DESIGN OF MODEL REFERENCE ADAPTIVE CONTROL FOR QUADROTOR UNMANNED AERIAL VEHICLE AT HOVERING CONDITION

M.Sc. THESIS

KIDUS GEBREMICHAEL GEBREGZIABHER

HAWASSA UNIVERSITY, HAWASSA, ETHIOPIA

JULY, 2020

DESIGN OF MODEL REFERENCE ADAPTIVE CONTROL FOR QUADROTOR UNMANNED AERIAL VEHICLE AT HOVERING CONDITION

KIDUS GEBREMICHAEL GEBREGZIABHER

A THESIS SUBMITTED TO THE
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING,
HAWASSA INSTITUTE OF TECHNOLOGY,

SCHOOL OF GRADUATE STUDIES

HAWASSA UNIVERSITY
HAWASSA, ETHIOPIA

IN PARTIAL FULFILLMENT OF THE

REQUIREMENTS FOR THE

DEGREE OF

MASTER OF SCIENCE IN ELECTRICAL AND COMPUTER ENGINEERING
(CONTROL AND INSTRUMENTATION ENGINEERING)

JULY, 2020

Declaration

I hereby declare that this MSc thesis entitled "**Design of Model Reference Adaptive Control For Quadrotor Unmanned Aerial Vehicle at Hovering Condition**" is my original work and has not been presented for a degree in any other university, and will not be presented by me to any other university for similar or any other degree award, and all sources of material used for this thesis have been duly acknowledged.

Name: Kidus Gebremichael

Signature: _______________

This MSc thesis entitled **"Design of Model Reference Adaptive Control for Quadrotor Unmanned Aerial Vehicle at Hovering condition"** has been submitted for examination with my approval as thesis advisor.

Name: Dr.-Ing. Gebremichael Teame (PhD)

Signature: __________________

Place and Date of Submission: __________________

Acknowledgements

First and incessantly, all admiration and thanks to god, who gave me the strength, and persistence to carry out this work in this good manner. Second, I would like to thank Adigrat University for allowing me to get this chance and the financial sponsorship for the study program.

I would like to express my heartily gratitude and sincere thanks to my respected advisor Dr.-Ing Gebremichael Teame for his professional guidance, advice, motivation, endurance and encouragements during his supervision period. The present work would have never been possible without his vital supports and valuable assistance.

Also, I want to thank Hawassa University, especially school of Electrical and Computer Engineering, for allowing me to study and for creating different learning opportunities from beginning up to now.

Finally, I want to thank my family, my friends and my class mates for their supporting me continuously throughout the years of my study.

Abstract

In this thesis, we have considered the detail dynamic and kinematics mathematical modeling of 6DOF quadrotor UAV VTOL. Quadrotors are brush less direct current motors important device in terms of mechanical design and aerodynamic configuration. Quadrotors are under actuated vehicle which have four inputs and six outputs. The nonlinear model of the quadrotor unmanned aerial vehicle is linearized using the jacobian method of linearization at the selected operating points. For controlling of roll, pitch and yaw and altitude subsystems of quadrotor at hovering point, we have designed four model reference adaptive control (MRAC) based on the gradient method of adaptation mechanism. The reference model for each subsystem of the quadrotor unmanned aerial vehicle were chosen with a very small overshoot and zero steady state error. So that the output of the plant model tracks the reference model output. Finally, the sustainability and success of the designed controller (MRAC) are tested by MATLAB Simulink platform simulations was perform for different adaptation gain. The values of adaptation gain from 10 up to 200 gives a satisfactory result based on the desired performance.

Key words: Quadrotor, UAV, Modeling, DOF, VTOL, Gradient method and MRAC

Table of Contents

List of Figures

List of Tables

List of Abbreviations

UAV	Quadrotor unmanned aerial vehicle
6DOF	Six degree of freedom
BLDC	Brushless direct current
DC	Direct current
E-frame	Earth frame
B- frame	Body frame
MRAC	Model Reference Adaptive Control
VTOL	Vertical take-off landing
PID	Proportional Integral Derivative Controller
LQR	Linear quadratic regulator
X	Cross-product
MIMO	Multi input multi output
Mp	Maximum overshoot
tr	Rising time
ts	Settling time
MIT	Massachusetts Institute of Technology
m	Mass of the quadrotor
g	Gravity
K_d	Force constant
d	Torque constant
$\omega_{i,i=1,2,3,4}$	Angular speed of the i_{th} rotors
ω_t	Total angular speed of the four rotors
e	Error
yp	Output of the plant
ym	Output of the model reference
u_c	Reference input
τ	Torque
ϕ	Roll angle along x-axis
θ	Pitch angle along y-axis

