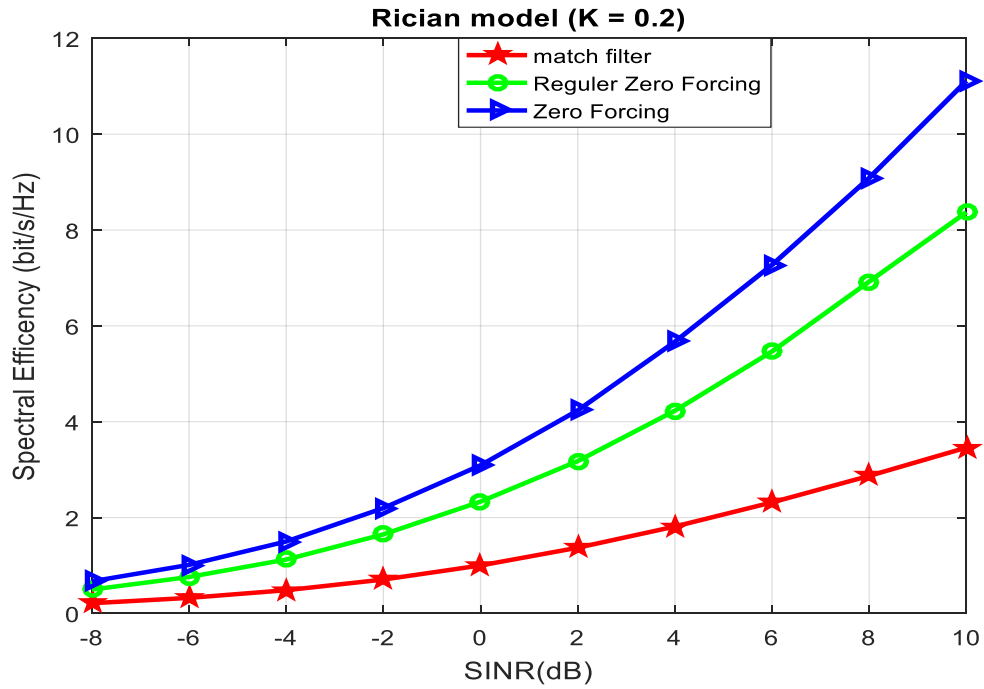


Figure 4-10 SE vs SINR for ZF, RZF and MF (R factor  $K=0.2$ ).  $M = 30$ ,  $K = 15$

