Analysis of Unhealthy Attitude on Marriage and its Impact on the Dynamics of Divorce using Mathematical Model

MSc. Thesis

By: Etsehiwot Gobezayewu

Hawassa, Ethiopia

June 27, 2024

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Under the guidance of

Dr. Admasu Tadesse

Submitted to: College of Natural and Computational Science, Department of Mathematics in Partial Fulfillment of the Requirement for the

Degree of Masters in

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Hawassa, Ethiopia

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Declaration

I declare that the thesis entitled " **Analysis of Unhealthy Attitude on Marriage and its Impact on the Dynamics of Divorce using Mathematical Model** " is my own original work carried out under the supervision of **Dr. Admasu Tadesse , Tesfaye Tadesse (MSc)** and has been submitted to the Department of Mathematics of Hawassa university in partial fulfillment of the requirements for the award of master of science in mathematics. I earnestly state that, this thesis has not been submitted or done by others for any academic degree in any other universities or institutions, and all the source materials that I have been used for this thesis are duly acknowledged and indicated by means of complete references.

Submitted By :

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Approval Sheet -1

This is to officially state that the study entitled "**Analysis of Unhealthy Attitude on Marriage and its Impact on the Dynamics of Divorce using Mathematical Model** " is an original work carried out by **Etsehiwot Gobezayewu, ID No. GPMASTR/0001/15** under my guidance and supervision. This work has been done by Etsehiwot Gobezayewu for the partial fulfillment of the award of the Degree of Master of Science in Mathematical and Statistical Modeling from Hawassa University. Daily acknowledgments are done during her course of investigation. Therefore, I recommend that it would be accepted as fulfilling the thesis requirements.

Approved By :

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Approval Sheet -2

We, the undersigned, members of the Board of Examiners of the final open defense by **Etsehiwot Gobezayewu** have read and evaluated her thesis entitled "**Analysis of Unhealthy Attitude on Marriage and its Impact on the Dynamics of Divorce using Mathematical Model**" and examined the candidate. This is, therefore, to certify that the thesis has been accepted in partial fulfillment of the requirement of the Degree of Master of Science in Mathematical and Statistical Modeling.

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Final approval and acceptance of the thesis is contingent upon the submission of the final copy of the thesis to the School of Graduate Studies (SGS) through the Department/School Graduate Committee (DGC/SGC) of the candidate's department.

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List of Abbreviations

Abbreviations	Meaning
R_0	Reproduction Number
MATLAB	Matrix Laboratory
ODE	Ordinary Differential Equation
UAFE	unhealthy attitude free equilibrium point
UAPE	Unhealthy attitude present equilibrium point
USA	United State America
INE	Spanish statistics Institution
FBSM	Forward-Backward sweep method
IVP	initial value problem
EEP	endemic equilibrium point

Abstract

Divorce is the dissolution of two partners marriage, which is a serious problem challenging the establishment of the family in a routine manner and causing severe impacts on the emotional and mental health of the individual. In this thesis, we modified and analyzed a mathematical model that describes the spread of unhealthy attitude on marriage and its impact on the dynamics of divorce. The population was divided in to six compartments and analyzed using a system of non-linear ordinary differential equations. The basic reproduction number R_0 is calculated using next generation matrix operator. We have found two equilibrium points, namely the unhealthy attitude free equilibrium point and the unhealthy attitude present equilibrium point. We have established the conditions for the stability of equilibrium points. The unhealthy attitude free equilibrium point is both locally and globally stable if $R_0 < 1$, and the unhealthy attitude present equilibrium point is also both locally and globally stable if $R_0 > 1$. The modified model exhibits a forward bifurcation whenever $R_0 < 1$, which indicates the threshold parameter plays an important role in reducing the spread of unhealthy attitude on marriage. Secondary data sources for divorce case were collected from Hawassa first instance court. The data were fitted to the model to estimate some parameter values using Least Square optimization method. Sensitivity analysis was performed to identify parameters which are sensitive to the reproduction number. According to this the contact rate between married and individuals who have unhealthy attitude β_1 and the rate of transfer from unhealthy attitude to healthy counseling ϕ plays an important role in reducing R_0 less than unity. Finally, we performed numerical simulations using ODE 45 codes to support the analytical results in agreement with numerical solutions.

Keywords: Marriage divorce, Parameter estimation, Unhealthy attitude, Least Square Fitting, Stability Analysis, Forward Bifurcation, Sensitivity Analysis.

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Chapter 1

1 Introduction

1.1 Background of study

Marriage is a socially and legally recognized union usually begins with a ceremony named weeding, which formally or legally unites the marriage partners [13]. It is regarded as a symbol of love, stability, and dedication. It's viewed as a means of cementing partner relationships, leaving a lasting legacy, bolstering families, creating a strong bond between them, and creating a stable atmosphere for raising kids [17]. Basically, marriage is a consensual or contractual agreement that can be recognized by law. Sometimes conflict between a husband and wife leads to divorce. Divorce is a legal way for married couples to end their marriage and the legal dissolution of a marriage whereby a couple are no longer legally bound to one another and divorce is only acknowledged in today's modern culture if it is lawful and sanctioned by legislation [23].

The dissolution of marriage contracted between spouses by the judgment of a court or by an act of the legislature is called divorce. Divorce is generally understood to be the dissolution of a marriage. The majority of divorces, according to experts, involve the following: minor arguments, serious arguments because of the recurrence of such minor arguments, separation or disinclination of one partner toward the other, repeated long-term sheer offs and the exhaustion caused by them, dispute, emotional divorce, and finally legal divorce [20].

Literature shows that various factors are contributing to divorce few of them are infidelity (unfaithfulness), employment status, drinking or drug use, lack of open discussion, physical or mental abuse and not meeting family obligations, marriage partner abuse, sexual incompatibilities, childlessness within the first marriage, interference from outside and loss of love [19, 28]. In Ethiopia both early age of marriage and childlessness has a significant impact on the risk of divorce [1]. The marital stability within a couple's social network also play a role in whether their union lasts. There is a 75% chance that a couple's own marriage will end if they have friends who have divorced. Because of this association, even couples who are two degrees away from divorce still have a 33% increased risk, some sociologists believe divorce is a social contagion [30].

Divorce is a typical occurrence in today's society. Divorce rates have risen considerably not only in developed countries but also in the developing nations. All over the world, the divorce rate among couples is increasing at an alarming rate. Based on statistics from 2021, Maldives tops the list with the highest divorce rate per 1,000 people, standing at 5.52 and followed by Kazakhstan, Russia, Belgium, and Belarus. United States and Ethiopia comes in midway, with a divorce rate of 2.7 and 2.6 per 1,000 peoples respectively.

According to Hawkins A.J. and Fackrell T.S. [14] in the United States 40 - 60 percent of all marriages end in divorce. Besides in USA couples marrying for the first time continue to

face a 50% chance of divorce during their life time.
The rate of divorce in many African homes is rising quickly as a result of a developing country that just accepted the externalities without critically analyzing them [32]. In the case of Ethiopia approximately 45 percent of all first marriages ends in divorce within 30 years [41]. These are national-level statistics that also include metropolitan regions, and Ethiopia's divorce rate has caught up to that of the United States. Data from the Vital Event Registration and Information Agency shows that between 2021 and 2022, there were 16,035 divorces in Addis Abeba alone.
In its broad sense, divorce has various negative effects on society as a whole. Its impacts are felt psychologically, physically, socially, and economically. Divorce is now a major issue that makes it difficult to establish a family in a regular way and causing severe impacts on the emotional and mental health of the individual especially, women. Divorce affects not only adults but also children. studies suggest that children are at higher risk of mental health issues and substance abuse due to their familial circumstances. children with divorced parents being more likely to drop out of school or fall victim to adult mental health problems or be angry, also affects health in different ways. Like most stressors, marital discord can lead to the production of stress hormones, which can lead to chronic systemic diseases, [35, 6, 45]. More commonly, anxiety, depression, and cardiovascular disorders are the commonly reported physical and mental health problems after marriage dissolution [9].
Divorce is an endemic issue that seriously affects the social and economic structure of contemporary society as much as any disease because of its adverse effect on personal stability. Children of divorced parents face various challenges, including healthy, social, psychological, and behavioral problem [12] .
It is the duty of the husband and wife to work out their issues and make their marriage stronger. The most successful strategies for preventing divorce in marriages include couple relationship education, reducing getting married too young, healthy consulting (anti-divorce therapy) to separators to renew their marriage (pre-divorce and post-divorce) [37]. Marriage counseling is proven to improve physical and emotional intimacy, increase communication, and establish an overall better connection between spouses, which enables to find solutions to divorce [15].
In epidemiology, mathematical modeling helps explain the fundamental processes that affect how diseases spread while also proposing preventative measures. Assumptions, variables, and parameters are clarified during the model formulation process. Additionally, models yield conceptual outcomes like replacement numbers, contact numbers, fundamental reproduction numbers, and thresholds. Mathematical models and computer simulations are useful experimental tools for building and testing theories, assessing quantitative conjectures, answering specific questions, determining sensitivities to changes in parameter values, and estimating the key parameters from data [3].
In a compartmental model, we assume that a person who is 18 years old or older is split into a number of different compartments, each individual belonging to exactly one compartment

at any point in time. An individual can change state by moving from one compartment to another.
According to different compartmental model divorce is treated as a social illness spread by divorced people and is considered a social epidemiology problem in academic research [21, 39, 31]. In this thesis, we proposed an improvement of the model [39]. We have formulated a mathematical model using deterministic techniques to predict the spread of unhealthy attitude on marriage by considering unhealthy attitude on a marriage as transmitted diseases or as a social contagion [30]. A side from that, we incorporate a class of married individual whose marriages are in a challenge to restore there stable marriage with a healthy counseling [18].

1.2 The scope and background of the study site

Hawassa city is located 273 km (170 mi) south of Addis Ababa via Bishoftu, 130 km (81 mi) east of Sodo and 75 km (47 mi) north of Dilla. The town serves as the capital of the Sidama Region. It lies on the Trans-African Highway 4 Cairo-Cape town and has a latitude and longitude of 7^o3'N 38^o28'E and an elevation of 1,708 meters (5,604 ft) above sea level. Hawassa's population is estimated to be 164,591 according to Ethiopian Statistics Agency 2023. The number of divorced case secondary data collected from this study area. In this area, we considered every four first instance court. Besides, we collected the total annual number of divorce cases that occurred between 2017 and 2023.

1.3 Statement of the problem

Divorce is a serious problem affecting an exponentially increasing number of people. In Ethiopia, 45% of all first marriages end in divorce or separation within 30 years [41], nearly 56% reported that their first marriage ended either because they were too young or "not interested "in the marriage. Nearly 52% of these dissolved marriages ended within 3 years [2]. Data from the Vital Event Registration and Information Agency shows that between 2021 and 2022, there were 16 035 divorces in Addis Abeba alone. It become a serious problem; challenging the establishment of the family in a routine manner and causing severe impacts on the emotional and mental health of the individual especially women lead to poorer health conditions and the risk of certain diseases, such as heart disease, depression, cardiovascular disorders and chronic pains [9]. Conflicts in marriages can affect family life in different ways. The effects of divorce is irreversible harm to all who involved in, but critical to the children.
The growth of divorce rate has also affect children as well. Children from divorced family are socially isolated, they have poor peer relationship, less sociable , fewer close friends and spend less time with friends. Economically children from disrupted family might not get enough health services, educational materials, books toys and other resources they need. In 2006 reported the number of street children total 600,000 in the country (in Ethiopia) and

more than 100,000 in the capital (Addis Ababa). In Ethiopia around 4,042,357 children are estimated to live under difficult condition because of families' inability to support children due to family dissolution[36].

Due to their experiences with a unhealthy perception of marriage and witnessing a failed relationship, young adults who have grown up through a single or family divorce may have changed their attitudes toward marriage and the family. They may also have become less optimistic about marriages. There family may also have unhealthy attitude on marriage to marry again due to there past experience of unhealthy marriage [29]. Different models available in the literature represent the dynamics of Marriage Divorce by system of nonlinear differential equations considering Divorce is often viewed as a social contagion between married people and divorced people, without taking into account the effects of person's with unhealthy marriage mindset on other married people. After identifying the key issues outlined in the problem statement, this thesis aims to explore innovative solution through rigorous analysis and empirical investigation. By addressing these challenges, we endeavor to contribute valuable insights to the existing body of knowledge in the field. We improved the work of Tessema haileyesus et al. [39] by including individual with unhealthy attitude on marriage and a healthy counseling in the model.

Hence the following basic research question are addressed in our study:

- What are the underlying influences that contribute to individuals adopting unhealthy attitude towards marriage?
- What conditions make the stability of unhealthy attitude free and unhealthy attitude present equilibrium points?
- Which parameters are more sensitive to spread of unhealthy attitude?
- How to solve the model numerically and make predictions using mathematical software?

1.4 Objective of the study

1.4.1 General objective

The general objective of this study is to analyze unhealthy attitude on marriage and its impact on the dynamics of divorce using mathematical model.

1.4.2 Specific objective of the study

The specific aims to achieve the following specific objectives:

- To formulate a modified model by considering Individual with unhealthy attitude on marriage.

- To establish condition for unhealthy attitude free and unhealthy attitude present equilibrium point
- To calculate a threshold parameters which determine whether unhealthy attitude on marriage expand or diminish in a population.
- To identify the sensitive parameter which has a great impact on divorce.
- To simulate the numerical solutions of the models and their graphs.

1.5 Significance of study

This study prompted us to conclude that the spread of unhealthy attitude about marriage in the society and its impact on the dynamics marriage divorce. For such real problems, planning constraints that should be placed on society can be supported by basic models and control divorce and it's negative consequences. Research on marriage divorce benefits society by identifying factors leading to marital dissolution, enabling interventions to strengthen relationships. It provides insights into effective counseling methods, fostering healthier marriages and reducing divorce rates. Through empirical evidence, it guides policymakers in crafting laws and policies that support marital stability and family well-being. Some of the benefits of our research work are detailed below:

- To control the spread of unhealthy attitude and its impact on the dynamics divorce in the society.
- To understand real world application of mathematical modeling in the dynamic of marriage divorce.
- To construct mathematical model for marriage divorce by considering individual with unhealthy attitude about marriage and healthy counseling.
- To analyze and interpret the solution provided by the model.
- To find the most sensitive parameters.

1.6 Organization of the Thesis

The thesis is organized as follows:

- Chapter (1)
  This chapter talks about the introduction to the topic. It contains the background to the study, statement of the problem, objectives and the study's significance, and also the chapter include the scope and background of the study site.

- Chapter (2)
  Under this chapter we reviewed various studies employing deterministic(compartmental) models, which can serve as valuable guides and reference for our work.
- Chapter (3)
  This chapter presents some epidemiological preliminaries, methodology of the study including the bifurcation analysis and sensitivity analysis.
- Chapter (4)
  Mathematical models, namely existing and modified are described and formulated in this chapter. Moreover models assumptions are included followed by showing the well-posedness of the model.
- Chapter (5)
  In this chapter we stated the model analysis including stability of equilibria, determining the reproduction number, bifurcation analysis and sensitivity analysis of the model.
- Chapter (6)
  we have discuss mathematical simulation of the model by using ODE45, with Matlab software, we use real data to simulate the outcomes and estimate the value of an unknown parameter.
- Chapter (7)
  The final chapter presents the concluding observation on the work that was done in previous chapters, and it does so by referring to those chapters. In addition to that, a summary of the overall objectives of the forthcoming research study on the topic is provided her followed by appendix.

Chapter 2

2 Review of related literature

In contemporary time, deterministic models have emerged as pivotal tools across various sectors, facilitating a deeper comprehension of critical phenomena mechanisms. Their utility lies in providing rapid, cost-effective and insightful evaluations through modeling and simulation techniques. Prior to the present study, extensive research has been conducted in this realm, serving as a foundation and guiding framework. In this review literature, we delve in to some of these previous studies, elucidating their methodologies and finding, while also elucidating how our own work builds upon and contributes to this existing body of knowledge.

Duato R. and Jódar L. in 2013 [10] developed mathematical modeling of divorce propagation allowing the estimation of the future divorced population using a real data are from the Spanish Statistics Institute (INE). They employ a linked discrete linear-quadratic difference system model with data from INE. A sensitivity analysis of the growth of the divorced population with respect to the contagion rate is included, and several different economic scenarios are considered. they estimated the divorced population in December 2015. Results show that the divorced population at the end of year 2015 will increase by around 50% with respect to the existing divorced population in 2010.

Mustapha Lhous et al in 2017 [24] considered a discrete-time marital status model, and assumed that individuals in the society can be classed in a compartments. Determined two controls which allow to reduce the number of virgin, divorced individuals and increase the number of married individuals. In their work the optimal control problem was obtained utilizing a progressive-regressive discrete schema that converges after a practical test connected to the Forward-Backward Sweep Method (FBSM) on the optimal control. The discrete version of Pontryagin's maximum principle served as the basis for the problem's numerical determination.

GWERYINA, R. I. et al. in 2021 [21] constructed an epidemiological model of divorce epidemic, using standard incidence function as force of marital disunity. The study examines qualitatively that the two equilibra (divorce-free and endemic equilibrium point) are globally stable by Lyapunov functions. Numerical results reveal that, anti-divorce protocols and reconciliation can jointly stabilize marriages, and married cases that survive divorce epidemic in 30 years period of marriage (twice the survival period of separation) cannot break again.

Hugo A. and Lusekelo E. M. in 2021 [22] focused on the dynamics of marriage and divorce scenarios. They evaluated stability analysis for the model and investigate counseling is useful to maintain peace and harmony among marriages. Justified the analytical results by using numerical verification on the model and observed that increasing counseling effort reduces the hardship life and complexity within marriage and or divorced individuals and suggest that harmony, peace, and happier life in families could be maintained and stained through

proper counseling and which in turn reduces divorce scenarios within marriages.
Muaraf.S et al. in 2021 [31] formulated a mathematical model of divorce by classify the model in to four population classes. The results showed that the rate of interaction between population M (married class) and populations other than H (the population class who experiences depression or stress due to divorce Hardship) is very influential on efforts to minimize divorce. Divorce can be minimized when the transmission rate is reduced to $R_0 < 1$. Reducing the transmission rate and increasing the rate of transfer from split bed class to married class can turn divorce endemic cases into non-endemic cases. A numerical simulation is given to confirm the analysis results.
Tessema, H. et al. in 2022 [39] considered a deterministic model for the marriage divorce in a population. They proposed and analyzed qualitatively using the stability theory of differential equations. In their work the basic reproduction number with respect to the divorce free equilibrium was obtained using next generation matrix approach. The model exhibits backward bifurcation and determined sensitivity indices of the parameters with respect to eradicating or spreading divorce in marriage. Performed numerical simulation and displayed graphically to justify the analytical results.
Haileyesus Tessema and his colleagues in 2022 [40] also expanded the deterministic model [39] in to an optimal control model. Qualitatively established the model positivity and boundlessness. They developed an optimal control model were by incorporating three time dependent control strategies (couple relationship education, reducing getting married too young and consulting separators to renew their marriage) on the deterministic model. They discovered that the best control for managing marriage divorce is using all three strategies simultaneously.
Karaagac, B. and Owolabi, K. M. in 2023 [23] considered a mathematical model of divorce with fractal fractional Caputo–Fabrizio derivative. They prove the existence and uniqueness of the solution of the model by the Banach Fixed point theorem. Evaluated behavior of the model using graphics for different values of the fractal dimension and fractional derivative by developed a numerical method for the model that contains the fractal fractional derivative. From the numerical results, for $R_0 < 1$, observed low or no divorce rate. However, a tiny number of marital populations degrade over time for various causes. For $R_0 > 1$, the number of divorced people appears to be increasing with time. They also analyzed experimental results for different instances of the key parameters that played major roles for each of the sub-population classes.
Gambrah, P. P. and Abdul-Rahaman, in 2023 [17] used a compartmental model. They simulated and analyzed transmission network of divorce spread in continuous time on networks (that is, analyzed married divorced-separated model on networks). According to their findings, divorced individuals or nodes can cause many marriage nodes to become separated. Again there were few divorced nodes at the peak and at the end of the epidemic, but most marriages resulted in separation at the end of the epidemic. At the end of the epidemic, 80% of married nodes became separated nodes, which is a very serious problem and suggested

that divorce or separation is contagious and can spread quickly through indirect contact. This makes it vital to examine the effect of divorce to the population so as reduce the number of broken homes at the end of the divorce epidemic.

Chapter 3

3 Methodology

The thesis employs a mathematical model, incorporating non-linear ordinary differential equation, to study the spread of unhealthy attitude on marriage and its impact on the dynamics of divorce. Stability conditions for equilibrium points are determined using Lyapunov function, Comparison theorem, or Routh Hurwitz criteria, while threshold parameters are computed to assess the spread or decline of unhealthy attitudes within the population. Utilizing mathematical software like MATLAB and Maple, numerical simulation are conducted to derive consistent results, supported by secondary data analysis from Hawassa city first instance courts using purposive sampling particularly total population sampling of divorce cases to validate parameter values and perform data fitting. To estimate the parameters from data, we used a least square. A nonlinear curve fitting method with the help of "fminsearch", builtin MATLAB function. Additionally, fundamental principles of dynamical systems and epidemiology are applied to further elucidate the problem and its solutions.

3.1 Autonomous System of Ordinary Differential Equations

Let us consider an n- dimensional autonomous system of the form :

$$x'(t) = f(t, x, p), \tag{3.1}$$

$$x(t_0) = x_0,$$

where ; x_0; x ∈ D ⊂ R^n and f : $R^n \to R^n$; with f is continuous at x ∈ D ⊂ R^n.

Definition 3.1 [46] (Well-posedness)
An initial value problem (IVP) given in (3.1) is mathematically said to be well-posed if the followings conditions hold:

- Its solution exists.
- Its solution is unique and continuously depends on the initial conditions.

Since in epidemiology we are dealing with populations, the following two additional conditions are quite important

- The solution should be non-negative over time.
- The solution should be bounded.

Theorem 3.1 [27] (Picard's theorem)
Consider the initial value problem given in (3.1), if the function f is continuous and that all its partial derivatives $\frac{\partial f_i}{\partial x_j}$, for i , j = 1 , 2 , 3 , ... , n are continuous for x in some open connected set D ⊂ R^n then , for x_0 ∈ D the problem (3.1) has a solution $x(t)$ on some time interval (-Ω , Ω), Ω > 0 about $t = 0$ and the solution is unique.

Definition 3.2 [43] (Positivity of Solution)
The solution of a given autonomous system of (3.1) is said to be positive , if all trajectories $x(t)$ is positive for any t ≥ 0.

Definition 3.3 [34] (Boundedness of Solution)
The positive solution of an autonomous system, (3.1) is said to be bounded if any solution $x(t, t_0, x_0)$, of (3.1) satisfies

$$||x(t, t_0, x_0)|| \leq C(||x_0||, t_0),$$

for all t $\geq t_0$ where, $C : \mathbb{R}^+$ x $\mathbb{R}^+ \rightarrow \mathbb{R}^+$ is a constant that depends on t_0 and x_0.

Theorem 3.2 [47] (Integrating factor method)
Let a linear ordinary differential equation is given by:

$$Y'(x) + P(x)y = q(x). \tag{3.2}$$

Then the integrating factor and its general solution, respectively, are given by:

$$I(x) = e^{\int p(x)d(x)}, \tag{3.3a}$$

$$Y(x) = \frac{1}{I(x)}\left[\int I(X)q(x)d(x) + c\right], \tag{3.3b}$$

where, C is an arbitrary constant of integration.