ψ	Yaw angle along z-axis
x	Position of the quadrotor along x -axis
y	Position of the quadrotor along y-axis
z	Altitude of the quadrotor along z- axis
u	Linear velocity of the quadrotor along x-axis
v	Linear velocity of the quadrotor along y-axis
w	Linear velocity of the quadrotor along w-axis
p	Rotational speed of the quadrotor along x-axis
q	Rotational speed of the quadrotor along y-axis
r	Rotational speed of the quadrotor along z-axis
γ_1, γ_2	Aadaptation gains
u_i, $_{i=1,2,3,4}$	Control inputs of the system
J_p	Inertia of the motor

CHAPTER ONE

1 Introduction

Unmanned aerial vehicle (UAV) is flying vehicle without human on board. Quadrotors are unmanned aerial vehicles which are designed to operate with high precision and growing robustness for happenstance interesting task necessities. These vehicles consist four identical brushless DC motors and propellers. The propellers are a special form of UAV that has two pairs of contra rotating rotors to provide lift and the directional control.

Quadrotors are applicable in civil domains, security, military, environmental researches, traffic monitoring, image processing over nuclear reactor, mapping, construction, etc. When we compare the quadrotor with the other types of the UAVs, it has better characteristics regarding of its structure, cost, maintenance, vertical take-off and landing capability, and rapid maneuvering.

Quadrotor have few advantages over conventional helicopters like possibility of control by varying the angular speeds of the rotors, simple in design, and have fixed-pitch propeller mechanisms due to mechanical and aerodynamic simplicity. It needs less kinetic energy per rotor, small area to obtain lift like the fixed wing air crafts and easy to control with fixed pitch propeller.

1.1 Background of the study

Quadrotors were among the first vertical take-off and landing vehicles (VTOL). The first helicopter was designed by Breguet Brothers in 1907 what we call it Gyroplane No. 1. This was a quadrotor. The Gyroplane No. 1 consists four long girders organized in horizontal cross [1]. The flights of this quadrotor was considered as the first flight of a helicopter with pilot on the bored, but it was not free flight. Because of lack of stability, precise control, uneconomic and inefficient the machine was never flew completely free.

In 1920, Etienne Omnichen developed Omnichen quadrotors which consists four identical motors and eight propellers to control the vehicle. The motor of this vehicle drives by single engine to overcome the problem of the helicopter. Omnichen quadrotors makes a thousand flight of tests at the middle of 1920. In 1923 it was hovered for several minutes at a time and in 1924 moves distance record for helicopters of 360 m [2].

Later, in 1922, Georges de Bothezat was designed helicopter of the time also a quadrotor. The flight of this helicopter was successfully flying many times with slow forward speeds and low altitudes. However, because of insufficient performance and high financial cost the project was cancelled [2].

In 1956, quadrotor was proposed the Convertawings Model A Quadrotor sample for a line of much larger military and civil quadrotor. The roll, pitch and yaw angles are controlled by varying the thrust rotors.[3]. This design was used two engines for driving four rotors and added wings for additional lift in roll down in the cross configuration. But the convertawings model A quadrotor project was canceled due to lack of order for commercial or military versions.

In 1958, the Curtiss-Wright was designed a VZ-7 vertical take-off landing air craft with four identical rotors for united states army (US). This vehicle was controlled by varying the angular speed of each rotors [3].

In last ten years, technology is highly attained such as electronics, primary elements of the measurement system, digital controllers, precise motors and software's are an opportunity of designing different types of UAVs became available in a world. This is the main reason of increasing in engineering research interest on quadrotor. Nowadays quadrotors are used mostly as models, items for teaching purposes in colleges, research and scientific areas and for view video recording [4].

In this thesis, the altitude of the quadrotor is adjusted by applying equal forces to each rotor, its yaw by applying more force to the rotors rotating in clockwise direction and less forces to the rotating in counterclockwise direction. The roll angle is adjusted by applying more forces to back rotor and less force to front rotor. The pitch angle is adjusted by applying more forces to left rotor and less force to right rotor.