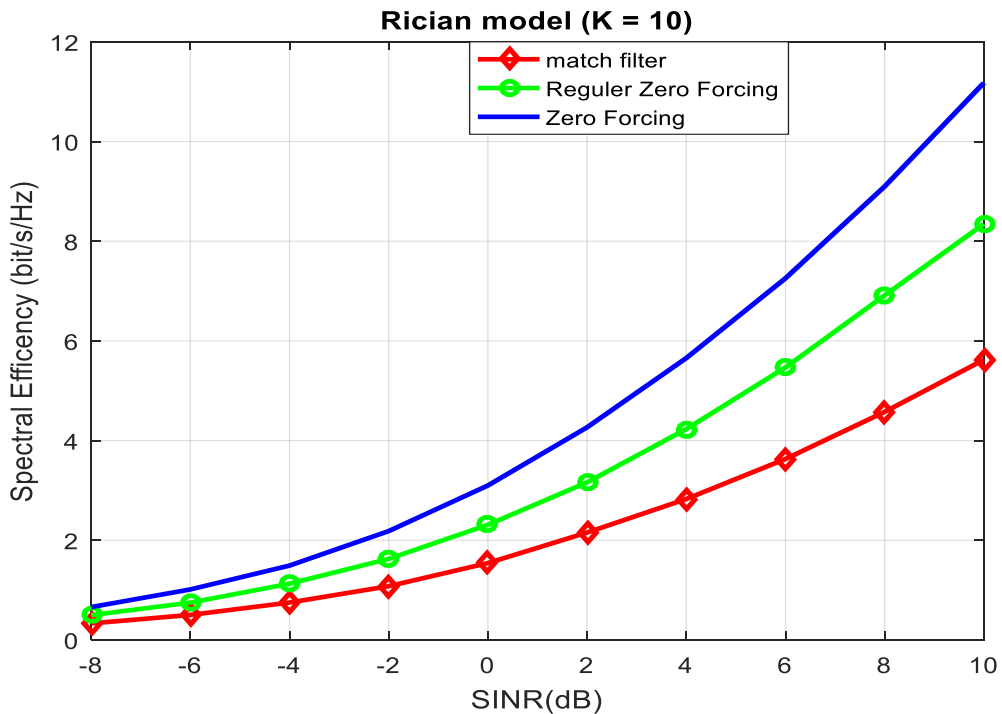


Figure 4-11 SE vs SINR for ZF, RZF and MF, (R factor  $K=10$ ).

Table 4-8 For Rician fading channel SE versus SINR for Rician Factor  $\mathbf{K}=0.2$  and  $\mathbf{K}=10$

SINR (dB)		Rician K factor	Spectral Efficiency (bits/s/Hz)		
			MF	RZF	ZF
-6	M=30	$\mathbf{K}=0.2$	0.2	0.5	0.9
	K=15	$\mathbf{K}=10$	0.32	0.61	1.1
4	M=30	$\mathbf{K}=0.2$	1.9	4.11	5.79
	K=15	$\mathbf{K}=10$	2.9	4.42	5.86
6	M=30	$\mathbf{K}=0.2$	2.3	5.16	7.12
	K=15	$\mathbf{K}=10$	3.5	5.7	7.89

From the **4.10** and **4.11** figure and table 4.8 all results show that the system performance is improved. The spectral efficiency of users increase when the number of antennas from 30 to 60. The spectral efficiency of ZF, RZF and MF are increased by increasing the number of antennas. If the Rician  $\mathbf{K}$  factor increase the spectral efficiency is most probably increased in MF while in RZF and ZF spectral efficiency highly increased. In these case M=30 and K=15 the same to under Rayleigh fading channel. Generally From figure 4.10 and 4.11, it can be observed that at low and high SINR and Rician K factor ZF pre-coder performs better than RZF and MF precoders. The results show that ZF gives better performance at all.