3.2 Stability analysis of equilibrium points

The equilibrium points to a system of first order differential equations are the points at which each differential equation is equal to zero.

Definition 3.4 [33] Given the autonomous system (3.1), a state x^* is said to be an equilibrium point of the system if $f(x^*) = 0$.

Definition 3.5 [26] The solution x^* is said to be stable if for every $\epsilon > 0$, there exists a $\delta = \delta(\epsilon) > 0$ such that $|x^* - x_0| < \delta \Rightarrow |x^* - x(t)| < \epsilon$, $t > t_0 \in \mathbb{R}$, for every solution of (3.1) with $x(t_0) = x_0$.

Definition 3.6 [26] An equilibrium point x^* is said to be globally asymptotically stable if it is asymptotically stable for all initial condition $x_0 \in \mathbb{R}^n$.

Definition 3.7 [26] An equilibrium point x^* of the model (3.1) is said to be locally asymptotically stable if it is locally stable and every trajectory that starts sufficiently close to x^* tend towards x^* as t $\rightarrow \infty$. A steady state x^* which is not stable is said to be unstable.

Definition 3.8 [26] An equilibrium point of a given dynamical system is stable means all solution curves of the equation attracts towards the equilibrium point, while an equilibrium point is unstable means all solution curves of the dynamic system go away from the equilibrium point.

Definition 3.9 [33] An equilibrium point x^* is globally stable if all trajectories converge to x^* , i.e $lim_{t\rightarrow\infty}x(t) = x^*$.

3.2.1 Local stability by linearization

Mathematically, the stability of equilibrium can be analyzed using the linearized system at the equilibrium point. The Jacobian matrix associated to the system (3.1) at the equilibrium point x^* , which is denoted by $Df(x^*)$ is given by the matrix:

$$Df(x^*) = \left[\frac{\partial f_i(x^*)}{\partial x_j}\right] = \begin{bmatrix} \frac{\partial f_1(x)}{\partial x_1} & \frac{\partial f_1(x)}{\partial x_2} & \cdots & \frac{\partial f_1(x)}{\partial x_n} \\ \frac{\partial f_2(x)}{\partial x_1} & \frac{\partial f_2(x)}{\partial x_2} & \cdots & \frac{\partial f_2(x)}{\partial x_n} \\ . & . & & . \\ . & . & & . \\ \frac{\partial f_n(x)}{\partial x_1} & \frac{\partial f_n(x)}{\partial x_2} & \cdots & \frac{\partial f_n(x)}{\partial x_n} \end{bmatrix}_{|x=x^*},$$

where i, j = 1, 2, ..., n.
An equilibrium point x^* of the dynamical system (3.1) is locally asymptotically stable if all the eigenvalues of the Jacobian $Df(X^*)$ evaluated at x^* are negative. The equilibrium x^* is unstable if at least one of the eigenvalues of $Df(X^*)$ is positive.

3.2.2 Global stability

The following two theorem are presented to establish the global stability of equilibrium points of mathematical model.

Theorem 3.3 *(*Lyapunov Stability Theorem)
Let x^* be an equilibrium solution of (3.1) and suppose that we can find a Lyapunov function i.e a continuously differentiable, real valued function $V(x)$ such that

i , $V(x) > 0$ for all $x \neq x^*$, and $V(x^*) = 0$. (V is positive definite),

ii , $\frac{dV(x)}{dt} < 0$ for all $x \neq x^*$,

then x^* is globally asymptotically stable: for all initial conditions, $x(t) \to x^*$ as $t \to \infty$ [7].

Theorem 3.4 (LaSalle's invariance principle)
Let $\bar{x}$ be an equilibrium point of (3.1) defined on $\Omega \in R^n$. Let V be a positive definite Liapunov function for $\bar{x}$ on the set Ω. Furthermore let $M = \{x \in \Omega : \dot{V}(x) = 0\}$ and if
$S = \{$the union of all trajectory that start and remain in M for all $t > 0\}$, that is S the largest positively invariant subset of M such that $S \in \Omega$, then $\bar{x}$ is globally asymptotically stable on Ω if and only if it is globally asymptotically stable on S [7].

Theorem 3.5 [4](Castillo Chavez theorem)
Assume that the system (3.1) can be rewritten in the form:

$$\frac{dZ_1}{dt} = F(Z_1, Z_2), \tag{3.4a}$$
$$\frac{dZ_2}{dt} = G(Z_1, Z_2), \tag{3.4b}$$

where the $Z_1 \in R^m$ and $Z_2 \in R^{n-m}$ represents the classes of non-infected individuals and infected individuals respectively. Assume that $G(Z_1, 0) = 0$ and let $E_0 = (Z_1^*, 0)$ be a steady state of (3.1) (the decrease free equilibrium point). If the following condition are satisfied.

a, For the system $\frac{dZ_1}{dt}$ = F(Z_1, 0), the steady state Z_1^* is globally asymptotically stable.

b, $G(Z_1, Z_2) = AZ_2$- $\hat{G}(Z_1, Z_2)$, $\hat{G}(Z_1, Z_2) \geq 0$ for $(Z_1, Z_2) \in \Omega$, where A is a Metzler matrix (the off diagonal elements of A are non-negative) and Ω is the region where the model makes biological sense, then the steady state $E_0 = (Z_1^*, 0)$ is a globally asymptotically stable for the system (3.4) provided that the basic reproduction number of the model is less than one.

3.2.3 Routh-Hurwitz stability criterion

Routh Hurwitz criterion is an important criterion that gives necessary and sufficient condition for all the roots of the characteristics polynomial (with real coefficients) to have negative real parts. It is used to determine asymptotic stability of an equilibrium point for non-linear system of differential equations. Consider the characteristic equation of degree n, given by:

$$p(\lambda) = \lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + ... + a_n, \quad a_i > 0, \quad \text{for} \quad i = 1,2...n, \tag{3.5}$$

where the coefficients a_i are real constants, we define the 'n x n' Hurwitz matrices using the coefficients a_i of the characteristic polynomial:

$$H_1 = (a_1), \quad H_2 = \begin{bmatrix} a_1 & 1 \\ 0 & a_2 \end{bmatrix}, ..., H_n = \begin{bmatrix} a_1 & 1 & 0 & ... & 0 \\ a_3 & a_2 & a_1 & ... & 0 \\ a_5 & a_4 & a_3 & ... & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & 0 & 0 & ... & a_n \end{bmatrix},$$

where $a_j = 0$ if j > n. All the roots of the polynomial p(λ) are negative or have negative real part if and only if the determinants of all Hurwitz matrices are positive.

$$\text{i.e} \quad \det(H_j) > 0, \quad \text{for all} \quad j = 1,2,...n.$$

For polynomials of degree n = 2,3,4, and 5 , the Routh-Hurwitz criteria are summarized as follows:

- n = 2: $a_1 > 0$, $a_2 > 0$ and $(a_1 a_2 > 0)$,
- n = 3: $a_1 > 0$ $a_3 > 0$ and $(a_1 a_2 > a_3)$,
- n = 4: $a_1 > 0$, $a_3 > 0$, $a_4 > 0$ and $(a_1 a_2 a_3 > a_3^2 + a_1^2 a_4)$,
- n = 5: $a_i > 0$, i = 1,2,3,4,5; $a_1 a_2 a_3 > a_3^2 + a_1^2 a_4$ and $(a_1 a_4 - a_5)(a_1 a_2 a_3 - a_3^2 - a_1^2 a_4) > (a_5(a_1 a_2 - a_3)^2 + a_1 a_5^2)$.

3.3 Bifurcation analysis

Definition 3.10 Bifurcation is defined as a change in the qualitative behavior of a given dynamical system when an associated parameter is varied.

In particular, fixed points can be created or destroyed, or their stability can change. These qualitative changes in the dynamics are called bifurcations, and the parameter values at which they occur are called bifurcation points [38]. We investigate the nature of the bifurcation by using the method introduced in [5], which is based on the use of the central manifold theory. To study the stability of endemic equilibrium points, the center manifold theory (described in by [5] Theorem 4.1) is used as an alternative. This theory is reproduced here for convenience.

3.3.1 Center manifold theory

Center manifold theory has been used to decide the local stability of a non-hyperbolic equilibrium (linearization matrix has at least one eigenvalue with zero real part). We shall describe a theory that not only can determine the local stability of the non-hyperbolic equilibrium but also settles the question of the existence of another equilibrium (bifurcated from the non-hyperbolic equilibrium). This theory is based on the general center manifold theory. To describe it, consider a general system of ODEs with a parameter ϕ:

$$\frac{dx}{dt} = f(x,\phi), \quad f : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n \quad \text{and} \quad f \in \mathbb{C}^2(\mathbb{R}^n \times \mathbb{R}), \tag{3.6}$$

without loss of generality, it is assumed that 0 is an equilibrium for system (3.6) for all values of the parameter ϕ, that is

$$f(0,\phi) = 0, \quad \text{for all} \quad \phi. \tag{3.7}$$

Theorem 3.6 [42] Assume

A1: $A = D_x f(0.0) = \left(\frac{\partial f_i}{\partial x_j}(0,0)\right)$, is the linearization matrix of system (3.6) around the equilibrium 0 with ϕ evaluated at 0. Zero is a sample eigenvalue of A and all other eigenvalues of A have negative real parts.

A2: Matrix A has a non-negative right eigenvector w and a left eigenvector v corresponding to the zero eigenvalue.

Let f_k be the k^{th} component of f and

$$a = \sum_{k,i,j=1}^{n} v_k w_i w_j \frac{\partial^2 f_k}{\partial x_i \partial x_j}(0,0), \tag{3.8}$$

$$b = \sum_{k,i=1}^{n} v_k w_i \frac{\partial^2 f_k}{\partial x_i \partial \phi}(0,0). \tag{3.9}$$

The local dynamics of system (3.6) around 0 is totally determined by the signs of a and b:

- If $a > 0$ and $b > 0$, then a backward bifurcation occurs at $\phi = 0$.
- If $a < 0$ and $b > 0$, then a forward bifurcation occurs at $\phi = 0$.

3.4 Sensitivity Analysis

Sensitivity Analysis is used to determine the relative importance of model parameters to disease transmission. We perform the analysis by calculating the Sensitivity Induces of the basic reproduction number R_0, because it determines whether or not the disease will spread in the population. Sensitivity analysis is commonly used to determine the robustness of model predictions to parameter values, since there usually errors in data collection and preassumed values. It also allows for the measurement of relevant changes in a state variable when a parameter changes.
In performing the sensitivity analysis, we apply the method called normalized forward sensitivity index of a variable that has been used quite commonly and it is defined as the ratio of relative change in the variable to the relative change in the parameter [11]. The sensitivity may also be defined using partial derivatives when the variable is a differentiable function of the parameter and defined as:

$$\Lambda_P^{R_0} = \frac{\partial R_0}{\partial P} x \frac{p}{R_0},$$

where p represents all the basic parameters.

3.5 Least-Square Estimation

The method of least squares is a parameter estimation method in regression analysis based on minimizing the sum of the squares of the residuals (a residual being the difference between an observed value and the fitted value provided by a model) made in the results of each individual equation.
In order to estimate the unknown parameters p, the state variable $x(t)$ is observed at T time instants $t_1, \ldots, t_T$, it given by

$$x(t_i) = \hat{x}(t_i) + E_i, \quad i = t_1....t_T,$$

where $x(t_i)$ are the experimentally observed values of the state variables at time instant t_i and E_i are independent measurement errors with zero mean. The objective is to determine appropriate parameter values so that errors between the outputs of the estimated model and the measured data are minimized.
to find the vector of least-square estimators, p, that minimizes

$$E(p) = \sum_{i=1} (x_i - \hat{x}_i)^2. \tag{3.10}$$

To find the values of parameters p that minimizes Eq(3.10) [25].

Chapter 4

4 Model Formulation

4.1 The Existing Mathematical model

A deterministic model for the marriage divorce in a population was proposed by Tessema Haileyesus et al. [39]. They considered the total number of population N (t) in a given time t. The model divides the entire population into four sub populations: those who reach the age of getting married are single individuals, S(t); those who got married individuals are denoted by, M(t); those who separate but not divorced are broken marriage individuals, B(t) and those who are divorced marriage are denoted by D(t) with a natural death rate μ in all the classes.
The following assumptions are considered under the existing work:

- Divorce is considered as a transmitted disease.
- Individuals who are not married consider as a single.
- There is a possibility for broken individual to get in to their marriage again.
- People who divorce can remain single.
- Members mix homogeneously (have the same interaction to the same degree).
- Sex, race and social status do not affect the probability of being divorced.
- Individuals in each group have the same natural death rate μ.

From the model π is the recruitment rate of individuals being single when he/she reaches the age of getting married. This individuals got married at rate of β. The married individuals got broken and move to the broken compartment due to the contact with the divorced individuals at a rate of α. The broken marriage recovered from their conflicts and renew their marriage at a rate of ϵ and live as the previous style. Some of these broken marriage got a permanent divorce at a rate of δ. This divorced people join the single sub population at a rate of ρ and some of the will die due to divorce at a rate of σ. The whole population has μ as an average death rate.

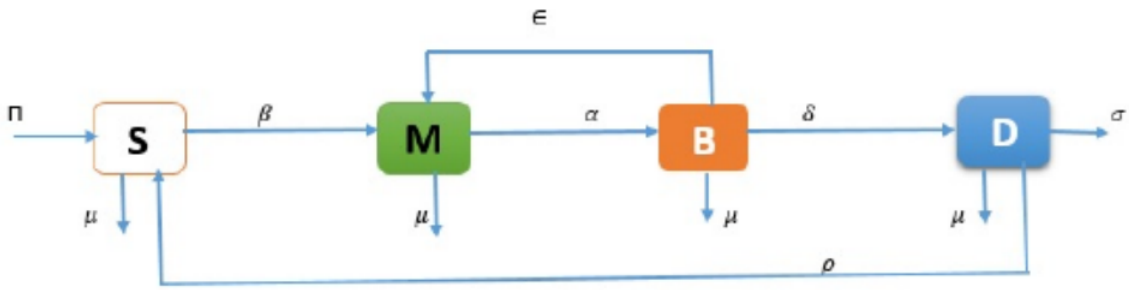


Figure 4.1.1: Flow chart of exist model.

Thus the mathematical existing model is described by the following system of ordinary non-liner differential equation:

$$\begin{aligned}
\frac{dS}{dt} &= \pi + \rho D - (\beta + \mu)S, \\
\frac{dM}{dt} &= \beta S + \epsilon B - (\alpha D + \mu)M, \\
\frac{dB}{dt} &= \alpha MD - (\delta + \epsilon + \mu)B, \\
\frac{dD}{dt} &= \delta B - (\sigma + \rho + \mu)D.
\end{aligned} \tag{4.1}$$

With non negative initial conditions are: $S(0) = S_0 \geq 0$; $M(0) = M_0 \geq 0$; $B(0) = B_0 \geq 0$; $D(0) \geq D_0$
The total number of populations in system (4.1) is given as:

$$N(t) = S(t) + M(t) + B(t) + D(t).$$

4.2 The Modified Mathematical Model

In this section, we consider an SMGUDH type Mathematical modeling for the spread of unhealthy attitude on marriage and its impact on the dynamics divorce. The total population N(t) is divided into six compartments: Single individuals S(t), Married individuals M(t), Married individuals with in challenge G(t), Unhealthy attitude on marriage U(t), Divorce individuals D(t) and Healthy counseling H(t) time $t \geq 0$. These categories are mutually exclusive.
Single individual are those who have reached to get married, but not yet get married. In the Ethiopian context a person whose age is above 18 and not married considered to be single and able to marry. Married individual are a legally and socially sanctioned union. Some time there maybe happen disagreement in marriage and the couple marriage get in challenge. Married people in challenge are those who are in sharp dispute and refuse to come together for an agreement at the time, but they are still together under one roof.
Individuals who have unhealthy attitude on marriage are those who have unhealthy attitude

for several reasons, this people's attitude about marriage is negative and turning you against your partner. Unhealthy attitude can come in the form of comparison trap, selfishness, ungratefulness, criticism about marriage, attacking others marriage, pessimism, perfectionism, and irritation(annoyance) [29, 8]. Healthy Counseling is a group of peoples or individual with a positive outlook on marriage who support peoples about marriage who are facing challenges on their marriage and unhealthy attitudes about marriage by providing them with appropriate Counseling [22, 15]. And divorce individuals are a married individual formally dissolves their legal marriage.

4.3 Model Assumption

- Unhealthy attitude on marriage can negatively influenced both single and married people.
- Single individual will be married or they will have unhealthy attitude on marriage.
- Single individual with negative attitude on marriage can influence married couples.
- Married peoples who are in challenge, may contract unhealthy attitude at a rate of α if they are not able to settle things on time.
- There is a possibility for married people who are in challenge to settle things by themselves.
- Both married individuals with challenge and Individuals with unhealthy attitude on marriage can take healthy counseling.
- Divorced individual will remarry after taking healthy counseling or will have unhealthy attitude about marriage.
- All individual who are taking Healthy counseling get married or renew their previous marriage.
- Individuals who have unhealthy attitude are found in the same compartment, but only those who are married commit divorce.
- Single individuals are whose age is greater than or equal to 18 and able to marry.
- Sex, race and social status do not affect the probability of acquiring negative attitude.
- Individuals in each group have the same natural death rate μ.

4.4 Description of Variables and Parameters

The descriptions on variables and parameters are depicted in the table (1) and (2) respectively

Table 1: Description of variables

Variables	Descriptions
S	Single individuals at t time.
M	Married individual at t time.
U	Individual with unhealthy attitude on marriage at t time.
G	Married individuals whose marriage is in a challenge.
H	Healthy Counseling.
D	Divorce individual.

Table 2: Parameters descriptions

Parameters	Descriptions
Λ	Recruitment rate of individuals to the age of single.
ϵ	Rate of single individuals he or she reach for marriage get married.
β_1	The contact rate between married individuals with whose attitude about marriage unhealthy.
β_2	The contact rate of individuals whose attitude about marriage is unhealthy with single individuals.
η	Rate of individuals whose marriage is in a challenge renew their previous marriage with out any counseling.
α	Rate of a marriage Individuals with in challenge who become unhealthy attitude about marriage.
δ	Rate of an individual whose attitude is unhealthy who get divorced.
ϕ	Rate of individuals attaining Healthy counseling due to unhealthy attitude about marriage .
ρ	Rate of individuals attaining Healthy counseling due to there marriage is in challenge.
π	Rate of divorced individuals attaining healthy counseling.
ω	Rate at which people return to married status after Healthy counseling.
θ	Rate of divorced individual who become an healthy attitude about marriage.
μ	Natural death rate of individuals.

By considering above assumption, the modified mathematical model which describe the spread of unhealthy attitude and its impact on divorce. The flow chart of the modified model is illustrated in Figure (4.4.1).

Based on our assumptions and the flow chart (4.4.1), the modified model given by the fol-

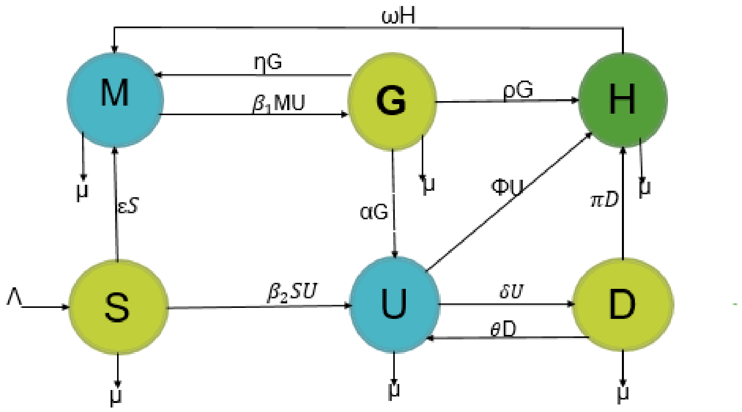


Figure 4.4.1: The flow diagram of modified model.

lowing deterministic system of non-linear differential equations:

$$\frac{dS}{dt} = \Lambda - \beta_2 US - (\epsilon + \mu)S, \tag{4.2}$$

$$\frac{dM}{dt} = \epsilon S + \eta G + \omega H - \beta_1 MU - \mu M, \tag{4.3}$$

$$\frac{dG}{dt} = \beta_1 MU - (\rho + \alpha + \eta + \mu)G, \tag{4.4}$$

$$\frac{dU}{dt} = \beta_2 SU + \alpha G + \theta D - (\delta + \phi + \mu)U, \tag{4.5}$$

$$\frac{dD}{dt} = \delta U - (\theta + \pi + \mu)D, \tag{4.6}$$

$$\frac{dH}{dt} = \phi U + \pi D + \rho G - (\omega + \mu)H. \tag{4.7}$$

The initial conditions are: $S(0) = S_0 > 0$; $M(0) = M_0 \geq 0$; $G(0) = G_0 \geq 0$; $U(0) = U_0 \geq 0$; $H(0) = H_0 \geq 0$; $D(0) = D_0 \geq 0$.

The total number of populations in system (4.2 - 4.7) is given as:

$$N(t) = S(t) + M(t) + U(t) + G(t) + H(t) + D(t).$$

4.5 Well-posedness of the Model

The dynamical system (4.2 - 4.7) is to be epidemiological meaningful and well-posed if its solution exist, unique and continually depends on the initial conditions. Since the model monitors changes in the human population the state variables and parameters are assumed to be positive for all $t \geq 0$. Thus, the existence and uniqueness of the solutions is establish by using Picard's theorem.