1.2 Statement of the problem

The UAV Quad rotor are very useful vertical takeoff landing (VTOL) concept mainly in terms of hovering, heavier than air, aerodynamic wing configurations, flight safety and simple in mechanical design. Quad rotors are brushless DC motors which are naturally unstable due to their dynamic model consists highly nonlinear, coupling, underactuated and uncertainty. quad rotors are also influenced by body dynamic gravity, wind, force which is driven by the propeller, gyroscopic impacts etc. Therefore, flight control of this system needs design of proper controller.

1.3 Objectives

1.3.1 General objective

The general objective of this thesis is to design model reference adaptive control for trajectory control of quadrotor unmanned aerial vehicle (UAV) at hovering point.

1.3.2 Specific objectives

The specific objectives of this thesis are:

- To Develop the dynamic mathematical model of the quadrotor UAV.
- To linearize the nonlinear mathematical model of quadrotor.
- To design MRAC based on gradient method.
- To control the attitude and altitude of quadrotor at hovering point.
- To Simulate the MRAC on MATLAB /Simulink platform.
- To Validate the results.

1.4 Methodology

To perform the specific objectives of this thesis, the following methodology are followed. The work of this begins with Collecting and reviewing related works with this problem to check whether already done or not. This stage includes gathering international journals, Msc. thesis and other different papers from various literatures and publications.

The final step is how to go through this work up to its final destination that is the steps that must be followed after it is decided to do this thesis.

To design the controller, developing mathematical model of the quadrotor system is the first step. The mathematical model of this vehicle is done using the Newton-Euler formalism law. This system is nonlinear multivariable system with four inputs and six outputs variables. Then in order to use linear controller it is obligatory to linearize the nonlinear multivariable quadrotor unmanned aerial vehicle air craft at the selected operating points.

Finally, MRAC is designed based on gradient method, simulate results to select the effective range of adaptation gain for the desired response at hovering point.

1.5 Scopes and limitations

In this thesis, we use linearized dynamic model of the quadrotor system and dynamic model of the motors, it has its own scope and limitation.

1.5.1 scope

The scope of this thesis is only limited on MATLAB Simulink platform level. To work on the environment, the dynamic model of quadrotor is simplified as much as possible by applying a a technique of small angle approximation, body symmetry and taking the model conventional DC motor at the steady state condition in terms of brushless DC motors. It is also controlling the controlled variables at hovering point.

1.5.2 Limitation

The limitation of this thesis is not considering of any disturbances.

1.6 Thesis outline

The structure of this thesis is organized on six chapters including the introduction part. The rest chapter of this thesis are organized as follow.

Chapter two: this chapter consists the literature review, brief introduction to the quadrotor UAV system, adaptive control system and actuator.

Chapter three: presents the detail mathematical modeling of the 6DOF quadrotor UAV using the Newton's Euler formalism of body dynamics and aerodynamic properties acting on the body of the quadrotor, linearization of the dynamic model, modeling of control inputs to the quadrotor, rotor velocity to control input and motor dynamic modeling.

chapter four: presents about the design of model reference adaptive control based on gradient method of adaptation mechanism.

Chapter five: presents the simulation results and discussion part.

Chapter six: presents conclusion and recommendation part.

CHAPTER TWO

2 Literature Review

Many active research areas are done during previous periods on desired target quadrotor UAV control. Several control approaches have been studied for control of the quadrotor UAV. Most quadrotor researchers are used the linear controllers. Here, we are reviewed some papers which are worked related to the quadrotor control system as follow:

Gaopeng Bo. et al., in [8-13], have done a centralized PID control to control the roll, pitch and yaw angles of the quadrotor system. They did a good work from modeling of the vehicle up to linearizing method and applying small angle approximation technique. However, the first problem of these paper is neglection the motor dynamics model. So that the system becomes simple. The second problem of these paper is less accuracy, oscillation and steady state error on the response of attitude parameters on variation of parameter of the quadrotor system due to environmental concerns.

Another work related to this topic is by Hossein Bolandi1, et al., in [14,15], which considered the theoretical analysis is made for general multi input multi output (MIMO) system. But, PID control scheme is not optimal and it also does not handle the operational constraints.