#### 4.4.2. Spectral Efficiency against number of BS antennas under Rician Channel

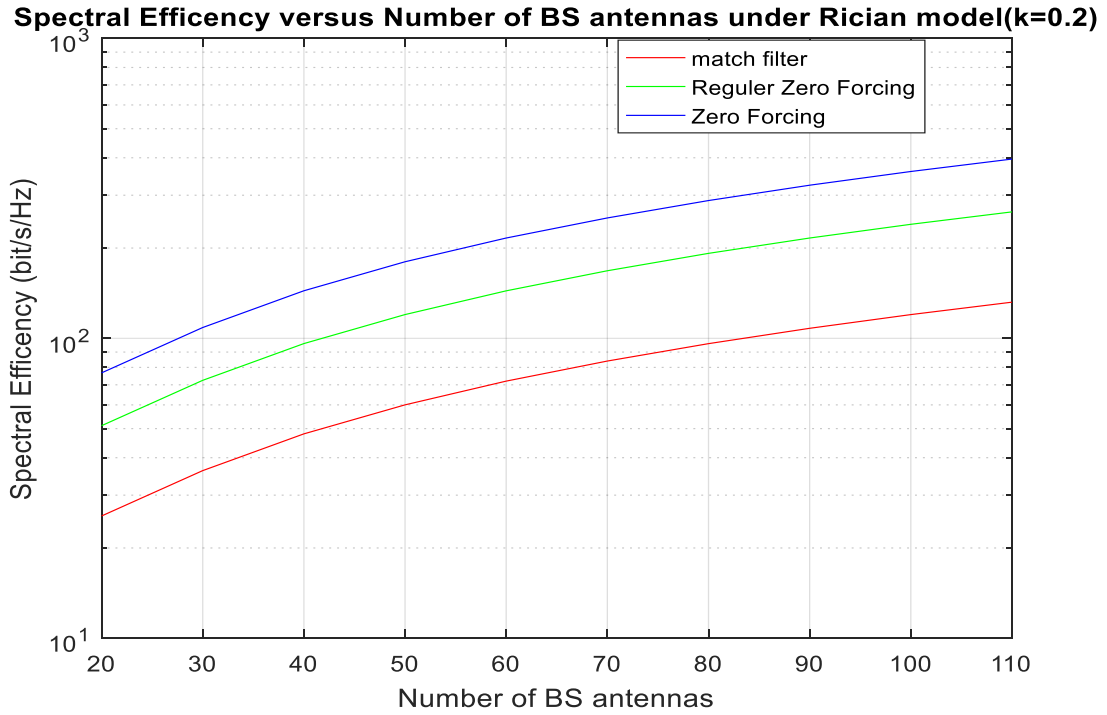


Figure 4-12 SE v BS antennas ZF, RZF and MF (R factor  $\mathbf{K}=0.2$ ).  $K = 15$ ,  $\text{SNR} = 10\text{dB}$ .

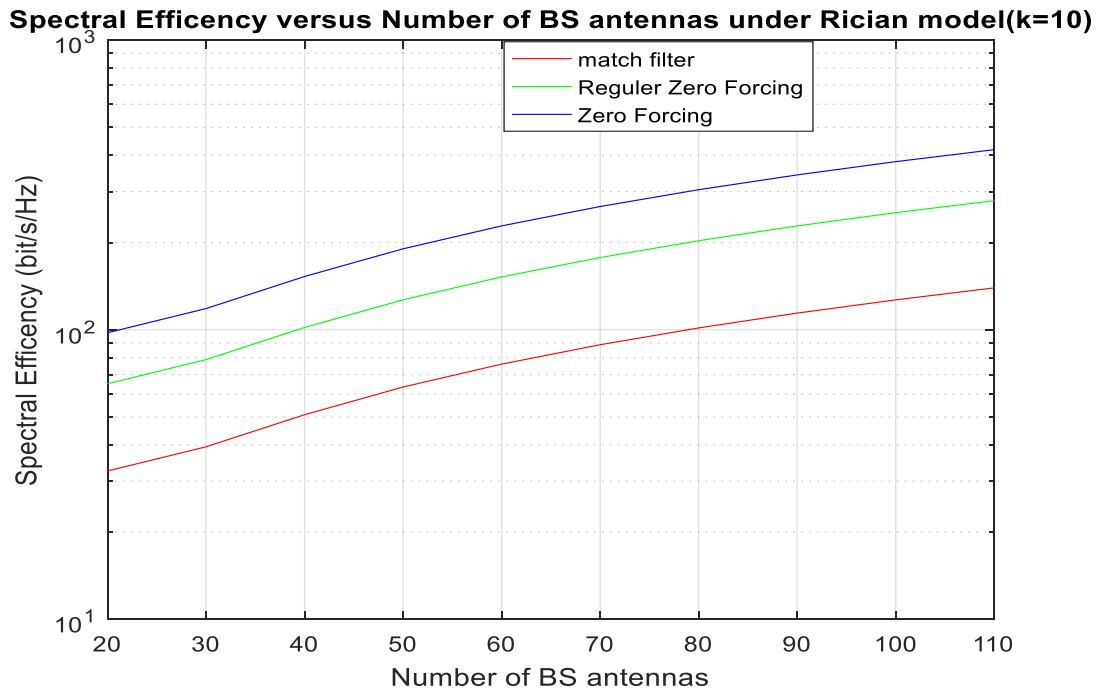


Figure 4-13 SE v BS antennas ZF, RZF and MF (R factor  $\mathbf{K}=10$ ).  $K = 15$ ,  $\text{SNR} = 10\text{dB}$ .