Theorem 4.1 (Existence and Uniqueness of Solution)

Let $\Omega = \left\{(S, M, U, G, H, D) \in \mathbb{R}^6_+\right\}$ denotes the region defined

$$|t - t_0| \leq a, \quad ||x - x_0|| \leq b, \quad x = (x_1, x_2, ...x_n), \quad x_0 = x_{01}, x_{02}, ...x_{0n} \tag{4.8}$$

and suppose that $f(t,x)$ satisfies the Lipschitz condition.

$$||f(t,x_1)-f(t,x_2)|| \quad \leq \quad L||x_1-x_2||, \tag{4.9}$$

when the pairs (t,x_1) and (t,x_2) belongs to Ω where L is a positive constant. Then, there exists a constant $\zeta > 0$ such that there exists a unique continuous vector solution $x(t)$ of condition (4.8) in the interval $4|t-t_0| < \zeta$. It is important to note that condition (4.8) is satisfied by the requirement that $\frac{\partial f_i}{\partial x_i}$ i,j = 1, 2, ...n is continuous and bounded in Ω.

proof

Let Ω denote the region for the above. It serves to show that $\frac{\partial f_i}{\partial x_i}$,i, j = 1, 2, 3, 4, 5 are continuous and bounded in Ω. At present, from (4.8), Ω = (S,M,U,G,H,D) where

$$|S-S_0| \leq b_1, |M-M_0| \leq b_2, |U-U_0| \leq b_3, |G-G_0| \leq b_4, |H-H_0| \leq b_5, |D-D_0| \leq b_5 \tag{4.10}$$

Let

$$\begin{aligned}
f_1 &= \frac{dS}{dt} = \Lambda - \beta_2 US - (\epsilon+\mu)S,\\
f_2 &= \frac{dM}{dt} = \epsilon S + \eta G + \omega H - \beta_1 MU - \mu M,\\
f_3 &= \frac{dG}{dt} = \beta_1 MU - (\rho+\alpha+\eta+\mu)G,\\
f_4 &= \frac{dU}{dt} = \beta_2 SU + \alpha G + \theta D - (\delta+\phi+\mu)U,\\
f_5 &= \frac{dD}{dt} = \delta U - (\theta+\pi+\mu)D,\\
f_6 &= \frac{dH}{dt} = \phi U + \pi D + \rho G - (\omega+\mu)H.
\end{aligned}$$

Consider the partial derivatives with respect to each state variable

$$\frac{\partial f_1}{\partial S} = \beta_2 U - (\epsilon+\mu), \frac{\partial f_1}{\partial M} = 0, \frac{\partial f_1}{\partial G} = 0, \frac{\partial f_1}{\partial U} = -\beta_2 S, \frac{\partial f_1}{\partial D} = 0, \frac{\partial f_1}{\partial H} = 0, \tag{4.11}$$

$$\frac{\partial f_2}{\partial S} = \epsilon, \frac{\partial f_2}{\partial M} = -\beta_1 U - \mu, \frac{\partial f_2}{\partial G} = \eta, \frac{\partial f_2}{\partial U} = -\beta_1 M, \frac{\partial f_2}{\partial D} = 0, \frac{\partial f_2}{\partial H} = \omega, \tag{4.12}$$

$$\frac{\partial f_3}{\partial S} = 0, \frac{\partial f_3}{\partial M} = \beta_1 U, \frac{\partial f_3}{\partial G} = -(\rho+\alpha+\eta+\mu), \frac{\partial f_3}{\partial U} = \beta_1 M, \frac{\partial f_3}{\partial D} = 0, \frac{\partial f_3}{\partial H} = 0, \tag{4.13}$$

$$\frac{\partial f_4}{\partial S} = \beta_2 U, \frac{\partial f_4}{\partial M} = 0, \frac{\partial f_4}{\partial G} = \alpha, \frac{\partial f_4}{\partial U} = \beta_2 S - (\delta+\phi+\mu), \frac{\partial f_4}{\partial D} = \theta, \frac{\partial f_4}{\partial H} = 0, \tag{4.14}$$

$$\frac{\partial f_5}{\partial S} = 0, \frac{\partial f_5}{\partial M} = 0, \frac{\partial f_5}{\partial G} = 0, \frac{\partial f_5}{\partial U} = \delta, \frac{\partial f_5}{\partial D} = -(\theta+\pi+\mu), \frac{\partial f_5}{\partial H} = 0, \tag{4.15}$$

$$\frac{\partial f_6}{\partial S} = 0, \frac{\partial f_6}{\partial M} = 0, \frac{\partial f_6}{\partial G} = \rho, \frac{\partial f_6}{\partial U} = \phi, \frac{\partial f_6}{\partial D} = \pi, \frac{\partial f_6}{\partial H} = -(\omega+\mu). \tag{4.16}$$

Now,by substituting (4.10) in to (4.11 - 4.16), we can get the following
For partial derivative (4.11)

$$\left|\frac{\partial f_1}{\partial S}\right| = |\beta_2 U - (\epsilon+\mu)| \leq |\beta_2 U| + |-(\epsilon+\mu)| \leq \beta_2(b_3+U_0)+\epsilon+\mu < \infty,$$
$$\left|\frac{\partial f_1}{\partial U}\right| = |-\beta_2 S| = \beta_2 S \leq \beta_2(b_1+S_0) < \infty,$$
$$|\frac{\partial f_1}{\partial M}| = |\frac{\partial f_1}{\partial G}| = |\frac{\partial f_1}{\partial D}| = |\frac{\partial f_1}{\partial H}| = 0 < \infty.$$

For partial derivative (4.12)

$$|\frac{\partial f_2}{\partial S}| = |\epsilon| = \epsilon < \infty, |\frac{\partial f_2}{\partial M}| = |-\beta_1 U - \mu| = \beta_1 U + \mu \leq \beta_1(b_3+U_0)+\mu < \infty,$$
$$|\frac{\partial f_2}{\partial G}| = |\eta| = \eta < \infty, |\frac{\partial f_2}{\partial U}| = |-\beta_1 M| = \beta_1 M \leq \beta_1(b_2+M_0) < \infty,$$
$$|\frac{\partial f_2}{\partial D}| = 0 < \infty, |\frac{\partial f_2}{\partial H}| = |\omega| = \omega < \infty.$$

And we continue in the same manner for the (4.13-4.16). Thus,the system of equation (4.2 - 4.7) are differentiable with respect to all state variables and all the partial derivatives of the system equation (4.2 - 4.7) are continuous and bounded.
Therefore, by the existence and uniqueness theorem the modified mathematical model of the equation system (4.2 - 4.7) has unique solution.

Theorem 4.2 (Positivity of Solutions)
If $S(0) = S_0 > 0$; $M(0) = M_0 \geq 0$; $G(0) = G_0 \geq 0$; $U(0) = U_0 \geq 0$; $H(0) = H_0 \geq 0$; $D(0) = D_0 \geq 0$, then the solution (S(t), M(t), G(t), U(t), H(t), D(t)) of the dynamical system (4.2 - 4.7) is non-negative for all time t ≥ 0.

Proof
To show positivity of solutions of the dynamical system (4.2 - 4.7), we will perform the proof by using contradiction. We assume that S(t) ≤ 0 for some $t \geq 0$, that is there exists small $t_0 > 0$ such that $S(t_0) = 0$, $S'(t_0) \leq 0$ and S(t) > 0 for $t \in [0, t_0)$. Then G(t) ≥ 0, U(t) ≥ 0 and D(t) ≥ 0.
If this is not the case, then there exists:

i, $t_1 \in [0, t_0]$ such that $M(t_1) = 0$, $M'(t_1) < 0$ and M(t) > 0 for $t \in [0, t_1)$. Then U(t) ≥ 0, G(t) ≥ 0 and D(t) ≥ 0 for $[0, t_1]$.

ii, $t_2 \in [0, t_1]$ such that $G(t_2) = 0$, $G'(t_2) < 0$ and G(t) > 0 for $t \in [0, t_2)$. Then U(t) ≥ 0 , M(t) ≥ 0 and D(t) ≥ 0 for $[0, t_2]$.

iii, $t_3 \in [0, t_2]$ such that $U(t_3) = 0$, $U'(t_3) < 0$ and U(t) > 0 for $t \in [0, t_3)$. Then D(t) ≥ 0, M(t) ≥ 0 and G(t) ≥ 0 for $[0, t_3]$.

iv, $t_4 \in [0, t_3]$ such that $D(t_4) = 0$, $D'(t_4) < 0$ and D(t) > 0 for $t \in [0, t_4)$. Then U(t) ≥ 0, M(t) ≥ 0 and G(t) ≥ 0 for $[0, t_4]$.

Now from equation (4.2), we have:

$$\frac{dS}{dt} = \Lambda - \beta_2 US - (\epsilon + \mu)S,$$

$$\frac{dS}{dt} \geq -(\beta_2 U + (\epsilon + \mu))S, \quad \text{since} \quad \Lambda \quad \text{is positive.}$$

By rearranging, we have

$$\frac{dS}{S} \geq -(\beta_2 U + (\epsilon + \mu))dt,$$

Integrating both sides of the inequality, we have;

$$\int \frac{dS}{S} \geq -\int (\beta_2 U + (\epsilon + \mu))dt,$$

$$lnS(t) \geq -(\beta_2 U + (\epsilon + \mu))t + c,$$
$$S(t) \geq e^{-(\beta_2 U + (\epsilon + \mu))t + c}.$$

At time $t = 0$,

$$S(0) \geq e^{c},$$
$$S(t) \geq S(0)e^{-(\beta_2 U + (\epsilon + \mu))t} > 0.$$

Since S(0) is initial value which is non-negative and $e^{-(\beta_2 U + (\epsilon + \mu))t}$ the exponential is always positive. Hence, it is clear that as $t \to \infty$ then the solution of S(t) is always non-negative for $t \geq 0$.

from equation (4.4), we have:

$$G(t_2) = \beta_1 M(t_2)U(t_2) - (\rho + \alpha + \eta + \mu)G(t_2),$$
$$\Rightarrow G(t_2) = \beta_1 M(t_2)U(t_2) \geq 0, \quad \text{since}, U(t_3) \geq 0, M(t_3) \geq 0 \quad \text{and} \quad G(t_3) = 0 \quad \text{from (ii)}$$

$\Rightarrow G'(t_2) \geq 0$ which contradicts the assumption that $G'(t_2) < 0$. Thus, G(t) ≥ 0 for all $t \geq 0$.

from equation (4.6), we have:

$$D'(t_4) = \delta U(t_4) - (\theta + \pi + \mu)D(t_4),$$
$$\Rightarrow D'(t_4) = \delta U(t_4) \geq 0 \quad \text{since}, U(t_4) \geq 0 \quad \text{and} \quad D(t_4) = 0 \quad \text{from (iv)}$$

$\Rightarrow D'(t_4) \geq 0$ which contradicts the assumption that $D'(t_4) < 0$. Thus, D(t) ≥ 0 for all $t \geq 0$.

from equation (4.5), we have;

$$U'(t_3) = \beta_2 S(t_3)U(t_3) + \alpha G(t_3) + \theta D(t_3) - (\delta + \phi + \mu)U(t_3),$$
$$\Rightarrow U'(t_3) = \alpha G(t_3) + \theta D(t_3) \geq 0 \quad \text{since}, G(t_3) \geq 0, D(t_3) \geq 0 \quad \text{and} \quad U(t_3) = 0 \quad \text{from (ii)}$$

$\Rightarrow U'(t_3) \geq 0$ which contradicts the assumption that $U'(t_3) < 0$. Thus, U(t) ≥ 0 for all $t \geq 0$. from equation (4.7), we have:

$$\frac{dH}{dt} = \phi U + \pi D + \rho G - (\omega + \mu)H.$$

This can be written as $\frac{dH}{dt} \geq -(\omega + \mu)H$, since u(t) ≥ 0 , D(t) ≥ 0 and G(t) ≥ 0 integrating both sides of the inequality, we get

$$\int \frac{dH}{H} \geq -\int (\omega + \mu) dt,$$
$$\Rightarrow ln(H) \geq -(\omega + \mu)t + c,$$
$$\Rightarrow H(t) \geq e^{-(\omega+\mu)t+c} \quad \text{and} \quad H(0) > 0 > e^c,$$
$$\Rightarrow H(t) \geq H(0)e^{-(\omega+\mu)t} \geq 0.$$

Thus H(t) ≥ 0 for all t > 0.
from equation (4.3), we have:

$$\frac{dM}{dt} = \epsilon S + \eta G + \omega H - \beta_1 MU - \mu M.$$

This can be written as $\frac{dM}{dt} \geq -(\beta_1 U - \mu)M$, since S(t) ≥ 0 , G(t) ≥ 0 , H(t) ≥ 0 and U(t) ≥ 0 integrating both sides of the inequality, we get

$$\int \frac{dM}{M} \geq -\int (\beta_1 U - \mu) dt,$$
$$\Rightarrow ln(M) \geq -(\beta_1 U - \mu)t + c,$$
$$\Rightarrow H(t) \geq e^{-(\beta_1 U-\mu)t+c} \quad \text{and} \quad M(0) > 0 > e^c,$$
$$\Rightarrow M(t) \geq M(O)e^{-(\beta_1 U-\mu)} \geq 0.$$

Thus M(t) ≥ 0 for all $t > 0$. Therefore, the solutions $\{S(t), M(t), G(t), U(t), H(t), D(t)\}$ of the system (4.2- 4.7) remains positive for all $t > 0$.

Theorem 4.3 (Boundedness of the Solution)
There exists a positively invariant region Ω in which the solution
(S(t), M(t) ,G(t) ,U(t) ,H(t) ,D(t)) of the dynamical system (4.2- 4.7) is bounded.

Proof
In order to show that the population sizes of each compartment is bounded, we prefer to show that the total population size, of the whole system is bounded. The total population size , is given by: N(t) = S(t) + M(t) + U(t) + G(t) + H(t) + D(t).

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dM}{dt} + \frac{dU}{dt} + \frac{dG}{dt} + \frac{dH}{dt} + \frac{dD}{dt}.$$

Then we obtain

$$\frac{dN}{dt} = \Lambda - \mu N.$$

By rearranging we get:

$$\frac{dN}{dt} + \mu N = \Lambda. \tag{4.17}$$

Equation (4.17) is first order non-linear homogeneous differential equation then, multiplies both by integrating factor $e^{\mu t}$.

$$\begin{aligned}
e^{\mu t}\frac{dN}{dt} + e^{\mu t}\mu N &= e^{\mu t}\Lambda \\
\frac{d}{dt}(Ne^{\mu t}) &= e^{\mu t}\Lambda \\
\int_0^t \frac{d}{dt}(Ne^{\mu t}) &= \Lambda \int_0^t e^{\mu t} \\
(Ne^{\mu t})|_0^t &= \frac{\Lambda}{\mu} e^{\mu t}|_0^t \\
N(t)e^{\mu t} - N(0) &= \frac{\Lambda}{\mu}(e^{\mu t} - e^0) \\
N(t)e^{\mu t} &= N(0) + \frac{\Lambda}{\mu}e^{\mu t} - \frac{\Lambda}{\mu} \\
N(t) &= (N(0) - \frac{\Lambda}{\mu})e^{-\mu t} + \frac{\Lambda}{\mu} \\
\text{as} t \to \infty, N(t) &= \frac{\Lambda}{\mu}.
\end{aligned}$$

Thus, the total population N(t) and each population classes are remain bounded for all time $t \geq 0$.

Therefore, the model (4.2 - 4.7) is well posed epidemiological and mathematically in a positively invariant set,

$$\Omega = \left\{ (S, M, U, G, H, D) \in \mathbb{R}^6_+ : N(t) = \frac{\Lambda}{\mu} \right\}.$$

Chapter 5

5 Qualitative Analysis of the Model

5.1 Equilibrium Point

In this section we identify the equilibrium points of the model system [4.3] equation. which are obtained by setting the right-hand sides of the system of equations of model to be zero. The system exhibits two types of equilibrium point they are unhealthy attitude-free equilibrium points (UAFE) and unhealthy attitude present equilibrium point (UAPE).

$$\Lambda - \beta_2 US - (\epsilon + \mu)S = 0, \tag{5.1}$$

$$\epsilon S + \eta G + \omega H - \beta_1 MU - \mu M = 0, \tag{5.2}$$

$$\beta_1 MU - (\rho + \alpha + \eta + \mu)G = 0, \tag{5.3}$$

$$\beta_2 SU + \alpha G + \theta D - (\delta + \phi + \mu)U = 0, \tag{5.4}$$

$$\delta U - (\theta + \pi + \mu)D = 0, \tag{5.5}$$

$$\phi U + \pi D + \rho G - (\omega + \mu)H = 0. \tag{5.6}$$

5.2 Unhealthy Attitude Free Equilibrium Point

Unhealthy attitude free equilibrium (UAFE) point , denoted by U_0, is a steady state solution of the model, indicating that there are no individuals with unhealthy attitude on marriage. This is obtained by setting the right hand side of the model equation equal to zero. Unhealthy attitude free equilibrium (UAFE) point of the model equation 4.3 be U_0, then it is obtained by setting:
$\frac{dS}{dt} = 0$, $\frac{dM}{dt} = 0$, $\frac{dG}{dt} = 0$, $\frac{dU}{dt} = 0$, $\frac{dD}{dt} = 0$, $\frac{dH}{dt} = 0$.
Unhealthy attitude free equilibrium point of our model is obtained by setting the unhealthy attitude state variable U = 0. Substituting this value at (5.1 - 5.6). Thus, Unhealthy attitude free equilibrium point is given by

$$U_0 = (S^0, M^0, G^0, U^0, D^0, H^0) = \left(\frac{\Lambda}{(\epsilon + \mu)}, \frac{\epsilon \Lambda}{\mu(\epsilon + \mu)}, 0, 0, 0, 0 \right).$$

5.3 Basic Reproduction Number

The basic reproduction number, denoted R_0, is the expected number of secondary cases produced, in a completely susceptible population, by a typical infective individual [44]. We calculate the basic reproduction number R_0 of the system by applying the next generation matrix approach as laid out by. The basic reproduction number R_0 can be calculated from the relation $R_0 = \rho(FV^{-1})$. Let f be the vector for the newly negatively influence with unhealthy attitude and v be the vector for the transfer of individuals into and out of the infected compartments. Let x = (G, U, D,H), then

$$\frac{dG}{dt} = \beta_1 MU - (\rho + \alpha + \eta + \mu)G,$$
$$\frac{dU}{dt} = \beta_2 SU + \alpha G + \theta D - (\delta + \phi + \mu)U,$$
$$\frac{dD}{dt} = \delta U - (\theta + \pi + \mu)D,$$
$$\frac{dH}{dt} = \phi U + \pi D + \rho G - (\omega + \mu)H.$$

Then by the principle, we obtained:

$$f(x) = \begin{bmatrix} \beta_1 MU \\ \beta_2 SU \\ 0 \\ 0 \end{bmatrix},$$

$$V(x) = \begin{bmatrix} (\rho + \alpha + \eta + \mu)G \\ -\alpha G - \theta D + (\delta + \phi + \mu)U \\ -\delta U + (\theta + \pi + \mu)D \\ -\phi U - \pi D - \rho G + (\omega + \mu)H \end{bmatrix}.$$

The Jacobian matrix to $f(x)$ and $v(x)$ are

$$F = \left[\frac{\partial f_i(U_0)}{\partial x_j}\right] = \begin{bmatrix} 0 & \beta_1 M^0 & 0 & 0 \\ 0 & \beta_2 S^0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$V = \left[\frac{\partial v_i(U_0)}{\partial x_j}\right] = \begin{bmatrix} k_1 & 0 & 0 & 0 \\ -\alpha & k_2 & -\theta & 0 \\ 0 & -\delta & k_3 & 0 \\ -\rho & -\phi & -\pi & (\omega + \mu) \end{bmatrix},$$

where $k_1 = (\rho + \alpha + \eta + \mu)$, $k_2 = (\delta + \phi + \mu)$ and $k_3 = (\theta + \pi + \mu)$
and

$$V^{-1} = \frac{adj(V)}{det(V)},$$

$$V^{-1} = \frac{1}{k_1(\omega + \mu)(k_2 k_3 - \delta\theta)} \begin{bmatrix} (\omega + \mu)(k_2 k_3 - \theta\delta) & 0 & 0 & 0 \\ k_3 k_4 \alpha & k_1 k_3 k_4 & k_1 k_4 \theta & 0 \\ k_4 \alpha\delta & \delta k_1 k_4 & k_1 k_2 k_4 & 0 \\ \alpha[\delta\pi + \phi k_3] + \rho[k_2 k_3 - \theta\delta] & k_1[\delta\pi + k_3\phi] & k_1[\pi k_2 + \theta\phi] & k_1[k_2 k_3 - \delta\theta] \end{bmatrix},$$

$$= \begin{bmatrix} \frac{1}{k_1} & 0 & 0 & 0 \\ \frac{k_3\alpha}{k_1(k_2k_3-\delta\theta)} & \frac{k_3}{(k_2k_3-\delta\theta)} & \frac{\theta}{(k_2k_3-\delta\theta)} & 0 \\ \frac{\alpha\delta}{k_1(k_2k_3-\delta\theta)} & \frac{\delta}{(k_2k_3-\delta\theta)} & \frac{k_2}{(k_2k_3-\delta\theta)} & 0 \\ \frac{\alpha[\delta\pi+\phi k_3]+\rho[k_2k_3-\theta\delta]}{k_1(\omega+\mu)(k_2k_3-\delta\theta)} & \frac{\delta\pi+k_3\phi}{(\omega+\mu)(k_2k_3-\delta\theta)} & \frac{\pi k_2+\theta\phi}{(\omega+\mu)(k_2k_3-\delta\theta)} & \frac{k_2k_3-\delta\theta}{(\omega+\mu)(k_2k_3-\delta\theta)} \end{bmatrix}.$$

Then the next generation matrix FV^{-1} is given by:

$$FV^{-1} = \begin{pmatrix} \frac{\alpha\beta_1k_3M^0}{k_1(k_2k_3-\delta\theta)} & \frac{\beta_1k_3M^0}{(k_2k_3-\delta\theta)} & \frac{\theta\beta_1M^0}{(k_2k_3-\delta\theta)} & 0 \\ \frac{\alpha\beta_2k_3S^0}{k_1(k_2k_3-\delta\theta)} & \frac{\beta_2k_3S^0}{(k_2k_3-\delta\theta)} & \frac{\theta\beta_2S^0}{(k_2k_3-\delta\theta)} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

The eigenvalues, λ of the matrix FV^{-1} , are calculated from the characteristic equation of $det(FV^{-1}-\lambda I)$,Where I is a 3x3 identity matrix,

$$= \begin{vmatrix} \frac{\alpha\beta_1k_3M^0}{k_1(k_2k_3-\delta\theta)}-\lambda & \frac{\beta_1k_3M^0}{(k_2k_3-\delta\theta)} & \frac{\theta\beta_1M^0}{(k_2k_3-\delta\theta)} & 0 \\ \frac{\alpha\beta_2k_3S^0}{k_1(k_2k_3-\delta\theta)} & \frac{\beta_2k_3S^0}{(k_2k_3-\delta\theta)}-\lambda & \frac{\theta\beta_2S^0}{(k_2k_3-\delta\theta)} & 0 \\ 0 & 0 & 0-\lambda & 0 \\ 0 & 0 & 0 & 0-\lambda \end{vmatrix},$$

$$\Rightarrow \lambda_1 = 0 \ , \ \lambda_2 = 0 \text{ and } \lambda^2 - \lambda(R_1+R_2) + \frac{\alpha\beta_1k_3M^0}{k_1(k_2k_3-\delta\theta)}\frac{\beta_2k_3S^0}{(k_2k_3-\delta\theta)} - \frac{\alpha\beta_1k_3S^0}{k_1(k_2k_3-\delta\theta)}\frac{\beta_2k_3M^0}{(k_2k_3-\delta\theta)} = 0,$$

$$\lambda^2 - \lambda(R_1+R_2) = 0,$$

where

$$R_1 = \frac{\alpha\beta_1k_3M^0}{k_1(k_2k_3-\delta\theta)} \quad \text{and} \quad R_2 = \frac{\beta_2k_3S^0}{(k_2k_3-\delta\theta)}, \tag{5.7}$$

$\lambda_3 = 0$ and $\lambda_4 = R_1 + R_2$ The basic reproduction number R_0 is the spectral radius (the largest eigenvalues in modulus) of FV^{-1} which is given by

$$R_0 = \lambda_4 = R_1 + R_2.$$

We can write the basic reproduction number as follows:

$$R_0 = \frac{(\theta+\pi+\mu)\Lambda}{(\delta(\pi+\mu)+(\phi+\mu)(\theta+\pi+\mu))(\epsilon+\mu)}\left(\frac{\alpha\epsilon\beta_1}{\mu(\rho+\alpha+\eta+\mu)}+\beta_2\right). \tag{5.8}$$

5.4 Stability analysis of Unhealthy Attitude Free Equilibrium Points

5.4.1 Local Stability of UAFE, U_0

Local stability of UAFE can be determined from the sign of the eigen values of the Jacobian matrix; that is if all the eigenvalues are negatives or have negative real parts, then the Unhealthy attitude free equilibrium point of the system is locally asymptotically stable and unstable if at least one of the eigenvalues has positive real part.