Khoi Nguyen Dangl, B. Bin Fan, et al., in [16-18], were designed Linear Quadratic Regulator (LQR) to attitude control of quadrotor aircraft. The work of these paper has a problem in modeling by neglection of the altitude system, so that the model is incomplete. As system model is incomplete it may be difficult to design accurate controller that works on the desired way. Therefore, applying of LQR to control unmodelled quadrotor UAV have a problem like fluctuation of the Euler angles in roll and pitch, cannot able to make good hovering motion. The parameter; mass, inertia and aerodynamic drag force of the quadrotor systems are variable but, LQR works only on the plant with constant parameter. So, this is taken as a second gap of those poets.

In contrary to the linear control methods, Hamid Saeed Khan, Shailaja R, et al., in [20-25] were studied nonlinear control of quadrotor. The common nonlinear control techniques used to control quadrotor are sliding mode control, feedback linearization, back-stepping control, integral back-stepping control methods. Th first problem of these paper are missing of actuator model, complexity of integration of nonlinear controllers. Second problem gyroscopic effects are not included in the dynamic equation of motion under Euler's second axiom and costly because of needing to two nonlinear controllers.

H. P. Whitaker, P. Parks, et al., in [26-29), were designed on the use of adaptive control for controlling Euler angles of the quadrotor system. These works are deal with attitude stabilization of the quadrotor. The gaps of those authors are neglecting of the altitude and actuator modeling.

Since, adaptive controller is unrelated from fixed gain controller, which is skilled to achieve with better performance characteristics in the existence of important parametric uncertainties, and even without having full knowledge on the process.

In this thesis we have chosen direct type adaptive control what we call model reference adaptive control (MRAC) based on the gradient method to control the Quadrotor unmanned aerial vehicle (UAV).

2.1 Introductions to Quadrotor

Quadrotor is an aerial vehicle its operation is based on the principle of flying and propulsion mode. Aerial vehicle / aircrafts can be categorized in to two namely, lighter than air and heavier than air. Each of them could be classified as motorized and not motorized as shown in figure.2.1.

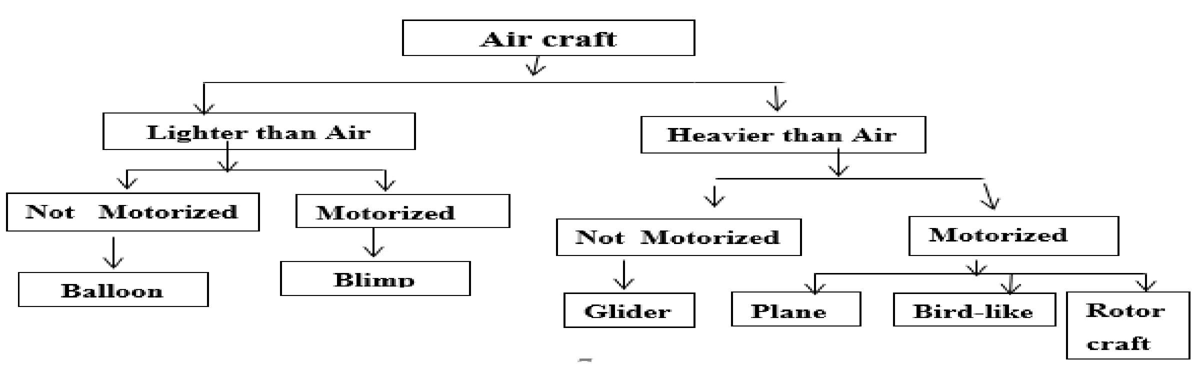


Figure 2. 1: Classification of Aircraft based on the flying principle and propulsion mode

2.2 Classification of Unmanned Aerial Vehicle

Unmanned aerial vehicles can be classified based on Size, range and endurance and Wing configuration [31].

2.2.1 Based on Size

Based on the size, UAVs are classified into:

a) **Very small UAVs**

These class are very small in size and in weight, have a size of from large insect up to 30-50 cm and they are used for fighting and snooping.

b) **Small UAVs**

Small UAVS are designed according to the fixed wing configuration type with at least one dimension must be higher than 50 cm and less than 2 meters.

c) **Medium UAVs**:

This class of UAVs are put on too heavy to be carried by one person but they are smaller than the light aircraft.

d) **Large UAVs**

Large UAVs are mainly applicable military for war operation purpose.