Table 4-9 For Rician fading channel Spectral Efficiency versus BS Antennas

No BS Antennas	Rician K Factor	Spectral Efficiency (bits/s/Hz)		
		MF	RZF	ZF
30	<b>K=0.2</b>	36.2	72.4	108
	<b>K=10</b>	39	78	118
60	<b>K=0.2</b>	72	144	216
	<b>K=10</b>	76	152	228
90	<b>K=0.2</b>	108	216	324
	<b>K=10</b>	144	228	341

From figure 4.12 and table 4.9 as Rician K factor and number of base station antennas are increased the spectral efficiency of all system precoding increases. It can be observed that the spectral efficiency increased to 10bits/s/Hz when the antennas was increased from 20 to 110 and Rician **K** factor 0.2 to 10, further more it can be observed that both the ZF and the RZF has a higher spectral efficiency and Rician **K** factor with increasing number of antennas as compared to the MF. The results show that ZF gives better performance at RZF and MF depending on Rician **K** factor. All spectral efficiencies with linear precoding increased very important by increasing the Rician k factor and number of Base Station antennas.

#### 4.4.3. SE against the number of users under Rician Fading Channel

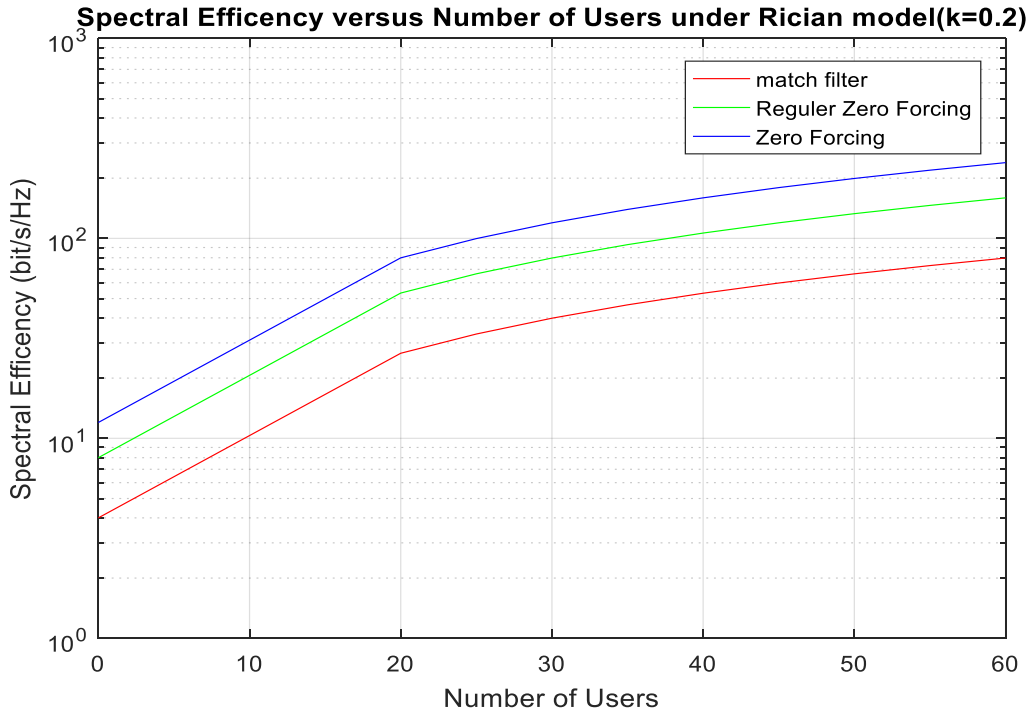


Figure 4-14 SE v number of users ZF, RZF and MF (factor  $K=0.2$ ).  $M = 60$ ,  $SNR = 10\text{dB}$