Theorem 5.1 Unhealthy attitude free equilibrium point U_0 of the system 4.2 - 4.7 is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.

Proof:
The Jacobian matrix of the system 4.2 - 4.7 at the Unhealthy attitude free equilibrium u_0 is given by

$$J(u_0) = \begin{bmatrix} -(\epsilon+\mu) & 0 & 0 & -\beta_2 S^0 & 0 & 0 \\ \epsilon & -\mu & \eta & -\beta_1 M^0 & 0 & \omega \\ 0 & 0 & -(\rho+\alpha+\eta+\mu) & \beta_1 M^0 & 0 & 0 \\ 0 & 0 & \alpha & \beta_2 S^0-(\delta+\phi+\mu) & \theta & 0 \\ 0 & 0 & 0 & \delta & -(\theta+\pi+\mu) & 0 \\ 0 & 0 & \rho & \phi & \pi & -(\omega+\mu) \end{bmatrix}.$$

Now, we need to find all the eigenvalues of $J(U_0)$ by solving $\det(J(U_0)-\lambda I) = 0$, where λ is an eigenvalue and I is 6x6 identity matrix,

$$J(u_0-\lambda I) = \left(\begin{array}{cc|cccc} -(\epsilon+\mu)-\lambda & 0 & 0 & -\beta_2 S^0 & 0 & 0 \\ \epsilon & -(\mu)-\lambda & \eta & -\beta_1 M^0 & 0 & \omega \\ \hline 0 & 0 & -k_1-\lambda & \beta_1 M^0 & 0 & 0 \\ 0 & 0 & \alpha & \beta_2 S^0-k_2-\lambda & \theta & 0 \\ 0 & 0 & 0 & \delta & -k_3-\lambda & 0 \\ 0 & 0 & \rho & \phi & \pi & -(\omega+\mu)-\lambda \end{array}\right).$$

The eigenvalue are λ_1, λ_2, λ_3, λ_4, λ_5 and λ_6. From the first block matrix of $J(u_0-\lambda I)$ we obtain λ_1 and λ_2. And λ_3, λ_4, λ_5 and λ_6 are eigenvalues of the fourth block matrix of $J(u_0-\lambda I)$.
From first block matrix of $J(u_0-\lambda I)$ given by

$$J_1 = \begin{bmatrix} -(\epsilon+\mu)-\lambda & 0 \\ \epsilon & -\mu-\lambda \end{bmatrix}.$$

We find the eigenvalues of $J_1(u_0-\lambda I)$ are $\lambda_1 = -(\epsilon+\mu)$ and $\lambda_2 = -\mu$
From the fourth block matrix of $J(u_0-\lambda I)$ is given by

$$J_4 = \begin{bmatrix} -k_1-\lambda & \beta_1 M^0 & 0 & 0 \\ \alpha & \beta_2 S^0-k_2-\lambda & \theta & 0 \\ 0 & \delta & -k_3-\lambda & 0 \\ \rho & \phi & \pi & -(\omega+\mu)-\lambda \end{bmatrix}.$$

Thus the eigenvalues λ_3, λ_4, λ_5 and λ_6 are obtain from characteristic equation of $J_4(u_0-\lambda I)$

$$-(\omega+\mu)-\lambda[\lambda^3+(k_1+k_2+k_3-\beta_2 S^0)\lambda^2+(k_1k_2+k_2k_3+k_1k_3-\beta_2 S^0 k_1-\beta_2 S^0 k_3 \\ -\delta\theta-\alpha\beta_1 M^0)\lambda-\beta_2 S^0 k_1 k_3+k_1k_2k_3-\delta\theta k_1-\alpha\beta_1 M^0 k_3] = 0.$$

From this equation, we obtain the values for $\lambda_3 = -(\omega+\mu)$ and the eigenvalues λ_4, λ_5 and λ_6 are the roots of the cubic polynomial:

$$p(\lambda) = \lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0.$$

By Routh-Hurwitz criteria for order polynomial leads to n = 3; the coefficient of the characteristics polynomial satisfies the conditions of

$$a_1 > 0, \quad a_3 > 0 \quad \text{and} \quad a_1a_2 > a_3.$$

Let us check the Routh-Hurwitz criteria

$$\text{For} \quad a_1,$$

$$a_1 = k_1 + k_2 + k_3 - \beta_2 S^0 > 0,$$
$$a_1 > 0 \quad \text{iff} \quad k_1 + k_2 + k_3 > \beta_2 S^0.$$

$$\text{For} \quad a_2,$$

$$\begin{aligned} a_2 &= k_1k_2 + k_2k_3 + k_1k_3 - \beta_2 S^0 k_1 - \beta_2 S^0 k_3 - \delta\theta - \alpha\beta_1 M^0, \\ &= k_1k_2 + k_1k_3 + \left(1 - \frac{R_0k_1}{k_3}\right)(k_2k_3 - \delta\theta) - \beta_2 S^0 k_3 > 0, \quad \text{iff} \\ k_1k_2 + k_1k_3 + (k_2k_3 - \delta\theta) &> \frac{R_0k_1}{k_3}(k_2k_3 - \delta\theta) + \beta_2 S^0 k_3. \end{aligned}$$

$$\text{For} \quad a_3,$$

$$a_3 = [1 - R_0]k_1(k_2k_3 - \delta\theta) > 0 \quad \text{if} \quad R_0 < 1.$$

Furthermore,
$a_1a_2 > a_3 = [k_1+k_2+k_3-\beta_2S^0][k_1k_2+k_1k_3+\left(1-\frac{R_0k_1}{k_3}\right)(k_2k_3-\delta\theta)-\beta_2S^0k_3] > [1-R_0]k_1(k_2k_3-\delta\theta)$,
$a_1a_2 > a_3$ if $R_0 < 1$ Therefore, we can concluded that the unhealthy attitude free- equilibrium point of the system (4.2 - 4.7) is locally asymptotically stable if $R_0 < 1$.

5.4.2 Global Stability of UAFE, U_0

Global stability describes behavior of the solution in the whole domain,that is in region Ω of the system (4.2 - 4.7). We use Castillo-Chavez theorem to prove the global stability of unhealthy attitude free equilibrium point.

Theorem 5.2 For $R_0 < 1$ unhealthy attitude free equilibrium U_0 of the system (4.2 - 4.7) is globally asymptotically stable in the feasible domain Ω.

Proof:
Let us rewrite our model system (4.2 - 4.7) as

$$\frac{dZ_1}{dt} = F(Z_1, Z_2),$$
$$\frac{dZ_2}{dt} = G(Z_1, Z_2), \quad G(Z_1, 0) = 0,$$

where, $Z_1 = (S, M) \in \mathbb{R}^3_+$ represent the compartment of uninfected individual or have positive attitude on marriage and $Z_2 = (G, U, D, H) \in \mathbb{R}^3_+$ represent the compartment of infected individual or have negative(unhealthy) attitude on marriage. Unhealthy attitude free equilibrium point of the model is denoted by $N_0 = (Z_1^*, 0)$, where $Z_1^* = \left(\frac{\Lambda}{(\epsilon+\mu)}, \frac{\epsilon\Lambda}{\mu(\epsilon+\mu)}, 0\right)$ Since unhealthy attitude free equilibrium point is locally asymptotically stable (proofed in theorem (5.1) to prove global stability, we will apply the Castillo-Chavez theorem (3.5). From the model system (4.2 - 4.7) we have,

$$\frac{dZ_1}{dt} = F(Z_1, Z_2) = \begin{bmatrix} \Lambda - \beta_2 US - (\epsilon+\mu)S \\ \epsilon S + \eta G + \omega H - \beta_1 MU - \mu M \end{bmatrix},$$

$$\frac{dZ_2}{dt} = G(Z_1, Z_2) = \begin{bmatrix} \beta_1 MU - (\rho+\alpha+\eta+\mu)G \\ \beta_2 SU + \alpha G + \theta D - (\delta+\phi+\mu)U \\ \delta U - (\theta+\pi+\mu)D \\ \phi U + \pi D + \rho G - (\omega+\mu)H \end{bmatrix}.$$

Now consider the following two conditions:

i, To show that Z_1^* is globally asymptotically stable for the system $\frac{dZ_1}{dt} = F(Z_1, 0)$, let us consider the reduced system.

$$\frac{dZ_1}{dt} = F(Z_1, 0) = \begin{bmatrix} \Lambda - (\epsilon+\mu)S \\ \epsilon S - \mu M \end{bmatrix}. \tag{5.9}$$

We can rewrite system (5.9) as:

$$\frac{dS}{dt} = \Lambda - (\epsilon+\mu)S, \tag{5.10}$$
$$\frac{dM}{dt} = \epsilon S - \mu M. \tag{5.11}$$

The system (5.10 - 5.11) is non-homogeneous linear system of ordinary differential equations. For (5.10) by applying an integrating factor method, we have

$$S(t) = \frac{\Lambda}{(\epsilon+\mu)} + \left(S(0) - \frac{\Lambda}{(\epsilon+\mu)}\right) e^{-(\epsilon+\mu)t}.$$

As t → ∞ , S(t) = $\frac{\Lambda}{(\epsilon+\mu)}$.
For (5.11), we have $\frac{dM}{dt} = \frac{\epsilon\Lambda}{(\epsilon+\mu)} - \mu M$,
by applying an integrating factor method, then we have

$$M(t) = \frac{\epsilon\Lambda}{\mu(\epsilon+\mu)} + \left(M(0) - \frac{\Lambda}{(\epsilon+\mu)}\right)e^{-\mu t}.$$

As t → ∞ , M(t) = $\frac{\epsilon\Lambda}{\mu(\epsilon+\mu)}$,
then, we obtain

$$Z_1^* = (S(t), M(t)) \to \left(\frac{\Lambda}{(\epsilon+\mu)}, \frac{\epsilon\Lambda}{\mu(\epsilon+\mu)}\right).$$

Therefor, Z_1^* is globally asymptotically stable for the system $\frac{dZ_1}{dt} = F(Z_1, 0)$.

ii, We now show that $G(Z_1, Z_2) = AZ_2$ - $\hat{G}(Z_1, Z_2)$, $\hat{G}(Z_1, Z_2) > 0$ for $(Z_1, Z_2) \in \Omega$ where $A = \frac{\partial G}{\partial Z_2}(Z_1^*, 0)$ is a Metzler matrix (the off diagonal elements of A are non-negative) and Ω is the region where the model makes biological sense. Consider a matrix,

$$A = \frac{\partial G}{\partial Z_2}(Z_1^*, 0) = \begin{bmatrix} -(\rho+\alpha+\eta+\mu) & \beta_1 M^0 & 0 & 0 \\ \alpha & \beta_2 S^0 - (\delta+\phi+\mu) & \theta & 0 \\ 0 & \delta & -(\theta+\pi+\mu) & 0 \\ \rho & \phi & \pi & (\omega+\mu) \end{bmatrix}.$$

Hence, A is a Metzer matrix (off diagonal elements are non-negative). Here,

$$\hat{G}(Z_1, Z_2) = AZ_2 - G(Z_1, Z_2),$$

$$\begin{pmatrix} -k_1 & \beta_1 M^0 & 0 & 0 \\ \alpha & \beta_2 S^0 - k_2 & \theta & 0 \\ 0 & \delta & -k_3 & 0 \\ \rho & \phi & \pi & -(\omega+\mu) \end{pmatrix} \begin{pmatrix} G \\ U \\ D \\ H \end{pmatrix} - \begin{pmatrix} \beta_1 MU - k_1 G \\ \beta_2 SU + \alpha G + \theta D - k_2 U \\ \delta U - k_3 D \\ \phi U + \pi D + \rho G - (\omega+\mu)H \end{pmatrix}.$$

After some simplification, we obtain

$$\begin{pmatrix} (M^0 - M)\beta_1 U \\ (S^0 - S)\beta_2 U \\ 0 \\ 0 \end{pmatrix} \geq 0.$$

Therefore by Castillo-Chavez theorem9(3.5) Unhealthy attitude free equilibrium point U_0 of the system (4.2 - 4.7) is globally asymptotically stable for $R_0 < 1$.

5.5 Unhealthy Attitude present Equilibrium Point

Unhealthy Attitude present Equilibrium Point (UAPE) is a steady state solution where the unhealthy attitude on marriage persists in the population. There exist an equilibrium point called endemic equilibrium point denoted by $E^* = (S^*, M^*, G^*, U^*, D^*, H^*)$. It can be obtained by setting each equation of the system (4.3) equal to zero:

$$\Lambda - \beta_2 US - (\epsilon + \mu)S = 0, \tag{5.12a}$$

$$\epsilon S + \eta G + \omega H - \beta_1 MU - \mu M = 0, \tag{5.12b}$$

$$\beta_1 MU - (\rho + \alpha + \eta + \mu)G = 0, \tag{5.12c}$$

$$\beta_2 SU + \alpha G + \theta D - (\delta + \phi + \mu)U = 0, \tag{5.12d}$$

$$\delta U - (\theta + \pi + \mu)D = 0, \tag{5.12e}$$

$$\phi U + \pi D + \rho G - (\omega + \mu)H = 0. \tag{5.12f}$$

From equation (5.12a), we obtain

$$S^* = \frac{\Lambda}{\beta_2 U^* + (\epsilon + \mu)}. \tag{5.13}$$

From equation (5.12c), we obtain

$$G^* = \frac{\beta_1 M^* U^*}{k_1}. \tag{5.14}$$

From equation (5.12e), we obtain

$$D^* = \frac{\delta U^*}{k_3}. \tag{5.15}$$

By substituting equations (5.14) and (5.15) into equation (5.12f), we get

$$U^* \left(\frac{\phi}{(\omega + \mu)} + \frac{\pi \delta}{(\omega + \mu) k_3} + \frac{\rho \beta_1 M^*}{(\omega + \mu) k_1} \right) = H^*. \tag{5.16}$$

By substituting equations (5.13), (5.14) and (5.16) into equation (5.12b), we get

$$k_1 \frac{\epsilon \Lambda k_3 (\omega + \mu) + k_3 \omega U^* \phi (\beta_2 U^* + (\epsilon + \mu)) + \omega U^* \pi \delta (\beta_2 U^* + (\epsilon + \mu))}{k_3 (\beta_2 U^* + (\epsilon + \mu))((\beta_1 U^* + \mu)(\omega + \mu) k_1 - \eta \beta_1 U^* (\omega + \mu) - \omega U^* \rho \beta_1)} = M^*. \tag{5.17}$$

By substituting equations (5.13), (5.14) and (5.15) into equation (5.12d), we get

$$U^* \left(\frac{\beta_2 \Lambda}{\beta_2 U^* + (\epsilon + \mu)} + \frac{\alpha \beta_1 M^*}{k_1} + \frac{\theta \delta}{k_3} - k_2 \right) = 0,$$

$$U^* = 0 \quad \text{and} \quad \frac{\beta_2 \Lambda}{\beta_2 U^* + (\epsilon + \mu)} + \frac{\alpha \beta_1 M^*}{k_1} + \frac{\theta \delta}{k_3} - k_2 = 0,$$

$$\frac{\beta_2 \Lambda}{\beta_2 U^* + (\epsilon + \mu)} + \frac{\alpha \beta_1 M^*}{k_1} + \frac{\theta \delta}{k_3} - k_2 = 0. \tag{5.18}$$

By substituting equation (5.17) into equation (5.18), we get

$$\frac{\beta_2 \Lambda}{\beta_2 U^* + (\epsilon + \mu)} + \alpha \beta_1 \left(\frac{\epsilon \Lambda k_3 (\omega + \mu) + k_3 \omega U^* \phi (\beta_2 U^* + (\epsilon + \mu)) + \omega U^* \pi \delta (\beta_2 U^* + (\epsilon + \mu))}{k_3 (\beta_2 U^* + (\epsilon + \mu))((\beta_1 U^* + \mu)(\omega + \mu) k_1 - \eta \beta_1 U^* (\omega + \mu) - \omega U^* \rho \beta_1)} \right) = k_2 - \frac{\theta \delta}{k_3} \tag{5.19}$$

$$\frac{\alpha\beta_1\epsilon\Lambda k_3(\omega+\mu)+\alpha\beta_1 k_3\omega U^*\phi\beta_2 U^*+\alpha\beta_1 k_3\omega U^*\phi(\epsilon+\mu)+\alpha\beta_1\omega U^*\pi\delta\beta_2 U^*+\alpha\beta_1\omega U^*\pi\delta(\epsilon+\mu)}{(\beta_1 U^*(\omega+\mu)k_1+\mu(\omega+\mu)k_1-\eta\beta_1 U^*(\omega+\mu)-\omega U^*\rho\beta_1)},$$

$$= k_3k_2\beta_2U^*+k_3k_2(\epsilon+\mu)-\theta\delta\beta_2U^*-\theta\delta(\epsilon+\mu)-\beta_2\Lambda k_3.$$

Finally we get

$$AU^2+BU+C=0, \tag{5.20}$$

where

$$A=\beta_1\beta_2(\omega\delta\mu(\alpha+\pi+\mu)+k_3\mu\omega(\alpha+\phi+\mu)+\mu(\rho+\alpha+\mu)(\delta(\pi+\mu)+(\phi+\mu)k_3)),$$

$$B=\beta_1k_1k_2(\omega+\mu)(\epsilon+\mu)(\rho+\alpha+\mu)+\beta_2\Lambda k_3\omega\rho\beta_1-(\alpha\beta_1k_3\omega\phi(\epsilon+\mu)+\alpha\beta_1\omega\pi\delta(\epsilon+\mu)+$$
$$\omega\rho\beta_1(\epsilon+\mu)(k_2k_3-\theta\delta)+\theta\delta\beta_2k_1\mu(\omega+\mu)+\theta\delta\beta_1(\epsilon+\mu)(\omega+\mu)(\rho+\alpha+\mu)+\beta_1\beta_2\Lambda k_2(\omega+\mu)(\rho+\alpha+\mu)),$$

$$C=(1-R_0)\mu(\omega+\mu)(\epsilon+\mu)k_1(k_2k_3-\delta\theta).$$

Using the quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}, \tag{5.21}$$

we have

$$U^*=\frac{-B\pm\sqrt{B^2-4AC}}{2A}=\Psi.$$

Hence

$$U^*>0,$$
$$S^*=\frac{\Lambda}{\beta_2\Psi+(\epsilon+\mu)},$$
$$G^*=\beta_1\Psi\left(\frac{\epsilon\Lambda k_3(\omega+\mu)+k_3\omega\Psi\phi(\beta_2\Psi+(\epsilon+\mu))+\omega\Psi\pi\delta(\beta_2\Psi+(\epsilon+\mu))}{k_3(\beta_2\Psi+(\epsilon+\mu))((\beta_1\Psi+\mu)(\omega+\mu)k_1-\beta_1\Psi(\eta(\omega+\mu)+\omega\rho))}\right),$$
$$D^*=\frac{\delta\Psi}{k_3},$$
$$H^*=\frac{\Psi}{(\omega+\mu)}\left(\phi+\frac{\pi\delta}{k_3}+\rho\beta_1\left(\frac{\epsilon\Lambda k_3(\omega+\mu)+k_3\omega\Psi\phi(\beta_2\Psi+(\epsilon+\mu))+\omega\Psi\pi\delta(\beta_2\Psi+(\epsilon+\mu))}{k_3(\beta_2\Psi+(\epsilon+\mu))((\beta_1\Psi+\mu)(\omega+\mu)k_1-\beta_1\Psi(\eta(\omega+\mu)+\omega\rho))}\right)\right),$$
$$M^*=k_1\frac{\epsilon\Lambda k_3(\omega+\mu)+k_3\omega\Psi\phi(\beta_2\Psi+(\epsilon+\mu))+\omega\Psi\pi\delta(\beta_2\Psi+(\epsilon+\mu))}{k_3(\beta_2\Psi+(\epsilon+\mu))((\beta_1\Psi+\mu)(\omega+\mu)k_1-\beta_1\Psi(\eta(\omega+\mu)+\omega\rho))}.$$

From the quadratic equation 5.21 we analyze the possibility of multiple equilibrium. It is important to note that the coefficient A is always positive with B and C having different signs. We realize that C is positive if R_0 is less than unity, and negative if R_0 is greater than unity. Hence, we have established the following results:

Proposition 5.1

1, Precisely one unique UAPE if B < 0 and C = 0 or B^2 - 4AC = 0,

2, Precisely one unique UAPE if C < 0 ⇔ $R_0 > 1$,

3, Precisely two UAPE if C > 0, B < 0 and B^2 - 4AC > 0,

3, No UAPE otherwise.

Theorem 5.3 The UAPE, E^* exists whenever $R_0 > 1$.

Proof

From the equation (5.20) $f(U) = AU^2 + BU + C = 0$, we solve for U the equation $AU^2 + BU + C = 0$, thus we obtain

$$U^* = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}.$$

From which it is clear that the unhealthy attitude is endemic when $U^* > 0$ that is

$$\frac{-B \pm \sqrt{B^2 - 4AC}}{2A} > 0.$$

It then follows that

$$\pm\sqrt{B^2 - 4AC} > B,$$

$$\Rightarrow B^2 - 4AC > B^2.$$

Then we have

$$B^2 - 4AH(1 - R_0) > B^2.$$

Where $H = \mu(\omega + \mu)(\epsilon + \mu)k_1(k_2 k_3 - \delta\theta)$,
leading to

$$4AH(1 - R_0) < 0.$$

This means that
$4AH(1 - R_0) < 0$, implying that $R_0 > 1$. This completes the theorem.
Thus, UAPE E^* exists if and only if $R_0 > 1$.

5.6 Stability Analysis of Unhealthy Attitude present Equilibrium Points

5.6.1 Local Stability of UAPE

We analyze the stability of unhealthy attitude present equilibrium point by linearizing the system of differential equations to get the Jacobian matrix. The Jacobian matrix is computed by differentiating each equation of system (4.2 - 4.7) with respect to the state variables at E^*. The local stability of E^* is determined by considering the sign of the eigenvalues of the Jacobian matrix of system (4.2 - 4.7) and finally, we will use Routh Hurwitz stability Criteria, which is used to determine asymptotic stability of equilibrium for a non-linear system of ordinary differential equations.