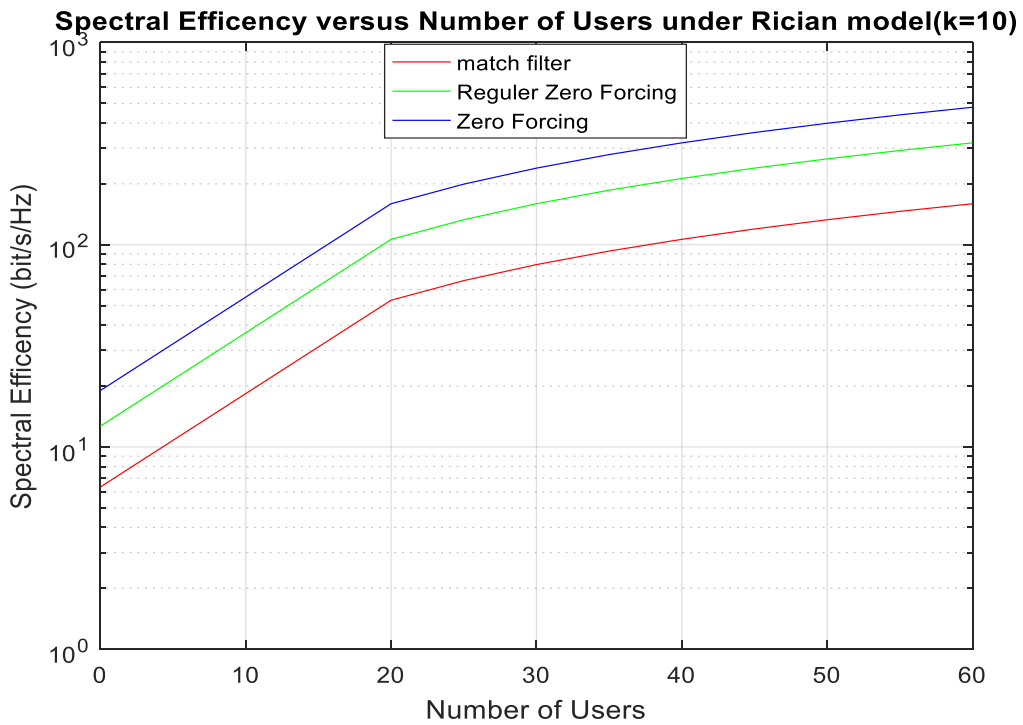


Figure 4-15 SE v number of users ZF, RZF and MF (R factor  $K=10$ ).  $M = 60$ ,  $SNR = 10\text{dB}$

Table 4-10 For Rician fading channel Spectral Efficiency versus Number of User

No Users	Rician $\mathbf{K}$ Factor	Spectral Efficiency (bits/s/Hz)		
		MF	RZF	ZF
10	$\mathbf{K}=0.2$	10	20	25
	$\mathbf{K}=10$	19	29	35
20	$\mathbf{K}=0.2$	26.6	53.2	79.8
	$\mathbf{K}=10$	53	106	159
50	$\mathbf{K}=0.2$	66.4	132.2	199
	$\mathbf{K}=10$	132	256	398

From figure 4.14 and 4.15 and table 4.10 the spectral efficiency versus the number of users with MF increases slowly related to ZF and RZF although the number of users and Rician  $\mathbf{K}$  factors increases. In fact, if Rician  $\mathbf{K}$  factor increase the spectral efficiency with MF is less than ZF and RZF. Generally the results show that ZF gives better performance over RZF and MF in rician. Results from simulations figure 4.10 up to figure 4.15 shows different Spectral Efficiency versus SINR, Spectral Efficiency versus Base station antenna and Spectral Efficiency versus number of Users for Rician k-factor values ( $\mathbf{K}=0.2$  and  $\mathbf{K}=10$ ). With Rician  $\mathbf{K}$  factor results obviously demonstrated that the higher the rician  $\mathbf{K}$  factor Values the higher the Spectral Efficiency recorded. Highest value of spectral efficiency was shown when  $\mathbf{K}$  factor is 10. Results also confirms that when  $\mathbf{K} \rightarrow \infty$  a Rician channel behaves like a Rayleigh channel.