Theorem 5.4 The positive equilibrium point, E^* of system (4.2 - 4.7) is locally asymptotically stable if $R_0 > 1$ and unstable if $R_0 < 1$.
Proof:
Stability of E^* of the model system is determined based on the signs of the eigenvalues of the Jacobian matrix. The Jacobean matrix of system (4.2 - 4.7) around the smoking-present equilibrium, E^* is given by:

$$J(E^*) = \begin{bmatrix} -(\beta_2 U^* + (\epsilon+\mu)) & 0 & 0 & -\beta_2 S^* & 0 & 0 \\ \epsilon & -(\beta_1 U^* + \mu) & \eta & -\beta_1 M^* & 0 & \omega \\ 0 & \beta_1 U^* & -k_1 & \beta_1 M^* & 0 & 0 \\ \beta_2 U^* & 0 & \alpha & \beta_2 S^* - (\delta+\phi+\mu) & \theta & 0 \\ 0 & 0 & 0 & \delta & -(\theta+\pi+\mu) & 0 \\ 0 & 0 & \rho & \phi & \pi & -(\omega+\mu) \end{bmatrix},$$

For simplicity, let $Z_1 = \beta_2 U^* + (\epsilon+\mu)$, $Z_2 = \beta_2 S^*$ $Z_3 = \beta_1 U^* + \mu$, $Z_4 = \beta_1 M^*$, $Z_5 = \beta_1 U^*$, $Z_6 = \beta_1 M^*$, $Z_7 = \beta_2 U^*$, $Z_8 = \beta_2 S^* - (\delta+\phi+\mu)$, $k_1 = (\rho+\alpha+\eta+\mu)$, $k_3 = (\theta+\pi+\mu)$, $k_4 = (\omega+\mu)$,

$$J(E^*) = \begin{bmatrix} -Z_1 & 0 & 0 & -Z_2 & 0 & 0 \\ \epsilon & -Z_3 & \eta & -Z_4 & 0 & \omega \\ 0 & Z_5 & -k_1 & Z_6 & 0 & 0 \\ Z_7 & 0 & \alpha & Z_8 & \theta & 0 \\ 0 & 0 & 0 & \delta & -k_3 & 0 \\ 0 & 0 & \rho & \phi & \pi & -K_4 \end{bmatrix},$$

$$det(J(E^*) - \lambda I) = 0,$$

$$= \begin{bmatrix} -Z_1-\lambda & 0 & 0 & -Z_2 & 0 & 0 \\ \epsilon & -Z_3-\lambda & \eta & -Z_4 & 0 & \omega \\ 0 & Z_5 & -k_1-\lambda & Z_6 & 0 & 0 \\ Z_7 & 0 & \alpha & Z_8-\lambda & \theta & 0 \\ 0 & 0 & 0 & \delta & -k_3-\lambda & 0 \\ 0 & 0 & \rho & \phi & \pi & -K_4-\lambda \end{bmatrix} = 0,$$

$$-Z_1-\lambda\begin{bmatrix}-Z_3-\lambda & \eta & -Z_4 & 0 & \omega\\ Z_5 & -k_1-\lambda & Z_6 & 0 & 0\\ 0 & \alpha & Z_8-\lambda & \theta & 0\\ 0 & 0 & \delta & -k_3-\lambda & 0\\ 0 & \rho & \phi & \pi & -K_4-\lambda\end{bmatrix}+ \tag{5.22}$$

$$Z_2\begin{bmatrix}\epsilon & -Z_3-\lambda & \eta & 0 & \omega\\ 0 & Z_5 & -k_1-\lambda & 0 & 0\\ Z_7 & 0 & \alpha & \theta & 0\\ 0 & 0 & 0 & -k_3-\lambda & 0\\ 0 & 0 & \rho & \pi & -K_4-\lambda\end{bmatrix}=0, \tag{5.23}$$

let first simplify (5.22) matrix

$$-Z_1-\lambda\begin{bmatrix}-Z_3-\lambda & \eta & -Z_4 & 0 & \omega\\ Z_5 & -k_1-\lambda & Z_6 & 0 & 0\\ 0 & \alpha & Z_8-\lambda & \theta & 0\\ 0 & 0 & \delta & -k_3-\lambda & 0\\ 0 & \rho & \phi & \pi & -K_4-\lambda\end{bmatrix}$$

$$=Z_1Z_3+\lambda(Z_1+Z_3)+\lambda^2\begin{bmatrix}-k_1-\lambda & Z_6 & 0 & 0\\ \alpha & Z_8-\lambda & \theta & 0\\ 0 & \delta & -k_3-\lambda & 0\\ \rho & \phi & \pi & -K_4-\lambda\end{bmatrix}+$$

$$Z_1Z_5+\lambda Z_5\begin{bmatrix}\eta & -Z_4 & 0 & \omega\\ \alpha & Z_8-\lambda & \theta & 0\\ 0 & \delta & -k_3-\lambda & 0\\ \rho & \phi & \pi & -K_4-\lambda\end{bmatrix}$$

$$=-K_4Z_1Z_3-\lambda(K_4(Z_1+Z_3)+Z_1Z_3)-\lambda^2(Z_1+Z_3+K_4)-\lambda^3\begin{bmatrix}-k_1-\lambda & Z_6 & 0\\ \alpha & Z_8-\lambda & \theta\\ 0 & \delta & -k_3-\lambda\end{bmatrix}$$

$$(-K_4Z_1Z_3-\lambda(K_4(Z_1+Z_3)+Z_1Z_3)-\lambda^2(Z_1+Z_3+K_4)-\lambda^3)(-k_1-\lambda)\begin{bmatrix}Z_8-\lambda & \theta\\ \delta & -k_3-\lambda\end{bmatrix}$$

$+$

$$K_4Z_1Z_3\alpha+\lambda\alpha(K_4(Z_1+Z_3)+Z_1Z_3)+\lambda^2\alpha(Z_1+Z_3+K_4)+\lambda^3\alpha\begin{bmatrix}Z_6 & 0\\ \delta & -k_3-\lambda\end{bmatrix}$$

$$\begin{aligned}&=(k_1K_4Z_1Z_3+\lambda(k_1K_4(Z_1+Z_3)+K_4Z_1Z_3+k_1Z_1Z_3)+\lambda^2(k_1(Z_1+Z_3+K_4)+K_4(Z_1+Z_3)+Z_1Z_3)\\&\quad+\lambda^3(k_1+Z_1+Z_3+K_4)+\lambda^4)(-(k_3Z_8+\delta\theta)+\lambda(k_3-Z_8)+\lambda^2)\\&=(Q+\lambda X+\lambda^2H+\lambda^3V+\lambda^4)(-(k_3Z_8+\delta\theta)+\lambda(k_3-Z_8)+\lambda^2)\end{aligned}$$

where

$$Q = k_1 K_4 Z_1 Z_3$$
$$X = k_1 K_4 (Z_1 + Z_3) + K_4 Z_1 Z_3 + k_1 Z_1 Z_3$$
$$H = k_1 (Z_1 + Z_3 + K_4) + K_4 (Z_1 + Z_3) + Z_1 Z_3$$
$$V = k_1 + Z_1 + Z_3 + K_4$$

$$= -(k_3 Z_8 + \delta\theta) Q + \lambda (Q(k_3 - Z_8) - (k_3 Z_8 + \delta\theta) X) + \lambda^2 ((k_3 - Z_8) X + Q - (k_3 Z_8 + \delta\theta) H)$$
$$+\lambda^3 ((k_3 - Z_8) H + X - (k_3 Z_8 + \delta\theta) V) + \lambda^4 ((k_3 - Z_8) V + H - (k_3 Z_8 + \delta\theta)) + \lambda^5 (k_3 - Z_8 + V) + \lambda^6 \tag{5.24}$$

+

$$\begin{aligned} -Z_6 k_3 \alpha K_4 Z_1 Z_3 - \lambda ((Z_6 k_3 \alpha (K_4 (Z_1 + Z_3) + Z_1 Z_3) + Z_1 Z_3 Z_6 k_4 \alpha) - \\ \lambda^2 \alpha Z_6 (k_3 (Z_1 + Z_3 + K_4) + K_4 (Z_1 + Z_3) + Z_1 Z_3) \\ -\lambda^3 Z_6 \alpha (k_3 + Z_1 + Z_3 + K_4)) - \lambda^4 Z_6 \alpha \end{aligned} \tag{5.25}$$

+

$$Z_1 Z_5 + \lambda Z_5 \begin{bmatrix} \eta & -Z_4 & 0 & \omega \\ \alpha & Z_8 - \lambda & \theta & 0 \\ 0 & \delta & -k_3 - \lambda & 0 \\ \rho & \phi & \pi & -K_4 - \lambda \end{bmatrix}$$

$$+Z_1 Z_5 \omega + \lambda Z_5 \omega \begin{bmatrix} \alpha & Z_8 - \lambda & \theta \\ 0 & \delta & -k_3 - \lambda \\ \rho & \phi & \pi \end{bmatrix} + (Z_1 Z_5 + \lambda Z_5)(K_4 + \lambda) \begin{bmatrix} \eta & -Z_4 & 0 \\ \alpha & Z_8 - \lambda & \theta \\ 0 & \delta & -k_3 - \lambda \end{bmatrix}$$

$$\begin{aligned} &+Z_1 Z_5 \omega \alpha k_3 \phi + Z_1 Z_5 \omega \alpha \delta \pi - Z_1 Z_5 \omega \rho (k_3 Z_8 + \theta\delta) - Z_1 Z_5 k_4 \eta (Z_8 k_4 + \delta\theta) - \alpha Z_1 Z_4 Z_5 k_3 k_4 \\ &+\lambda (Z_5 \omega \alpha \delta \pi + Z_5 \omega \alpha k_3 \phi + \phi Z_1 Z_5 \omega \alpha + Z_5 \omega \rho (k_3 Z_8 + \theta\delta) + (k_3 - Z_8) Z_1 Z_5 \omega \rho + Z_1 Z_5 k_4 \eta (k_3 - Z_8) \\ &-\eta (Z_1 Z_5 + Z_5 k_4)(Z_8 k_3 + \delta\theta) - k_3 Z_4 \alpha (Z_1 Z_5 + Z_5 k_4) - Z_1 Z_4 Z_5 \alpha k_4) \\ &+\lambda^2 (\phi Z_5 \omega \alpha + Z_5 \omega \rho (k_3 - Z_8) + Z_1 Z_5 \omega \rho + \eta (Z_1 Z_5 + Z_5 k_4)(k_3 - Z_8) + Z_1 Z_5 k_4 \eta - \\ &Z_5 \eta (Z_8 k_3 + \delta\theta) - Z_5 \alpha k_3 Z_4 - Z_4 \alpha (Z_1 Z_5 + Z_5 k_4)) \\ &+\lambda^3 (Z_5 \omega \rho + Z_5 \eta (k_3 - Z_8) + \eta (Z_1 Z_5 + Z_5 k_4) - Z_5 \alpha) + \lambda^4 Z_5 \eta \end{aligned} \tag{5.26}$$

+

$$+Z_2 \begin{bmatrix} \epsilon & -Z_3 - \lambda & \eta & 0 & \omega \\ 0 & Z_5 & -k_1 - \lambda & 0 & 0 \\ Z_7 & 0 & \alpha & \theta & 0 \\ 0 & 0 & 0 & -k_3 - \lambda & 0 \\ 0 & 0 & \rho & \pi & -K_4 - \lambda \end{bmatrix},$$

Now, let simplify (5.23) matrix and add with the simplification (5.22) matrix

$$+Z_2\epsilon\begin{bmatrix} Z_5 & -k_1-\lambda & 0 & 0 \\ 0 & \alpha & \theta & 0 \\ 0 & 0 & -k_3-\lambda & 0 \\ 0 & \rho & \pi & -K_4-\lambda \end{bmatrix} + Z_2Z_7\begin{bmatrix} -Z_3-\lambda & \eta & 0 & \omega \\ Z_5 & -k_1-\lambda & 0 & 0 \\ 0 & 0 & -k_3-\lambda & 0 \\ 0 & \rho & \pi & -K_4-\lambda \end{bmatrix}$$

$$+k_3\alpha K_4 Z_2 Z_5\epsilon + \lambda(k_3\alpha Z_2 Z_5\epsilon + \alpha K_4 Z_2 Z_5\epsilon) - Z_2 Z_5\epsilon\alpha\lambda^2 \tag{5.27}$$

+

$$+Z_2Z_7\begin{bmatrix} -Z_3-\lambda & \eta & 0 & \omega \\ Z_5 & -k_1-\lambda & 0 & 0 \\ 0 & 0 & -k_3-\lambda & 0 \\ 0 & \rho & \pi & -K_4-\lambda \end{bmatrix}$$

$$+(k_1Z_2Z_7Z_3 + \lambda(Z_2Z_7Z_3 + k_1Z_2Z_7) + Z_2Z_7\lambda^2)(k_3k_4 + \lambda(k_3+k_4) + \lambda^2)$$

$$\begin{aligned} &k_1Z_2Z_7Z_3k_3k_4 + \lambda(k_3k_4(Z_2Z_7Z_3 + k_1Z_2Z_7) + (k_3+k_4)k_1Z_2Z_7Z_3) \\ &+\lambda^2(k_1Z_2Z_7Z_3 + Z_2Z_7k_3k_4 + (k_3+k_4)(Z_2Z_7Z_3 + k_1Z_2Z_7)) \\ &+\lambda^3((Z_2Z_7Z_3 + k_1Z_2Z_7) + Z_2Z_7(k_3+k_4)) + Z_2Z_7\lambda^4 \end{aligned} \tag{5.28}$$

$$-k_3k_4\eta Z_2Z_5Z_7 + k_3Z_2Z_5Z_7\rho\omega + \lambda(-(k_3+k_4)\eta Z_2Z_5Z_7 + \omega Z_2Z_5Z_7\rho) - \lambda^2\eta Z_2Z_5Z_7 \tag{5.29}$$

By adding all equation (5.24- 5.29) we obtain

$$\lambda^6 + \lambda^5 a_1 + \lambda^4 a_2 + \lambda^3 a_3 + \lambda^2 a_4 + \lambda a_5 + a_6 = 0.$$

Where

$a_1 = k_3 - Z_8 + V,$

$a_2 = ((k_3 - Z_8)V + H - (k_3Z_8 + \delta\theta) - Z_6\alpha + Z_5\eta + Z_2Z_7),$

$a_3 = ((k_3 - Z_8)H + X - (k_3Z_8 + \delta\theta)V - Z_6\alpha(k_3 + Z_1 + Z_3 + K_4)) + Z_5\omega\rho + Z_5\eta(k_3 - Z_8),$
$\quad + \eta(Z_1Z_5 + Z_5k_4) - Z_5\alpha + (Z_2Z_7Z_3 + k_1Z_2Z_7) + Z_2Z_7(k_3 + k_4),$

$a_4 = ((k_3 - Z_8)X + Q - (k_3Z_8 + \delta\theta)H - \alpha Z_6(k_3(Z_1 + Z_3 + K_4) + K_4(Z_1 + Z_3) + Z_1Z_3) + \phi Z_5\omega\alpha +$
$\quad Z_5\omega\rho(k_3 - Z_8) + Z_1Z_5\omega\rho + \eta(Z_1Z_5 + Z_5k_4)(k_3 - Z_8) + Z_1Z_5k_4\eta - Z_5\eta(Z_8k_3 + \delta\theta) - Z_5\alpha k_3Z_4 -$
$\quad Z_4\alpha(Z_1Z_5 + Z_5k_4) - Z_2Z_5\epsilon\alpha + k_1Z_2Z_7Z_3 + Z_2Z_7k_3k_4 + (k_3 + k_4)(Z_2Z_7Z_3 + k_1Z_2Z_7) - \eta Z_2Z_5Z_7),$

$a_5 = Q(k_3 - Z_8) - (k_3Z_8 + \delta\theta)X - (Z_6k_3\alpha(K_4(Z_1 + Z_3) + Z_1Z_3) + Z_1Z_3Z_6k_4\alpha + Z_5\omega\alpha\delta\pi + Z_5\omega\alpha k_3\phi$
$\quad + \phi Z_1Z_5\omega\alpha + Z_5\omega\rho(k_3Z_8 + \theta\delta) + (k_3 - Z_8)Z_1Z_5\omega\rho + Z_1Z_5k_4\eta(k_3 - Z_8) - \eta(Z_1Z_5 + Z_5k_4)(Z_8k_3 + \delta\theta) -$
$\quad k_3Z_4\alpha(Z_1Z_5 + Z_5k_4) - Z_1Z_4Z_5\alpha k_4 + k_3\alpha Z_2Z_5\epsilon + \alpha K_4Z_2Z_5\epsilon + k_3k_4(Z_2Z_7Z_3 + k_1Z_2Z_7) +$
$\quad (k_3 + k_4)k_1Z_2Z_7Z_3 - (k_3 + k_4)\eta Z_2Z_5Z_7 + \omega Z_2Z_5Z_7\rho,$

$a_6 = -(k_3Z_8 + \delta\theta)Q - Z_6k_3\alpha K_4Z_1Z_3 + Z_1Z_5\omega\alpha k_3\phi + Z_1Z_5\omega\alpha\delta\pi - Z_1Z_5\omega\rho(k_3Z_8 + \theta\delta) - Z_1Z_5k_4\eta(Z_8k_4$
$\quad + \delta\theta) - \alpha Z_1Z_4Z_5k_3k_4 + k_3\alpha K_4Z_2Z_5\epsilon + k_1Z_2Z_7Z_3k_3k_4 - k_3k_4\eta Z_2Z_5Z_7 + k_3Z_2Z_5Z_7\rho\omega.$

The eigenvalue signs are determined by the Routh Hurwiztz array. Using the characteristic polynomial representation the following table to see the sign of eigenvalues.

Table 3: shows the Routh array of the 6th order

λ^6	1	a_2	a_4	a_6
λ^5	a_1	a_3	a_5	
λ^4	$b_1 = \frac{a_1a_2-a_3}{a_1}$	$b_2 = \frac{a_1a_4-a_5}{a_1}$	$b_3 = \frac{a_1a_6-a_7}{a_1}$	
λ^3	$c_1 = \frac{b_1a_3-b_2a_1}{b_1}$	$c_2 = \frac{b_1a_5-b_3a_1}{b_1}$		
λ^2	$d_1 = \frac{c_1b_2-c_2b_1}{c_1}$	$d_2 = \frac{c_1b_3-c_3b_1}{c_1}$		
λ^1	$e_1 = \frac{d_1c_2-d_2c_1}{d_1}$			
λ^0	a_6			

To accept whether the equilibrium point is stable or unstable, it should be satisfy one of the two cases:

Case 1 : If all the first column elements of the Routh-Hurwitz array are the same algebraic sign and the coefficients of the characteristic polynomial should be positive, then the equilibrium point is stable which means that the unhealthy attitude on marriage stay in the society and thus the UAPE exist.

Case 2 : If all the first column elements of the Routh-Hurwitz array are not the same algebraic sign and the coefficients of the characteristic polynomial should be negative, then the UAPE is unstable which means that the unhealthy attitude on a marriage diminish in the society and thus the UAPE is does not exist.

5.6.2 Global Stability of UAPE

Theorem 5.5 If $R_0 > 1$, then the UAPE E^* of the system (4.2 - 4.7) is globally asymptotically stable in Ω.

proof:

To prove the global asymptotic stability of E^* we use the method of Lyapunov functions. To do this, we define the Lyapunov function as follow

$$V = (S - S^* - S^* lnS) + (M - M^* - M^* lnM) + (G - G^* - G^* lnG) + (U - U^* - U^* lnU) + (D - D^* - D^* lnD) + (H - H^* - H^* lnH). \tag{5.30}$$

Differentiate equation (5.30) we obtain that;

$$\frac{dV}{dt} = \left(1 - \frac{S^*}{S}\right)\frac{dS}{dt} + \left(1 - \frac{M^*}{M}\right)\frac{dM}{dt} + \left(1 - \frac{G^*}{G}\right)\frac{dG}{dt} + \left(1 - \frac{U^*}{U}\right)\frac{dU}{dt} + \left(1 - \frac{D^*}{D}\right)\frac{dD}{dt} + \left(1 - \frac{H^*}{H}\right)\frac{dH}{dt}.$$

Now substituting equations of model (4.2 - 4.7), we get

$$\begin{aligned}\frac{dV}{dt} &= \left(1-\frac{S^*}{S}\right)(\Lambda-\beta_2 US-(\epsilon+\mu)S)\\ &+\left(1-\frac{M^*}{M}\right)(\epsilon S+\eta G+\omega H-\beta_1 MU-\mu M)\\ &+\left(1-\frac{G^*}{G}\right)(\beta_1 MU-(\rho+\alpha+\eta+\mu)G)\\ &+\left(1-\frac{U^*}{U}\right)(\beta_2 SU+\alpha G+\theta D-(\delta+\phi+\mu)U)\\ &+\left(1-\frac{D^*}{D}\right)(\delta U-(\theta+\pi+\mu)D)\\ &+\left(1-\frac{H^*}{H}\right)(\phi U+\pi D+\rho G-(\omega+\mu)H).\end{aligned}$$

Then by simplifying equation, we get;

$$\begin{aligned}\frac{dV}{dt} = \Lambda-\mu S-\frac{\Lambda S^*}{S}+\beta_2 US^*+(\epsilon+\mu)S^*-\mu M-\frac{\epsilon SM^*}{M}-\frac{\eta GM^*}{M}-\frac{\omega HM^*}{M}+M^*\beta_1 U+\mu M^*-\\ \mu G-\frac{\beta_1 MUG^*}{G}+(\rho+\alpha+\eta+\mu)G^*-\mu U-\beta_2 SU^*-\frac{\alpha GU^*}{U}-\frac{\theta DU^*}{U}+(\delta+\phi+\mu)U^*-\mu D\\ -\frac{\delta UD^*}{D}+(\theta+\pi+\mu)D^*-\mu H-\frac{\phi UH^*}{H}-\frac{\pi DH^*}{H}-\frac{\rho GH^*}{H}+(\omega+\mu)H^*.\end{aligned} \tag{5.31}$$

Let us take P the positive terms and Q as the negative terms of equation (5.31), Such that:

$$\begin{aligned}P = \Lambda+\beta_2 US^*+(\epsilon+\mu)S^*+M^*\beta_1 U+\mu M^*+(\rho+\alpha+\eta+\mu)G^*+(\delta+\phi+\mu)U^*\\ +(\theta+\pi+\mu)D^*+(\omega+\mu)H^*,\end{aligned}$$

and

$$\begin{aligned}Q = \frac{\Lambda S^*}{S}+\frac{\epsilon SM^*}{M}+\frac{\eta GM^*}{M}+\frac{\omega HM^*}{M}+\frac{\beta_1 MUG^*}{G}+\beta_2 SU^*+\frac{\alpha GU^*}{U}+\frac{\theta DU^*}{U}\\ +\frac{\delta UD^*}{D}+\frac{\phi UH^*}{H}+\frac{\pi DH^*}{H}+\frac{\rho GH^*}{H}+\mu N.\end{aligned}$$

Then equation (5.31) becomes;

$$\frac{dV}{dt} = P-Q.$$

If P < Q, then $\frac{dV}{dt} < 0$ and also $\frac{dV}{dt} = 0$ if and only if $S = S^*$, $M = M^*$, $G = G^*$, $U = U^*$, $D = D^*$, $H = H^*$.
The largest compact invariant set in $E\left\{(S^*, M^*, G^*, U^*, D^*, H^*) \in \Omega : \frac{dV}{dt} = 0\right\}$ is the singleton of E^*.
It implies that E^* globally asymptotically stable in Ω if P < Q by LaSalle's invariant principle [3.4].

5.7 Bifurcation Analysis

Theorem 5.6 The model in system (4.2 - 4.7) exhibits forward bifurcation at $R_0 = 1$.