Table 4-11 Summary of Rayleigh and Rician fading Spectral Efficiency v Number of SINR

		Rayleigh fading Channel						Rician Fading Channel				
SINR	No BS	Spectral Efficiency (bit/s/Hz)										
		Perfect CSI			Imperfect CSI							
		MF	RZF	ZF	MF	RZF	ZF	K factor	MF	RZF	ZF	
-6	M=30	0.321	0.51	1.01	0.3223	0.5017	0.7613	M = 30	K= 0.2	0.2	0.5	0.9
	M=60	0.4	0.7536	1.3	0.334	0.7536	1.1		K= 10	0.32	0.61	1.1
2	M=30	1.36	2.128	4.231	0.7057	1.1	1.648	M = 30	K= 0.2	1.37	3.186	4.196
	M=60	1.368	3.232	4.235	0.757	1.3	2.196		K= 10	2.134	3.2	4.23
8	M=30	2.87	4.62	9.095	2.6	4.15	6.843	M = 30	K= 0.2	2.87	6.4	8.114
	M=60	2.95	6.865	9.999	2.7	4.9	9.184		K= 10	3.615	6.554	9.297

From table 4.11 the spectral efficiency of users increase when the number of antennas from 30 to 60. Regarding the spectral efficiency versus the number of SINR MF increases slowly related to ZF and RZF, although the number of Base station antenna increases. Increasing the number of antennas makes ZF able to be used at high SINR. Due to imperfect CSIT, the capacity of all precoder go down hastily from high place or position than the capacity with perfect CSIT. As increase the number of base station antennas the spectral efficiency of all system precoding increases. In this case, the performance of MF is less affected by arbitrary assumption than ZF and RZF. If the Rician k factor increase the spectral efficiency is most probably increased in MF while in RZF and ZF spectral efficiency highly increased. At low and high SINR and Rician K factor ZF pre-coder performs better than RZF and MF precoders. As Rician K factor and number of base station antennas are increased the spectral efficiency of all system precoding increases. The results show that ZF gives better performance at RZF and MF depending on Rician K factor.

## Chapter 5

### 5. Conclusion and Recommendations for Future Work

#### 5.1. Conclusion

Now a day, massive multiple-input multiple-output (MIMO) system has attracted many research interests in wireless communications. Among many performance aspects and design, modeling problems regarding massive MIMO, precoding at the base station (BS) is one of the most important function components to ensure reliable downlink transmission. This thesis provides a thesis of linear precoding techniques for downlink transmission under both perfect channel state information and imperfect channel state information. In both cases, a ZF, RZF and MF linear precoding techniques are reviewed, mathematically modeled and performance of ZF, RZF and MF linear precoding where compared under Rayleigh and Rician fading channel. Due to imperfect channel state information at the transmitter, the capacity of all precoders reductions than the capacity with perfect channel state information at the transmitter number of BS antenna is crucial to the performance. On the other hand, number of terminals antenna reduce the performance due to user interference. As the number of terminals increases the mathematical complexity of SINR increases the deterministic equivalent expressions of the spectral efficiency were obtained for the MF, ZF and RZF linear precoding schemes. It is found that the ZF and RZF linear precoding schemes attain a higher spectral efficiency than MF at the high SINR item. Therefore, a massive MIMO system is a fundamental in next wireless system. It presents advantages in terms of the achievable rate and the spectral efficiency.

Generally, ZF gives better performances at all transmission power over RZF and MF in perfect and imperfect channel state information under Rayleigh fading channel. This conclusion also works in Rician Fading Channel.

## 5.2. Recommendations for Future Work

Here is list of recommendations to the possible extensions of the works of this thesis:

- Linear precoding schemes developed in this thesis consider Rayleigh fading channel and Rician Fading Channel, but it can be possible to extend these schemes for a system with the Nakagami fading channel.
- Linear precoding schemes that developed in this thesis mainly focused on ZF, RZF and MF, but Future works can also work non linear precoding.
- In this thesis work Linear precoding was assumed at downlink transmission. Therefore it is desirable to study and incorporate adaptive transmit beam forming which is a key to increase spectral and energy efficiency for very large antenna arrays.
- This thesis focused primarily on Massive MIMO system but Future works can be able to extend to Point-to-Point MIMO and Multiuser MIMO.
- This thesis does not considered modulation and demodulation techniques.
- For future work this thesis can be work with finite-alphabet inputs.
- The thesis consider spectral efficiency versus SINR, spectral efficiency versus Base station antenna and spectral efficiency versus User but Future works can extend spectral efficiency against BER under Rayleigh fading channel and Rician Fading Channel.

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