Proof:
We proved the theorem using the concept center manifold theorem (3.6) the possibility of bifurcation at $R_0 = 1$. Then the following change of variables was made S = x_1, M = x_2, G = x_3,U = x_4, D = x_5 and H = x_6 In addition, using vector notation x = $(x_1, x_2, x_3, x_4, x_5, x_6)^T$, then model in system (4.2 - 4.7) can be written in the form $\frac{dx}{dt} = F(x)$, with F = $(f_1, f_2, f_3, f_4, f_5, f_6)^T$, rewritten in the form:

$$f_1 = \frac{dx_1}{dt} = \Lambda - \beta_2 x_1 x_4 - (\epsilon + \mu)x_1, \tag{5.32a}$$

$$f_2 = \frac{dx_2}{dt} = \epsilon x_1 + \eta x_3 + \omega x_6 - \beta_1 x_2 x_4 - \mu x_2, \tag{5.32b}$$

$$f_3 = \frac{dx_3}{dt} = \beta_1 x_2 x_4 - (\rho + \alpha + \eta + \mu)x_3, \tag{5.32c}$$

$$f_4 = \frac{dx_4}{dt} = \beta_2 x_1 x_4 + \alpha x_3 + \theta x_5 - (\delta + \phi + \mu)x_4, \tag{5.32d}$$

$$f_5 = \frac{dx_5}{dt} = \delta x_4 - (\theta + \pi + \mu)x_5, \tag{5.32e}$$

$$f_6 = \frac{dx_6}{dt} = \phi x_4 + \pi x_5 + \rho x_3 - (\omega + \mu)x_6. \tag{5.32f}$$

We consider the contact and transmission rate β_1 as a bifurcation parameters so that $R_0 = 1$ iff

$$R_0 = \left(\frac{(\theta + \pi + \mu)\Lambda\alpha\epsilon\beta_1 + (\theta + \pi + \mu)\Lambda\mu(\rho + \alpha + \eta + \mu)\beta_2}{\mu(\rho + \alpha + \eta + \mu)(\delta(\pi + \mu) + (\phi + \mu)(\theta + \pi + \mu))(\epsilon + \mu)} \right),$$

$$\beta_1 = \beta_1^* = \frac{(\delta(\pi + \mu) + (\phi + \mu)k_3)k_1\mu(\epsilon + \mu) - k_3\Lambda\mu k_1\beta_2}{k_3\Lambda\alpha\epsilon}.$$

By calculating the eigenvalues of the Jacobian matrix at Unhealthy attitude-free equilibrium point by substituting $\beta_1 = \beta_1^*$. The Jacobian matrix of the system (5.32) evaluated at unhealthy attitude free equilibrium U_0 with $\beta_1 = \beta_1^*$ is given by

$$J^* = \begin{bmatrix} -(\epsilon + \mu) & 0 & 0 & -\beta_2 S^0 & 0 & 0 \\ \epsilon & -\mu & \eta & -\beta_1^* M^0 & 0 & \omega \\ 0 & 0 & -k_1 & \beta_1^* M^0 & 0 & 0 \\ 0 & 0 & \alpha & \beta_2 S^0 - k_2 & \theta & 0 \\ 0 & 0 & 0 & \delta & -k_3 & 0 \\ 0 & 0 & \rho & \phi & \pi & -(\omega + \mu) \end{bmatrix}.$$

Where $k_1 = (\rho + \alpha + \eta + \mu)$, $k_2 = (\delta + \phi + \mu)$ and $k_3 = (\theta + \pi + \mu)$.
The Jacobian matrix J^* of the linearized system has a simple zero eigenvalue with all other eigenvalues having negative real part, hence the center manifold theory will be used to analyse the dynamics of the system near $\beta_1 = \beta_1^*$. Thus, U_0 is a non-hyperbolic equilibrium,

when $\beta_1 = \beta_1^*$. Now, we calculate a right eigenvector $w = (w_1, w_2, w_3, w_4, w_5, w_6)^T$ of J^* associated with the zero eigenvalue.

$$\begin{bmatrix} -(\epsilon+\mu) & 0 & 0 & -\beta_2 S^0 & 0 & 0 \\ \epsilon & -\mu & \eta & -\beta_1^* M^0 & 0 & \omega \\ 0 & 0 & -k_1 & \beta_1^* M^0 & 0 & 0 \\ 0 & 0 & \alpha & \beta_2 S^0 - k_2 & \theta & 0 \\ 0 & 0 & 0 & \delta & -k_3 & 0 \\ 0 & 0 & \rho & \phi & \pi & -(\omega+\mu) \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

This implies that,

$$-(\epsilon+\mu) w_1 - \beta_2 S^0 w_4 = 0, \tag{5.33a}$$

$$\epsilon w_1 - \mu w_2 + \eta w_3 - \beta_1^* M^0 w_4 + \omega w_6 = 0, \tag{5.33b}$$

$$-k_1 w_3 + \beta_1^* M^0 w_4 = 0, \tag{5.33c}$$

$$\alpha w_3 + (\beta_2 S^0 - k_2) w_4 + \theta w_5 = 0, \tag{5.33d}$$

$$\delta w_4 - k_3 w_5 = 0, \tag{5.33e}$$

$$\rho w_3 + \phi w_4 + \pi w_5 - (\omega+\mu) w_6 = 0. \tag{5.33f}$$

Solving system of equation (5.33a), we obtain

$$\begin{aligned} w_1 &= \frac{-\beta_2 \Lambda}{(\epsilon+\mu)^2} w_4, \\ w_2 &= \frac{-A+B}{\alpha(\epsilon+\mu)^2 k_3(\omega+\mu)} w_4, \\ w_3 &= \frac{(\delta(\pi+\mu) + (\phi+\mu)k_3)(\epsilon+\mu) - B_2 \Lambda k_3}{k_3 \alpha(\epsilon+\mu)} w_4, \\ w_4 &= w_4 > 0, \\ w_5 &= \frac{\delta}{k_3} w_4, \\ w_6 &= \frac{\rho((\delta(\pi+\mu) + (\phi+\mu)k_3)(\epsilon+\mu) - B_2 \Lambda k_3) + \alpha\phi k_3(\epsilon+\mu) + \alpha\pi\delta(\epsilon+\mu)}{\alpha k_3(\omega+\mu)(\epsilon+\mu)} w_4, \end{aligned}$$

where

$A = \alpha\epsilon\beta_2\Lambda k_3(\omega+\mu) + (\delta(\pi+\mu) + (\phi+\mu)k_3)(\epsilon+\mu)^2(\omega(\alpha+\mu) + \mu(\rho+\alpha+\eta)) + \beta_2\Lambda\omega\rho k_3(\epsilon+\mu)$,
$B = \omega\alpha\phi k_3(\epsilon+\mu)^2 + \alpha\pi\delta\omega(\epsilon+\mu)^2 + \beta_2\Lambda k_3(\omega+\mu)(\epsilon+\mu)(\rho+\alpha+\mu)$.
The left eigenvectors of J^* associated with the zero eigenvalue is given by $v = (v_1, v_2, v_3, v_4, v_5, v_6)^T$, is calculated as:

$$\begin{bmatrix} -(\epsilon+\mu) & \epsilon & 0 & 0 & 0 & 0 \\ 0 & -\mu & 0 & 0 & 0 & 0 \\ 0 & \eta & -k_1 & \alpha & 0 & \rho \\ -\beta_2 S^0 & -\beta_1^* M^0 & \beta_1^* M^0 & \beta_2 S^0 - k_2 & \delta & \phi \\ 0 & 0 & 0 & \theta & -k_3 & \pi \\ 0 & \omega & 0 & 0 & 0 & -(\omega+\mu) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{5.34}$$

Solving system of equation (5.34), we obtain

$$v_1 = v_2 = v_6 = 0, \quad v_4 = v_4 > 0, \quad v_3 = \frac{\alpha}{k_1} v_4, \quad v_5 = \frac{\theta}{k_3} v_4.$$

Since the first, second and six component of v are zero we don't need the partial derivatives of f_1, f_2 and f_6. From the partial derivatives of f_3 , f_4 and f_5 at the Unhealthy attitude free equilibrium point, the only ones that are nonzero are the following:

$$\frac{\partial^2 f_3}{\partial x_2 \partial x_4} = \frac{\partial^2 f_3}{\partial x_4 \partial x_2} = \beta_1^* = \frac{k_1(k_2k_3 - \delta\theta)\mu(\epsilon+\mu) - k_3\Lambda\mu k_1\beta_2}{k_3\Lambda\alpha\epsilon},$$

$$\frac{\partial^2 f_4}{\partial x_1 \partial x_4} = \frac{\partial^2 f_4}{\partial x_4 \partial x_1} = \beta_2,$$

$$\frac{\partial^2 f_4}{\partial x_1 \partial \beta_1} = M^0,$$

and all the other partial derivatives are zero. The signs of the bifurcation coefficients a and b, obtained from the above partial derivatives, given respectively by

$$a = \sum_{k,i,j=1}^{6} v_k w_i w_j \frac{\partial^2 f_k}{\partial x_i \partial x_j}(S^0, M^0, 0, 0, 0, 0),$$

$$a = 2v_3w_2w_4\frac{\partial^2 f_3}{\partial x_2 \partial x_4} + 2v_4w_2w_4\frac{\partial^2 f_4}{\partial x_1 \partial x_4} = 2v_3w_2w_4\beta_1^* + 2v_4w_1w_4\beta_2$$
$$= 2w_4(v_3w_2\beta_1^* + 2v_4w_1\beta_2),$$

$$= 2w_4^2v_4\left(\frac{-A+B}{\alpha(\epsilon+\mu)^2k_3(\omega+\mu)}\right)\left(\frac{k_1(k_2k_3-\delta\theta)\mu(\epsilon+\mu) - k_3\Lambda\mu k_1\beta_2}{k_3\Lambda\alpha\epsilon}\right) + \left(\frac{-\Lambda}{(\epsilon+\mu)^2}\right),$$

$$= 2w_4^2v_4\left(\frac{-(k_1(\epsilon+\mu)WA + \Lambda k_3k_1\mu\beta_2B + \Lambda^2\epsilon\alpha^2k_3^2(\omega+\mu)) + \Lambda\mu k_1k_3\beta_2A + k_1(\epsilon+\mu)WB}{\Lambda\epsilon\alpha^2(\epsilon+\mu)^2k_3^2(\omega+\mu)}\right),$$

where $W = \mu(k_2k_3 - \delta\theta)$.

Suppose $X = k_1(\epsilon+\mu)WA + \Lambda k_3k_1\mu\beta_2B + \Lambda^2\epsilon\alpha^2k_3^2(\omega+\mu)$ and $Y = \Lambda\mu k_1k_3\beta_2A + k_1(\epsilon+\mu)WB$.

Since all parameter are positive, we conclude that a < 0 if X > Y and a > 0 if X < Y.

And

$$b = v_k\sum_{k,i=1}^{6} w_i\frac{\partial^2 f_k}{\partial x_i \partial \beta_1}(S^0, M^0, 0, 0, 0, 0),$$
$$= v_3w_4\frac{\partial^2 f_3}{\partial x_4 \partial \beta_1} = w_4v_4\frac{\alpha\epsilon\Lambda}{k_1\mu(\epsilon+\mu)} > 0.$$

Since a < 0 and b > 0 at $\beta_1 = \beta_1^*$ B based on Theorem [3.6], the system (4.2 - 4.7) undergoes a forward bifurcation at $R_0 = 1$ and the unique unhealthy attitude present equilibrium E^* is locally asymptotically stable for $R_0 > 1$.

5.8 Sensitivity Analysis

It is critical to comprehend how modifying certain parameters impacts our system's stability. Sensitivity analysis notifies us how significant each parameter to the transmission of unhealthy attitude on marriage.

Definition 5.1 The normalized forward sensitivity index of a variable to a parameter is a ratio of the relative change in the variable to the relative change in the parameter. When a variable is a differentiable function of the parameter, the sensitivity index may be alternatively defined using partial derivatives:

$$\Lambda_P^{R_0} = \frac{\partial R_0}{\partial P} x \frac{p}{R_0}.$$

The threshold parameter $R_0 = 0$ which determines stability is a function of the parameters Λ ,β_1, β_2, ϵ, η, α, δ, ϕ, ρ, π, ω, θ and μ. The sensitivity of these parameters show us how they are important to the spread of unhealthy attitude on marriage. In order to study the effect of this parameters on R_0 we performed a sensitivity analysis on R_0 with respect to these parameters. We recall that the basic reproduction number R_0 is given by:

$$R_0 = \frac{(\theta + \pi + \mu)\Lambda\alpha\epsilon\beta_1 + (\theta + \pi + \mu)\Lambda\beta_2\mu(\rho + \alpha + \eta + \mu)}{\mu(\epsilon + \mu)(\rho + \alpha + \eta + \mu)(\delta(\pi + \mu) + (\phi + \mu)(\theta + \pi + \mu))}. \tag{5.35}$$

Let's check this by computing the partial derivative of reproduction number with respect to each parameter.

$$S_\Lambda^{R_0} = \frac{\partial R_0}{\partial \Lambda} x \frac{\Lambda}{R_0} = 1 > 0,$$

$$S_{\beta_1}^{R_0} = \frac{\partial R_0}{\partial \beta_1} x \frac{\beta_1}{R_0} = \frac{(\theta + \pi + \mu)\Lambda\alpha\epsilon\beta_1}{(\theta + \pi + \mu)\Lambda\alpha\epsilon\beta_1 + (\theta + \pi + \mu)\Lambda\beta_2\mu(\rho + \alpha + \eta + \mu)} > 0,$$

$$S_{\beta_2}^{R_0} = \frac{\partial R_0}{\partial \beta_1} x \frac{\beta_1}{R_0} = \frac{(\theta + \pi + \mu)\Lambda\mu(\rho + \alpha + \eta + \mu)\beta_2}{(\theta + \pi + \mu)\Lambda\alpha\epsilon\beta_1 + (\theta + \pi + \mu)\Lambda\beta_2\mu(\rho + \alpha + \eta + \mu)} > 0,$$

$$S_\theta^{R_0} = \frac{\partial R_0}{\partial \theta} x \frac{\theta}{R_0} = \frac{\theta\delta(\pi + \mu)}{(\delta(\pi + \mu) + (\phi + \mu)(\theta + \pi + \mu))(\theta + \pi + \mu)} > 0,$$

$$S_\pi^{R_0} = \frac{\partial R_0}{\partial \pi} x \frac{\pi}{R_0} = \frac{-\pi\theta\delta}{(\delta(\pi + \mu) + (\phi + \mu)(\theta + \pi + \mu))(\theta + \pi + \mu)} < 0,$$

$$S_\alpha^{R_0} = \frac{\partial R_0}{\partial \alpha} x \frac{\alpha}{R_0} = \frac{\alpha((\epsilon\beta_1 + \beta_2\mu)k_1 - N)}{Nk_1} > 0,$$

$$S_\rho^{R_0} = \frac{\partial R_0}{\partial \rho} x \frac{\rho}{R_0} = \frac{-\rho\alpha\epsilon\beta_1}{(\alpha\epsilon\beta_1 + \beta_2\mu(\rho + \alpha + \eta + \mu))(\rho + \alpha + \eta + \mu)} < 0,$$

$$S_{\eta}^{R_0} = \frac{\partial R_0}{\partial \eta} x \frac{\eta}{R_0} = \frac{-\eta\alpha\epsilon\beta_1}{(\alpha\epsilon\beta_1 + \beta_2\mu(\rho+\alpha+\eta+\mu))(\rho+\alpha+\eta+\mu)} < 0,$$

$$S_{\epsilon}^{R_0} = \frac{\partial R_0}{\partial \epsilon} x \frac{\epsilon}{R_0} = \frac{\epsilon(\alpha\beta_1\mu - \beta_2\mu(\rho+\alpha+\eta+\mu))}{(\alpha\epsilon\beta_1 + \beta_2\mu(\rho+\alpha+\eta+\mu))(\epsilon+\mu)},$$

$$S_{\phi}^{R_0} = \frac{\partial R_0}{\partial \phi} x \frac{\phi}{R_0} = \frac{-\phi(\theta+\pi+\mu)}{\delta(\pi+\mu)+(\phi+\mu)(\theta+\pi+\mu)} < 0,$$

$$S_{\phi}^{R_0} = \frac{\partial R_0}{\partial \phi} x \frac{\phi}{R_0} = \frac{-\delta(\pi+\mu)}{\delta(\pi+\mu)+(\phi+\mu)(\theta+\pi+\mu)} < 0,$$

$$S_{\mu}^{R_0} = \frac{\partial R_0}{\partial \mu} x \frac{\mu}{R_0} = \frac{N\Lambda\mu k_1\delta(\epsilon+\mu)(\pi+\mu) - k_3(\lambda\alpha\epsilon\beta_1(k_1+\mu)L + N\Lambda\mu k_1 M + \Lambda\alpha\epsilon\beta_1\mu(\epsilon+\mu)k_1\delta)}{K_1 k_3 L\lambda N},$$

where

$k_1 = (\rho+\alpha+\eta+\mu)$,

$k_3 = (\theta+\pi+\mu)$,

$N = \alpha\epsilon\beta_1 + \beta_2\mu k_1$,

$L = (\epsilon+\mu)\delta(\pi+\mu) + k_3(\phi+\mu)(\epsilon+\mu)$,

$M = \delta(\pi+\mu) + k_3(\epsilon+\mu) + k_3(\phi+\mu)$.

The sensitivity indices of the basic reproductive number with respect to the embedded parameters are arranged in the table (4). Those parameters which have positive indices on R_0 are β_1, α, β_2 and θ, they have great impact on expanding unhealthy attitude on marriage. Whenever we increase or decrease the value of these parameters the reproduction number increases or decreases respectively. That means if we decrease the value of β_1 from 0.0006 to 0.00001 the reproduction number decrease from 21.0450 to 5.8391. On the other hand if we increase this parameter from 0.00001 to 0.0006 the reproduction number increase from 5.8391 to 21.0450. That means the change in increasing β_1 by 98.33% and will increase the reproduction number by 72.25%. Additionally increasing the rate of α from 0.002 to 0.2 the reproduction number increase from 5.8998 to 21.0450 which show us that α increase by 99% and the reproduction number will increase by 71.97%.

Furthermore, those parameters in which their sensitivity indices are negative, η, δ, ϕ, ρ and π have become more valuable, they may have an impact on reducing the individual with unhealthy attitude burden on the society. And also as their values increase, the basic reproduction number decreases, which leads to minimizing the endemic nature of unhealthy attitude on marriage in the community. Parameter values are taken from Table (6).

Table 4: Sensitivity indices table for deterministic model

Parameters	Sensitivity indices
Λ	+ 1
β_1	+ 0.7348
α	+ 0.3531
θ	+ 0.2678
β_2	+ 0.2652
ϵ	+ 0.0075
μ	- 1.1402
ϕ	- 0.4775
δ	- 0.4713
ρ	- 0.2863
π	- 0.2149
η	- 0.0382

Whenever we increase or decrease the value of these parameters the reproduction number decrease or increase respectively. That means if we increase the value of ϕ from 0.14 to 0.74 the reproduction number decrease from 21.0450 to 6.9079. That means the change in increasing ϕ by 81.1% and will decrease the reproduction number by 67.18%. Additionally increasing the rate of δ from 0.32 to 0.678 the reproduction number decrease from 21.0450 to 13.7793 which show us that δ increase by 52.8% and the reproduction number will decrease by 34.5%

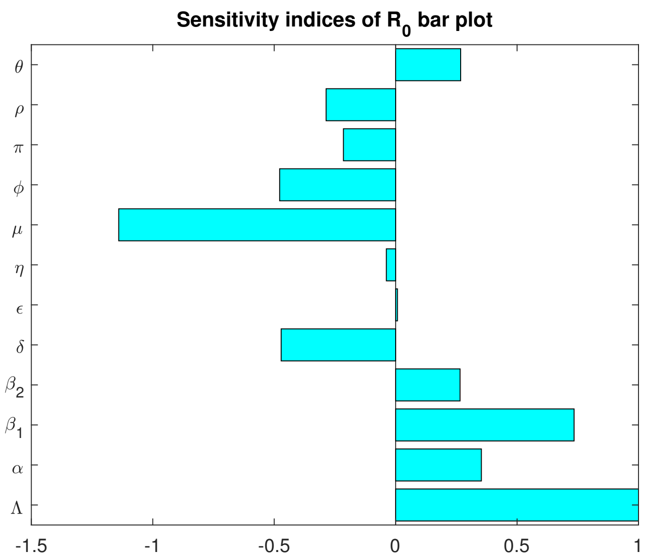
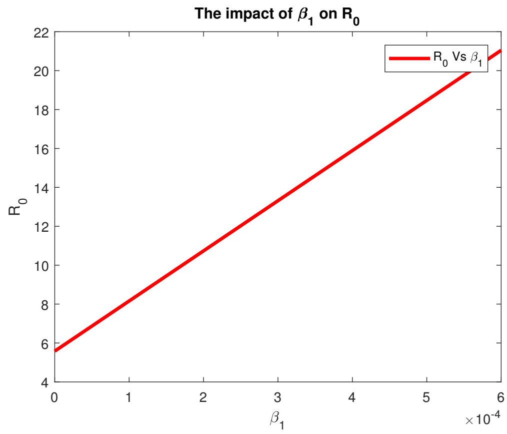


Figure 5.8.1: The sensitivity indices of R_0 with respect to the parameters.



(a)

Figure 5.8.2: Sensitivity analysis of reproduction number with respect to the contact rate β_1.

Figure5.8.2, shows that when the interaction rate between married individuals and individuals with unhealthy attitude (β_1) increase and reproduction number(R_0) also increases. Similarly, β_1 decreases and reproduction number(R_0) also decreases.

Chapter 6

6 Numerical Simulation

6.1 Numerical Simulation of the System

In this section, we use the numerical simulation to show the dynamic behavior of our model. The numerical simulations of the model system (4.2-4.7) were carried out to graphically illustrate with help of the ODE45 MATLAB tool. The sources of these parameters are mainly from parameter estimation using real data, as well as assumptions and literature.

6.2 Parameter Estimations

In this section, we fit the proposed model using real data of divorce case from Hawassa city, Ethiopia, and estimate the unknown model parameters. The simulation process are performed using MATLAB software. Table [5] depicts the yearly real data of total confirmed cases of divorce in Hawassa from 2017 to 2023 extracted from every four first instance court.

Table 5: Divorce case from 2017 to 2023

Time(year)	Number of Divorce case
2017	379
2018	388
2019	378
2020	329
2021	431
2022	464
2023	469

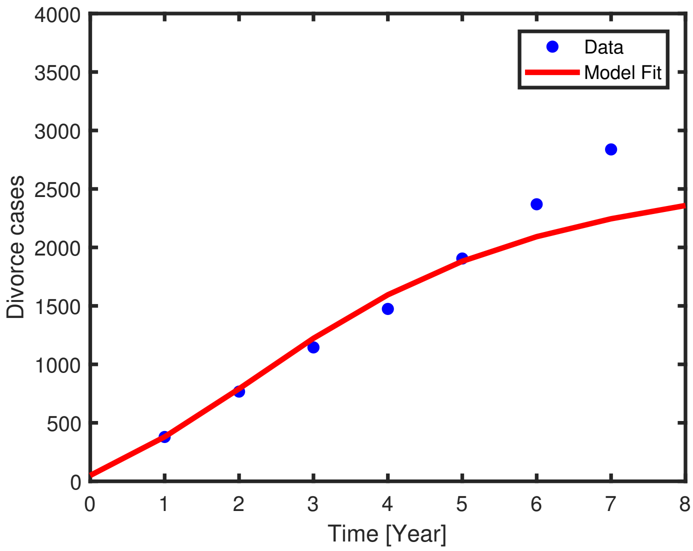
To solve the dynamic parameter estimation problem, we formulate system (4.2 - 4.7) in the form of (3.1) here x is the vector of dependent variables and p is the vector of unknown parameters. The error is sum of squares error and is represented by

$$E(p) = \sum_{i=1}(x_i - \hat{x}_i)^2, \tag{6.1}$$

$\hat{x}_i(t)$ represents the actual total confirmed cases of Divorce and $x_i(t)$ are the corresponding model solutions at time t_i. The aim is minimizing objective function to obtain our parameter estimates.

$$\min E(p) \text{ subject to Equation 6.1.} \tag{6.2}$$

In the process of least-squares fitting, we are looking for a value $\hat{p}$ of model parameter p such that the squared sum of errors is the minimum. Clearly, such a problem is nonlinear least squares problem, since the dependence of a solution $x(t, p)$ on the parameter p is through a highly nonlinear system of differential equations. Using this method we have fitted the data to the model and estimate the parameters and the result is shown in fig (6.2.1).



(a)

Figure 6.2.1: SMGUDH model fit with cumulative of real data on the number of Divorce cases in Hawassa city.

6.3 Simulation Results and Discussion

In this section, we simulated the spread of unhealthy attitude and its impact on the dynamics of divorce through numerical simulations by using the fitted values of parameters in the proposed model with appropriate initial conditions. The numerical simulations results to support our analytical results using the ode45 Matlab tool and with the initial conditions S(0) = 4000, M(0) = 1000, G(0) = 1500, U(0) = 500, D(0) = 379, H(0) = 50 in the model equation (4.2 - 4.7) and the total population of Hawassa city is N(0) = 164,591 populations according to Ethiopian Statistics Agency 2023. The natural death rate is computed as $\mu = \frac{1}{66.71}$ where 66.71 years is the average life expectancy in Ethiopia [16]. We approximated the recruitment rate $\Lambda = 300$.

According to our modified model, and the parameters values, the basic reproduction number is greater than unity, which shows more intervention strategies must be applied to reduce unhealthy attitude on marriage. The level of current healthy counseling and conversation between married couples is not enough to reduce divorce to the desired level. The result of R_0 shows that one individual who have unhealthy attitude on marriage introduce in to married and single individuals is able to influence more than 21 peoples in his duration of staying with his or her unhealthy attitude.

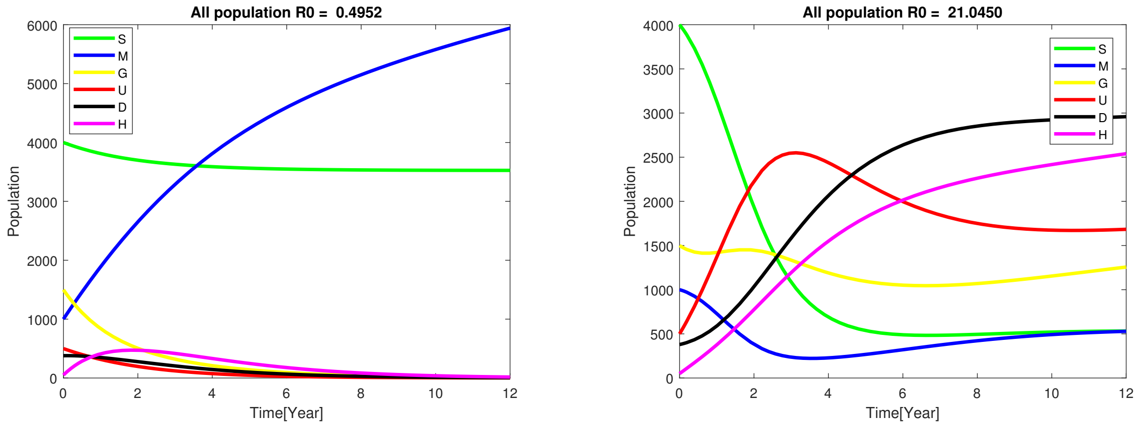
$$R_0 = \frac{(\theta+\pi+\mu)\Lambda\alpha\epsilon\beta_1 + (\theta+\pi+\mu)\Lambda\beta_2\mu(\rho+\alpha+\eta+\mu)}{\mu(\epsilon+\mu)(\rho+\alpha+\eta+\mu)(\delta(\pi+\mu)+(\phi+\mu)(\theta+\pi+\mu))} = 21.0450 > 1. \quad (6.3)$$

Table 6: The set of parameter values

Parameters	Value	Source	Parameters	Value	Source
Λ	300	Assumed	ϕ	0.14	Fitted
ϵ	0.04	[24]	ρ	0.15	[22]
β_1	0.0006	Fitted	π	0.061	[21]
β_2	0.0003	Fitted	ω	0.2	Fitted
η	0.02	Fitted	θ	0.1	Fitted
α	0.2	Fitted	δ	0.32	Fitted
μ	0.015	Assumed			

According to fig6.3.1(a), when the threshold parameter R_0 is less than one, a married individuals with challenge, unhealthy attitude of marriage and divorce case reduced and goes to extinction. From our qualitative analysis of bifurcation, we have found that R_0 is an important parameter, working to reduce it will affect the dynamics the trajectories. There are a number of ways to reduce this reproduction number. One of them is reducing interaction between married individuals and individuals who have unhealthy thinking about marriage which is represented by β_1 in our model. The other possible measurement can be taken is couples who have come to some challenges must experience settling down disagreement by themselves through discussion or ground rules. This indicate that the unhealthy attitude free equilibrium point is locally and globally asymptotically stable for the value when the reproduction number is less than unit and which support the result in theorem 5.1. As a result this provides absence of unhealthy attitude in population or the unhealthy attitude dies out and divorce decrease in the society.

Figure 6.3.1(b) Shows that the dynamics of the model when the threshold parameter R_0 is greater than 1. This describe the current daynamics of marriage and divorce in hawassa city. In this case, the number of single individuals reduces either by marriage or by unhealthy attitude. When individuals from unhealthy attitude compartment affect both single and married people, couples who have been staying in healthy environment start to struggle with different challenges. This may be occur whenever married individuals start to communicate

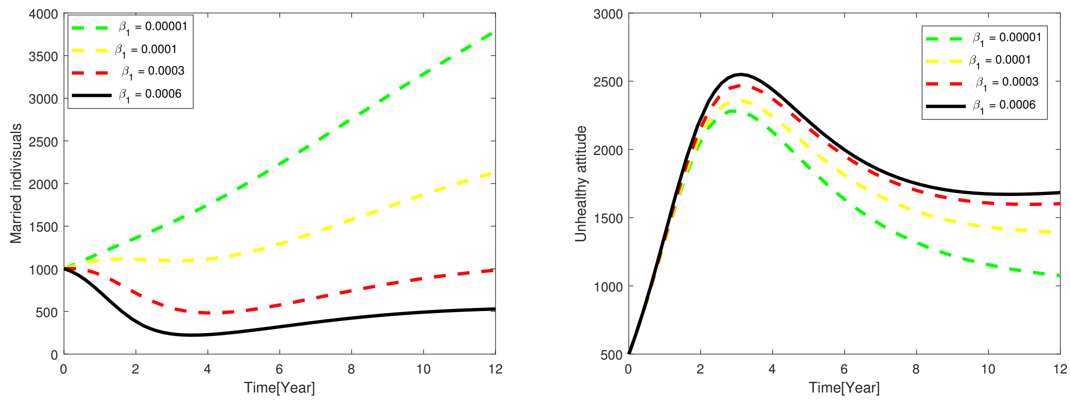


(a) (b)

Figure 6.3.1: Trajectories of state variables for $R_0 < 1$ and $R_0 > 1$

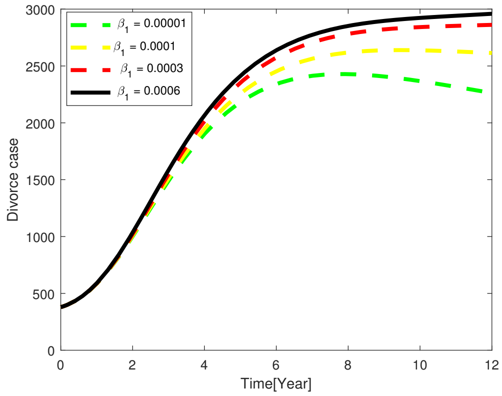
with another individual or group that undermine or underestimate the concept of marriage. This condition can leave majority of married couples with day to day conflict and which leads to unhappy life and join the group of individuals who have unhealthy attitude towards marriage. From the figure we observe that while married individuals decreases significantly, on the other hand divorce case increases as time goes. The simulation also show that even though the number of healthy counseling is greater than the number of unhealthy attitude unless otherwise other methods are taken, the number of couples who stay in challenge and divorce cases cannot be reduced.

The value of the basic reproductive number greater than unity shows all compartments converge to the unhealthy attitude present equilibrium point. These indicate that the UAPE is locally asymptotically stable for the value of $R_0 > 1$ and which support the result in theorem (5.4)



(a) (b)

One of the best intervention strategies that can be taken by the community is reducing the



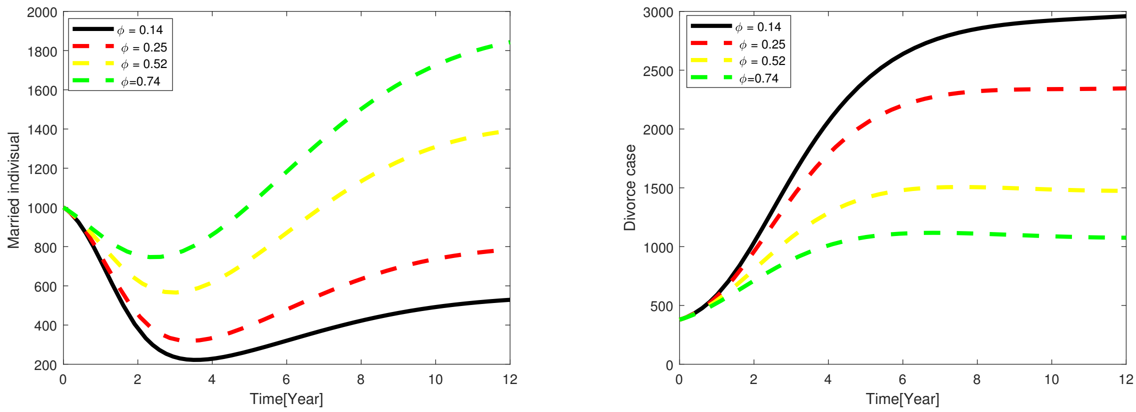
(c)

Figure 6.3.2: The effect of β_1 on marriage, unhealthy attitude and divorce.

contact rate(β_1) between individuals who have unhealthy attitude on marriage and individuals free from unhealthy attitude. Fig 6.3.2(a-c) shows the impact of β_1 on married groups, individuals with unhealthy attitude and divorced compartments.

As individual with unhealthy attitude on marriage interact with married individuals, unhealthy attitude and divorce increase in the society. β_1 is the most sensitive parameter of our model which was confirmed from sensitivity analysis.

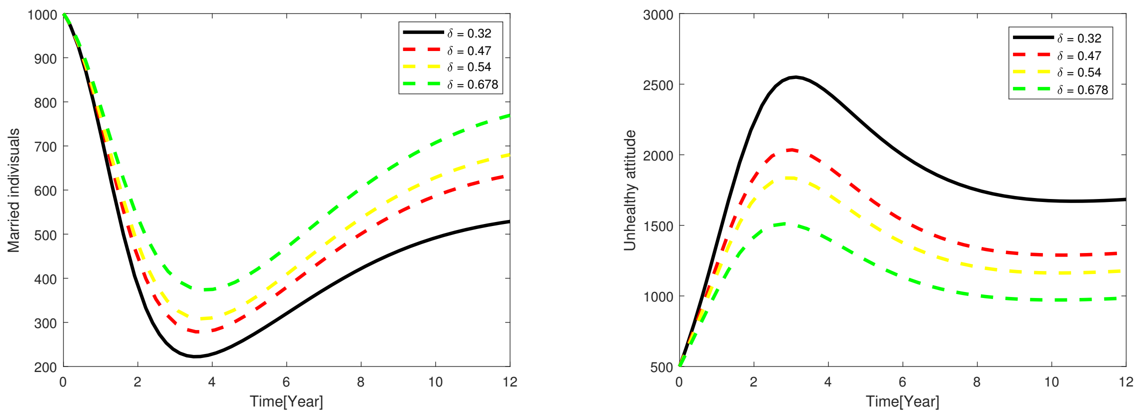
We observe that as the contact rate β_1 increases, unhealthy attitude and divorced individuals increases and the basic reproduction number is greater than one which means unhealthy attitude on marriage persists in the population.



(a) (b)

Figure 6.3.3: Effect of ϕ on marriage and divorce.

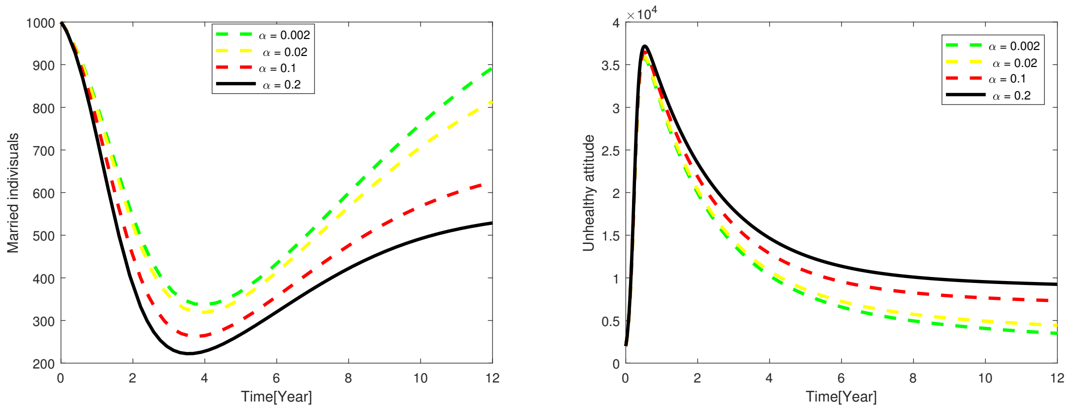
From figure6.3.3, we investigate the rate of healthy counseling (ϕ) on unhealthy attitude on marriage. This support individuals who have unhealthy attitude to change their mindset about marriage. By increasing the level of healthy counseling, this sensitive parameter will help to decrease the amount of people with unhealthy attitudes who negatively influence married couples and single people. As this rate of treatment increases, we can observe that from fig6.3.3(a) married individuals increase and from fig6.3.3(b) the rate of divorce decrease.



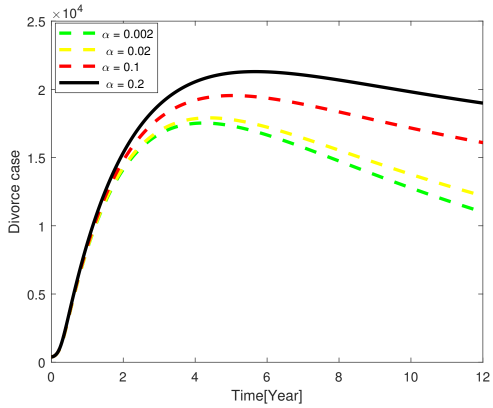
(a) (b)

Figure 6.3.4: Effect of δ on marriage and unhealthy attitude.

In Figure 6.3.4 shows the effect of δ (the rate of individuals with unhealthy attitude get divorce). From the simulation we observed that divorce is preferable mechanize rather than remaining with unhealthy attitude on marriage. So from the graph 6.3.4(a) we observe that when the rate of δ increases the number of married individuals become increase, but from the graph 6.3.4(b) we see that when the rate of δ increases the number of individual with unhealthy attitude on marriage decreases. at the same time the basic reproduction number start to decrease. This will result in decreasing the number of individual with unhealthy attitude on marriage.



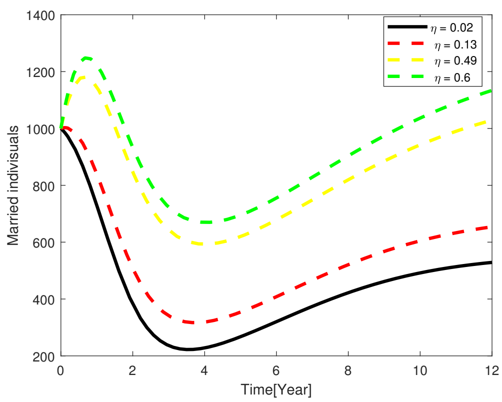
(a) (b)



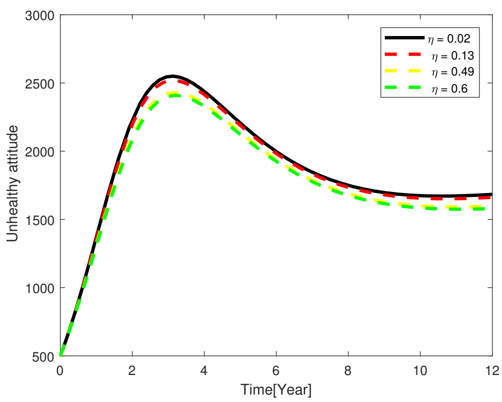
(c)

Figure 6.3.5: Effect of α on marriage, Unhealthy attitude and divorce.

According to figure 6.3.5 show the sensitivity of α which is in our sensitivity analysis. As we decrease alpha there is high probability for married people to stay in their marriage. That means keeping married couples in the compartment of challenge can have a probability of moving them to healthy counseling or solving the challenge by themselves. This also can be done by reducing communication between married couples who are in challenge and individuals who have unhealthy attitude to protect from further challenges. Married couples who are in challenge are more vulnerable to develop unhealthy attitude than any other individuals. Protecting such couples from further pain is very significant to save the marriage and related families, at the same time we observe that from 6.3.5(b & c), decreasing the rate of alpha reduce the number of individuals with unhealthy attitude and divorce individuals.

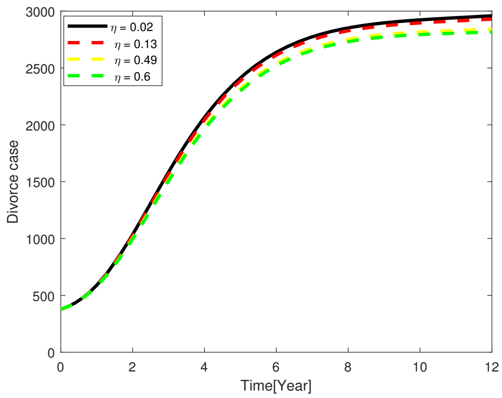


(a)



(b)

In figure 6.3.6 above we can observe that as the rate of η increase (rate of resolving the marital challenge by them selves) married individual increase and unhealthy attitude on marriage and divorced individuals decrease. These graphs also demonstrates that the rate η has high impact



(c)

Figure 6.3.6: Effect of η on marriage, unhealthy attitude and divorce.

to control the spread of unhealthy attitude and divorce in the society. Hence to increase the rate of η the reproduction number decrease.

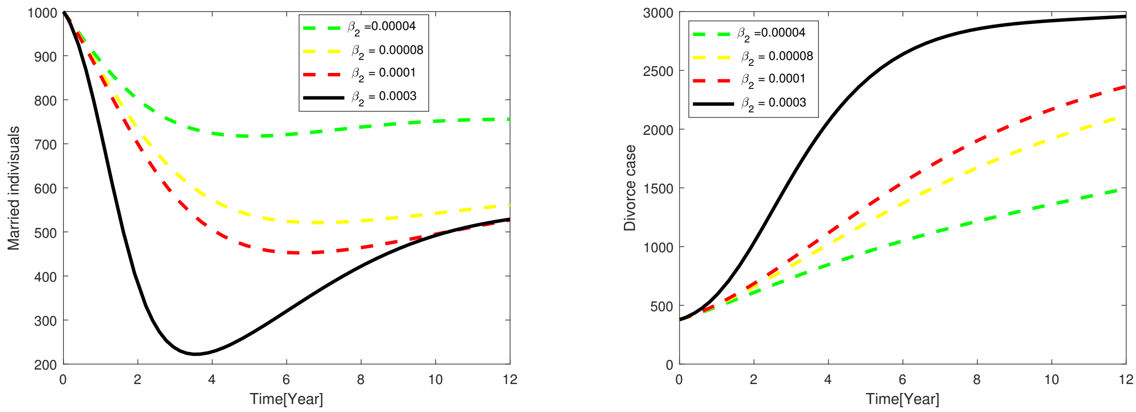
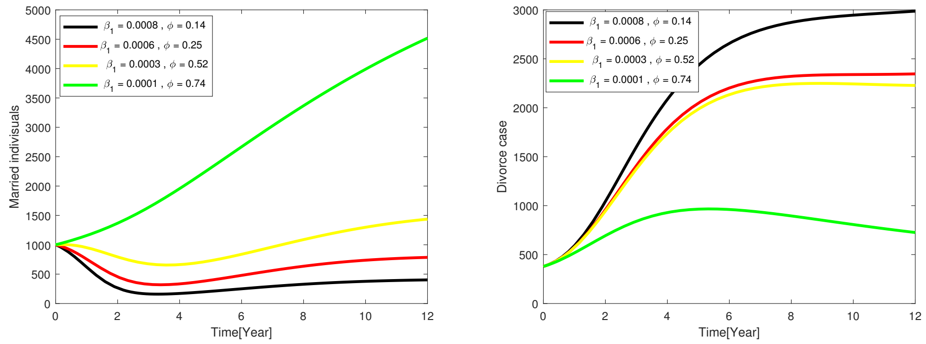


Figure 6.3.7: Effect of β_2 on marriage and divorce.

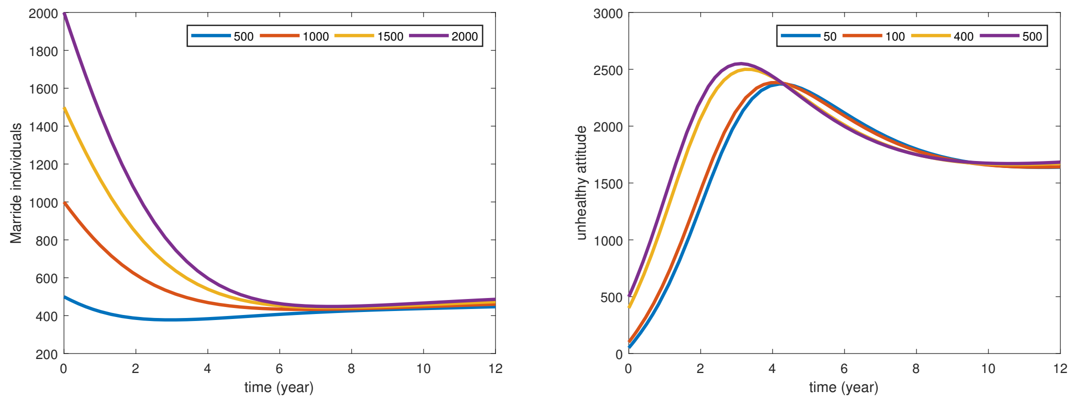
According to figure 6.3.7 we observe that as the transmission rate β_2(a contact rate between single individual and unhealthy attitude) increases, married individual decrease and divorce individual increases. As of the contact rate increase, the basic reproduction number is increase as they have direct proportionality due to this the unhealthy attitude on a marriage persist in the population. This graph also demonstrates the contact rate β_2 has positively induced parameter and it has impact on the spread of unhealthy attitude though out the population. Hence to decrease the number of reproduction number one can decrease the contact rate between single individual and unhealthy attitude.



(a) (b)

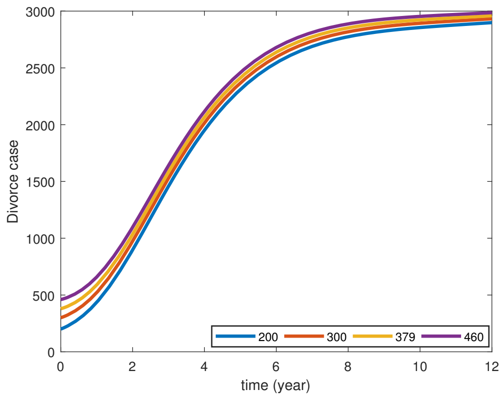
Figure 6.3.8: Combined intervention of β_1 and ϕ on marriage and divorce.

In figure 6.3.8, we have shown the effect of combined innervation the contact rates β_1 and the rate of attending healthy counseling ϕ on marriage and divorce. It was shown that when the rate of β_1 decrease and the rate of ϕ increase the married couples increase and divorce decrease.



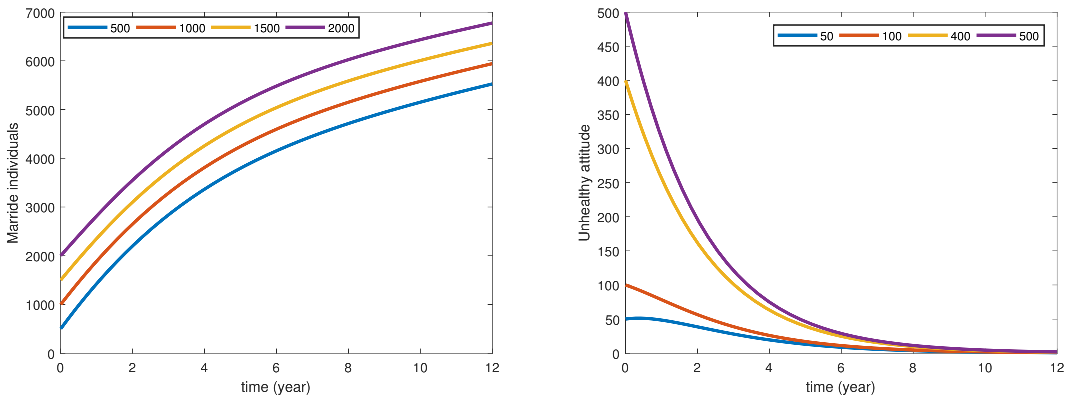
(a) (b)

From figure 6.3.9, we observe that the value of basic reproduction number $R_0 = 21.0450$ all the three compartments in the above figure shows goes away from the unhealthy attitude free equilibrium point. These indicate that the U_0 is unstable for the value of $R_0 > 1$ .The value of the basic reproduction number is greater than one show all compartment converges to the UAPE. These indicate that the UAPE is locally asymptotically stable for the initial which supports the theorem.



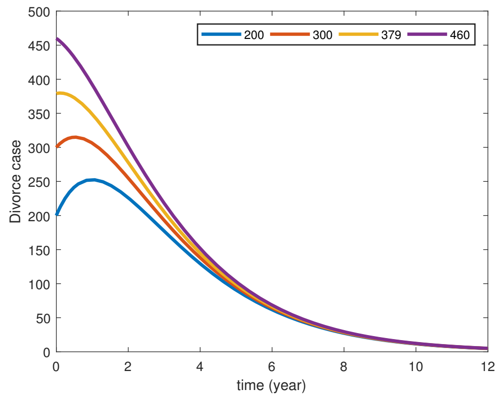
(c)

Figure 6.3.9: : Graph of state variables for $R_0 = 21.0450 > 1$ with different initial populations.



(a) (b)

In figure 6.3.10 (a-c) with $R_0 = 0.4952 < 1$ and using the given different initial human populations for marriage, unhealthy attitude and divorce , all state variables go to their components of the unhealthy attitude free equilibrium point U_0. These indicates that the unhealthy attitude free equilibrium point is locally and globally asymptotically stable for $R_0 < 1$.



(c)

Figure 6.3.10: : Graph of state variables for $R_0 = 0.4952 < 1$ with different initial populations.

Chapter 7

7 Conclusion and Recommendation

Here under, we gave the general conclusion of the thesis along with some recommendations to be given to the concerned bodies.

7.1 Conclusion

In this study, we have formulated and analyzed nonlinear deterministic mathematical model to reduce the spread of unhealthy attitude on marriage and its impact on the dynamics of divorce. The modified model was an extension of the existing model by including unhealthy attitude on marriage and healthy counseling. The six dimensions system of ordinary differential equation was developed. To achieve the objectives of the thesis, we gathered secondary data sources related to divorce from Hawassa first instance court and some parameter values are from related published articles and reasonable assumptions. The required data for the study were analyzed by using a MATLAB computer program and the results are presented in the form of tables and graphs. Both the qualitative and numerical analysis of the model was done. We have established the well-posedness of the modified model by proving the existence, uniqueness, positivity, and boundedness of the solutions. We computed the steady states and the basic reproduction number R_0. Based on the reproduction number R_0, it becomes clear that whenever $R_0 < 1$, the system has only unhealthy attitude free equilibrium U_0 which is locally as well as globally asymptotically stable. This implies that unhealthy attitude can not persist in the society. When $R_0 > 1$, the system has a unique unhealthy attitude present equilibrium point E^* which is locally and globally asymptotically stable and unhealthy attitude free equilibrium U_0 becomes unstable. It means that unhealthy attitude persist in the society. Using center manifold theory, bifurcation analysis of the proposed model was proven and the model exhibits forward bifurcation at $R_0 = 1$.

We used the real data that was gathered to undergo numerical experimentation on the dynamical system formulated, and estimate the unknown parameters using MATLAB. We evaluated the numerical value of the basic reproduction numbers $R_0 = 21.045$, which show that unhealthy attitude spread in the community. Sensitivity analysis was performed on different parameters with the basic reproduction numbers, R_0 to understand how sensitive the model is to the different parameter values and its structure dynamics using the normalized forward sensitivity index. The sensitivity analysis shows that the rate of interaction between married individuals with unhealthy attitude (β_1), the rate of individuals attending healthy counseling due to unhealthy attitude on marriage (ϕ), the rate of individuals who have unhealthy attitude get divorce (δ) and the rate of married individuals who are in a challenge get unhealthy attitude (α) are the most sensitive parameters to the reproduction number (R_0).
Hence, the results indicate that increasing the contact rate between married individuals and individuals with unhealthy attitude on marriage (β_1), and the rate of alpha are positively affects both the spread of unhealthy attitude and the threshold parameter R_0. Whereas, increasing rate of δ and ϕ has a great negative impact on the spread of unhealthy attitude in the society. Furthermore model shows that those individuals with unhealthy attitude are worse than any individuals from any other compartments. From the finding of our work result we can conclude that getting divorce is preferable to being a married person with a unhealthy attitude on a marriage. Increasing the rate of δ and ϕ have gratefully affects the output of the reproduction number and the general result of the model.

7.2 Recommendation

Considering the finding of the study and the previously draw conclusion, the following recommendations were made.

- From the above results we recommend stakeholders and policy makers to give a positive feedback for parameters that have negative indices (δ, η, ρ, π and ϕ) and put negative feedback on positive indices (β_1, β_2, α and θ) in order to control the spread of unhealthy attitude and divorce in a society.

- Married individuals should reduce the rate of communication with individuals who have unhealthy attitude on marriage (β_1) to protect their healthy marriage.

- Constant practice discussion between a married couples without interference of other will help to reduce divorce in marriage.

- We advise that the married couple to have healthy counseling whenever; there is a misunderstanding in the family which helps to motivate healthy marriage and happy life among the marriages.

7.3 Limitation of the study

The following are some limitations of this study:

- The collected data that we used for our study covered only the Hawassa city. The study has not covered at lest the whole region of sidama. As marriage and divorce issue is serious problem throughout the country, it was not able to study at lest the expected geographical region due to the simplicity and availability of the required data.
- The other limitation of the study was difficult to find real data for married couples.

7.4 Future Work

- Optimal control can be considered in the model.
- More geographical area will be considered with better techniques of data collection and greater human resource.
- Stochastic process can be included in the model.

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Appendices

Listing 1: MATLAB codes for sensitivity analysis bar graph:

```
            %%%MATLAB codes for sensitivity analysis bar graph
               :
clear all
 clc
 Lambda = 300; epsilon = 0.04; mu = 0.015;
eta = 0.02; beta1 = 0.0006; beta2 = 0.0003;
omega = 0.2; alpha =0.2; pi = 0.061;
phi = 0.14; delta = 0.32; theta = 0.1; rho = 0.15;
 k1 = (rho+alpha+eta+mu);
 k3=(theta+pi+mu);
 L=((epsilon+mu)*delta*(pi+mu)+k3*(phi+mu));
 M=(delta*(pi+mu)+k3*(epsilon+mu)+k3*(phi+mu));
N=(alpha*epsilon*beta1+beta2*mu*k1);
R0=((theta+pi+mu)*Lambda*alpha*epsilon*beta1+Lambda*mu*beta2
   *(theta+pi+mu)*(rho+alpha+eta+mu))...
  /(mu*(epsilon+mu)*(rho+alpha+eta+mu)*(delta*(pi+mu)+(phi+mu
     )*(theta+pi+mu)));
sLambda = 1;
salpha = (alpha*((epsilon*beta1+beta2*mu)*k1-N))/(k1*N);
stheta =(theta*delta*(pi+mu))/((delta*(pi+mu)+(phi+mu)*k3)*k3
   );
sbeta1 = ((k3*Lambda*alpha*epsilon*beta1))/((k3*Lambda*alpha*
   epsilon*beta1+...
k3*Lambda*beta2*mu*k1));
sbeta2 = (k3*Lambda*mu*k1*beta2)/(k3*Lambda*alpha*epsilon*
   beta1+...
k3*Lambda*beta2*mu*k1);
% % % % % % % i
%A = alpha*beta1*beta2*k3*omega*phi + alpha*beta1*omega*pi*
   delta*beta2*beta1*...
 %   beta2*omega +mu*((delta+phi+mu)*k3-delta*theta)*(rho +
    alpha + mu)+omega*rho*beta1*beta2*(delta+phi+mu)*...
  %  k3*((delta+phi+mu)-Lambda);
% % % % % % % % l
spi =(-pi*theta*delta)/((delta*(pi+mu)+(phi+mu)*k3)*k3);
srho= (-rho*alpha*epsilon*beta1)/((alpha*epsilon*beta1+beta2*
   mu*k1)*k1);
```

```
seta=(-(eta*alpha*epsilon*beta1))/((alpha*epsilon*beta1+beta2
   *mu*k1)*k1);
sepsilon= (epsilon*(alpha*beta1*mu-beta2*mu*k1))/...
((alpha*epsilon*beta1+beta2*mu*k1)*(epsilon+mu));
sphi=(-phi*k3)/(delta*(pi+mu)+(phi+mu)*k3);
sdelta=(-delta*(pi+mu))/(delta*(pi+mu)+(phi+mu)*k3);
smu=(N*Lambda*mu*(epsilon+mu)*k1*delta*(pi+mu)-k3*((k1+mu)*
   Lambda*alpha*epsilon*beta1*L+N*Lambda*mu*k1*M+Lambda*alpha
   *epsilon*beta1*mu*(epsilon+mu)*k1*delta))...
    \k1*L*k3*Lambda*N;
senv=[sLambda sbeta1 sbeta2 salpha stheta  spi sphi srho seta
    sepsilon sdelta smu ];
% barh(senv,'c') %horizontal par
%    names={'\Lambda','\beta_1','\beta_2', '\alpha','\theta
   '...
%  ,'\pi','\phi','\rho','\eta', '\epsilon','\delta','\mu'};
% set(gca,'yticklabel',names)
% %set(gca,'linewidth',0.5)
% title('Sensitivity indices of R_0 bar plot')
 barh(senv,'c') %horizontal par
   names={'\Lambda','\beta_1','\beta_2', '\alpha','\theta'...
 ,'\pi','\phi','\rho','\eta', '\epsilon','\delta','\mu'};
set(gca,'yticklabel',names)
%set(gca,'linewidth',0.5)
title('Sensitivity indices of R_0 bar plot')
```

Listing 2: MATLAB codes for The impact of β_1 on R_0:

```
clear all
 clc
 Lambda = 300; epsilon = 0.04; mu = 0.015;
eta = 0.02; beta1 = 0:0.0001:0.0006;
beta2 = 0.0003;
omega = 0.2; alpha =0.2; pi = 0.061;
phi = 0.14; delta = 0.32; theta = 0.1; rho = 0.15;
R0=((theta+pi+mu)*Lambda*alpha*epsilon*beta1+Lambda*mu*beta2
   *(theta+pi+mu)...
*(rho+alpha+eta+mu))/(mu*(epsilon+mu)*(rho+alpha+eta+mu)*(
   delta*(pi+mu)+(phi+mu)*(theta+pi+mu)));
plot(beta1,R0,'r','linewidth',2.5);
legend ('R_{0} Vs \beta_1');
```

```
xlabel('\beta_1');
ylabel('R_{0}');
title ('The impact of \beta_1 on R_{0}')
```

Listing 3: MATLAB codes for stability analysis of function:

```
function f = Etsehiwot(t,y,Lambda,epsilon,mu,eta,beta1,beta2,
    omega,alpha,pi,phi,delta,theta,rho)
S = y(1);
M = y(2);
G = y(3);
U = y(4);
D = y(5);
H = y(6);
dS = Lambda-beta2*U*S-(epsilon+mu)*S;
dM = epsilon*S+ eta*G + omega*H-beta1*M*U-mu*M;
dG = beta1*M*U-(rho+alpha+eta+mu)*G;
dU = beta2*S*U+alpha*G+theta*D-(delta+phi+mu)*U;
dD = delta*U-(theta+pi+mu)*D;
dH = phi*U+pi*D+rho*G-(omega+mu)*H;
f = [dS;dM;dG;dU;dD;dH];
```

Listing 4: MATLAB codes for stability analysis of unhelthy attitude free equilibrium point:

```
C = ['g ';'b ';'y ';'r ';'k ';'m '];
% Lambda = 300; epsilon = 0.04; mu = 0.016;
% eta = 0.032; beta1 = 0.00013; beta2 = 0.00011;
% omega = 0.3; alpha = 0.0124; pi = 0.418;
% phi = 0.016; delta = 0.678; theta = 0.00123; rho = 0.019;
Lambda = 300; epsilon = 0.07; mu = 0.015;
eta = 0.5; beta1 = 0.0002; beta2 = 0.0001;
alpha =0.02; pi = 0.6;
phi = 0.4; delta = 0.5; theta = 0.01; rho = 0.15;omega = 0.7;
R0=((theta+pi+mu)*Lambda*alpha*epsilon*beta1+Lambda*mu*beta2
    *(theta+pi+mu)...
*(rho+alpha+eta+mu))/(mu*(epsilon+mu)*(rho+alpha+eta+mu)*(
    delta*(pi+mu)+(phi+mu)*(theta+pi+mu)));
tspan = [0 12];
y0 =  [4000 1000 1500 500 379 50];
[t, y] = ode45(@Etsehiwot,tspan,y0,[],Lambda,epsilon,mu,eta,
    beta1,beta2,omega,alpha,pi,phi,delta,theta,rho);
for i = 1:6
```

```
plot(t,y(:,i),C(i,:),'LineWidth',2.5)
legend('S','M','G','U','D','H')
xlabel('Time[Year]')
ylabel('Population')
hold on
title('All population R0 =  0.4952')
end
```

Listing 5: MATLAB codes for stability analysis of endemic equilibrium point:

```
C = ['g ';'b ';'y ';'r ';'k ';'m '];
Lambda = 300; epsilon = 0.04; mu = 0.015;
eta = 0.02; beta1 = 0.0006; beta2 = 0.0003;
omega = 0.2; alpha =0.2; pi = 0.061;
phi = 0.14; delta = 0.32; theta = 0.1; rho = 0.15;
R0=((theta+pi+mu)*Lambda*alpha*epsilon*beta1+Lambda*mu*beta2
   *(theta+pi+mu)...
*(rho+alpha+eta+mu))/(mu*(epsilon+mu)*(rho+alpha+eta+mu)*(
   delta*(pi+mu)+(phi+mu)*(theta+pi+mu)));
tspan = [0 12];
%y0 = [40800 10000 8000 2000 379 500];
y0 =  [4000 1000 1500 500 379 50];
[t, y] = ode45(@Etsehiwot,tspan,y0,[],Lambda,epsilon,mu,eta,
   beta1,beta2,omega,alpha,pi,phi,delta,theta,rho);
for i = 1:6
plot(t,y(:,i),C(i,:),'LineWidth',2.5)
legend('S','M','G','U','D','H')
xlabel('Time[Year]')
ylabel(' Population ')
title('All population R0 =  21.0450')
hold on
end
```

Listing 6: Matlab codes for different values of some parameters on marriage, unhelthy attitude and divorce:

```
C = ['b  ';'R: ';'y  ';'r  ';'g--';'m  '];
Lambda = 300; epsilon = 0.04; mu = 0.015;
eta = 0.02;
%   beta1=0.0001;
%    beta1=0.0003;
%  beta1 = 0.0006;
```

```
beta1 = 0.0008;
beta2 = 0.0003;
omega = 0.2; alpha =0.2; pi = 0.061;
phi = 0.14; delta = 0.32; theta = 0.1; rho = 0.15;
R0=((theta+pi+mu)*Lambda*alpha*epsilon*beta1+Lambda*mu*beta2
    *(theta+pi+mu)...
*(rho+alpha+eta+mu))/(mu*(epsilon+mu)*(rho+alpha+eta+mu)*(
    delta*(pi+mu)+(phi+mu)*(theta+pi+mu)));
tspan = [0 12];
y0 =  [4000 1000 1500 500 379 50];
[t, y] = ode45(@Etsehiwot,tspan,y0,[],Lambda,epsilon,mu,eta,
    beta1,beta2,omega,alpha,pi,phi,delta,theta,rho);
for i = 2
plot(t,y(:,i),C(i,:),'LineWidth',2.5)
%legend('S','M','G','U','D','H')
legend(' \beta_1 = 0.0001 ','\beta_1 = 0.0003 '...
    ,'  \beta_1 = 0.0006',' \beta_1 = 0.0008','D','H')
xlabel('Time[Year]')
ylabel('Married indivisuals')
hold on
end
```

Listing 7: Matlab codes for different values of some parameters on marriage, unhelthy attitude and divorce:

```
 C = ['r--';'r--';'y  ';'K  ';'G--';'m  '];
Lambda = 300; epsilon = 0.04; mu = 0.015;
eta = 0.02; beta1 = 0.0006; beta2 = 0.0003;
omega = 0.2; alpha =0.2; pi = 0.061;
    %  phi = 0.14;
   %  phi = 0.25;
  %  phi=0.52;
    phi = 0.74;
delta = 0.32; theta = 0.1; rho = 0.15;
R0=((theta+pi+mu)*Lambda*alpha*epsilon*beta1+Lambda*mu*beta2
    *(theta+pi+mu)...
*(rho+alpha+eta+mu))/(mu*(epsilon+mu)*(rho+alpha+eta+mu)*(
    delta*(pi+mu)+(phi+mu)*(theta+pi+mu)));
tspan = [0 12];
%y0 = [40800 10000 8000 2000 379 500];
y0 =  [4000 1000 1500 500 379 50];
```

```
[t, y] = ode45(@Etsehiwot,tspan,y0,[],Lambda,epsilon,mu,eta,
   beta1,beta2,omega,alpha,pi,phi,delta,theta,rho);
for i = 5
plot(t,y(:,i),C(i,:),'LineWidth',3)
%legend('S','M','G','U','D','H')
legend('\phi = 0.14'...
    ,' \phi = 0.25',' \phi = 0.52',' \phi=0.74 ' ,'D','H')
xlabel('Time[Year]')
ylabel('Divorce case')
hold on
end
```

Listing 8: Matlab codes for different values of by combine two parameters on marriage and divorce:

```
C = ['b  ';'G  ';'y  ';'r  ';'g--';'m  '];
Lambda = 300; epsilon = 0.04; mu = 0.015;
eta = 0.02;
%      beta1 = 0.0008;    phi = 0.14;
%          beta1 = 0.0006;          phi = 0.25;
%            beta1=0.0003;         %  phi=0.52;
          beta1=0.0001;              phi = 0.74;
beta2 = 0.0003;
omega = 0.2; alpha =0.2; pi = 0.061;
 delta = 0.32; theta = 0.1; rho = 0.15;
R0=((theta+pi+mu)*Lambda*alpha*epsilon*beta1+Lambda*mu*beta2
   *(theta+pi+mu)...
*(rho+alpha+eta+mu))/(mu*(epsilon+mu)*(rho+alpha+eta+mu)*(
   delta*(pi+mu)+(phi+mu)*(theta+pi+mu)));
tspan = [0 12];
y0 =  [4000 1000 1500 500 379 50];
[t, y] = ode45(@Etsehiwot,tspan,y0,[],Lambda,epsilon,mu,eta,
   beta1,beta2,omega,alpha,pi,phi,delta,theta,rho);
for i = 2
plot(t,y(:,i),C(i,:),'LineWidth',2.5)
%legend('S','M','G','U','D','H')
legend(' \beta_1 = 0.0008 , \phi = 0.14 ','\beta_1 = 0.0006 ,
    \phi = 0.25 '...
    ,'  \beta_1 = 0.0003 , \phi = 0.52',' \beta_1 = 0.0001 ,
       \phi = 0.74 ','D','H')
xlabel('Time[Year]')
```

```
ylabel('Married indivisuals')
hold on
end
```

Listing 9: Matlab codes for different intital population:

```
%% for R_0>1
close all;clear all;clc
global Lambda beta1 beta2 epsilon omega delta eta theta  mu
   phi pi rho alpha
Lambda = 300; epsilon = 0.04; mu = 0.015;
eta = 0.02; beta1 = 0.0006; beta2 = 0.0003;
omega = 0.2; alpha =0.2; pi = 0.061;
phi = 0.14; delta = 0.32; theta = 0.1; rho = 0.15;
R0=((theta+pi+mu)*Lambda*alpha*epsilon*beta1+Lambda*mu*beta2
   *(theta+pi+mu)...
*(rho+alpha+eta+mu))/(mu*(epsilon+mu)*(rho+alpha+eta+mu)*(
   delta*(pi+mu)+(phi+mu)*(theta+pi+mu)));
 R0
ts = [0 12];
W=2.5;%line width
%figure(1)
x0=[4000 1000 1500 500 200 50];
[t x]=ode45('etsehiwot',ts,x0);
plot(t,x(:,5),'LineWidth',W)
xlabel ('time (year)')
ylabel ('Divorce case')
hold on
%%
x0=[4000 1000 1500 500 300 50];
[t x]=ode45('etsehiwot',ts,x0);
plot(t,x(:,5),'LineWidth',W)
hold on
%%
x0=[4000 1000 1500 500 379 50];
[t x]=ode45('etsehiwot',ts,x0);
plot(t,x(:,5),'LineWidth',W)
%%
x0=[4000 1000 1500 500 460 50];
[t x]=ode45('etsehiwot',ts,x0);
plot(t,x(:,5),'LineWidth',W)
```

```
legend ('200','300' ,'379' ,'460' )
```