2.2.2 Based on range and endurance

According to the rang and endurance time UAVs can be categorized in to:

a) **Very low cost, close range UAVs**

This class of UAVs includes that have five km range, 20 to 45 minutes endurance time and cost of about 10000$.

b) **Close range UAVs**

Are UAVs that have 50 km range, 1 to 6 hours endurance time which are applicable for observation and investigation purposes.

c) **Short range UAVs**

The short range of UAV includes range of flight 150 km and 8 to 12 hours endurance time which are used for scrutiny and inspection tasks.

d) **Mid-range UAVs**

This rang of UAV consist that have super high speed and 650 km working radius of 650 km. They are used for collecting metrological data, surveillance and reconnaissance.

e) **Endurance UAVs**

The endurance range of UAV class includes that have endurance time 36 hours and working radius of 300 km. They can operate at altitude of 30,000 feet and they are used for surveillance and scouting purposes.

2.2.3 Based wing type configuration

Based on the aerodynamic configuration (type of wings); UAVs can be classified in to three groups as shown in figure 2.2.

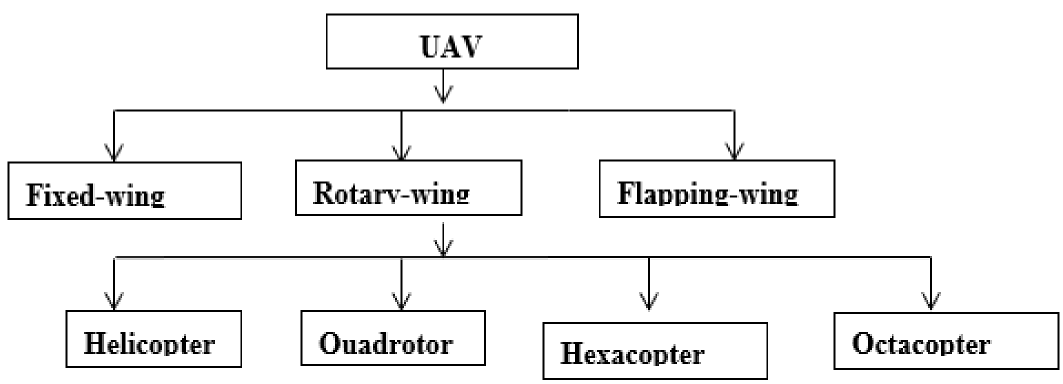


Figure 2. 2: Classification of UAV based on the type of wing

a) **Fixed wing UAV**

Are usually used for long range, distance and high-altitude tasks of civil fields and in scientific research areas like meteorological investigation and environmental observing.

b) **Rotary wing UAV**

These vehicles are called VTOL aircrafts, which are used on the missions that require hovering flight. The rotary wing type of UAV is less inclined to air confusion related to fixed wing UAV.

c) **Flapping wing**

Those types of UAVs are trying to matching the way insects or birds. Most of them are still now under development. The flapping-wing has low payload capability, low endurance, and high-power consumption and it can perform the VTOL.

The comparison between fixed-wing, rotary wing and flapping-wings are summarize in table 2.1.

Table 2. 1: Comparison of types of UAV based of wing structure

Characteristics UAV	Types of UAV		
	Fixed-wing	Rotary-wing	Flapping-wing
Maneuverability	Low	High	Medium
Cost	Low	Medium	High
Construction and repairing	Low	Medium	High
Civilian application	Low	High	High
Military application	Low	Medium	Medium
Power consumption	Low	Low	High
Flight safety	Low	Medium	Low
Range	Low	Medium	Low

In this comparison, the characteristics of rotary-wings UAVs are better from the other type of UAV as a shown in table 2.1[5]. Based on table 2.1, this thesis focused on the rotary type VTOL because of having better comparison characteristics compare to other type of UAV. Due to the criteria of the VTOL shown in table2.2, in this thesis we have chosen quadrotor type from the other rotary (VTOL) types of UAV. The comparison of different kinds of VTOL of UAV is summarized as shown in table 2.2.

Table 2. 2: comparisons of types of rotary-wing UAV

Criteria's of the VTOL	Types of rotary-wing (VTOL) of UAV							
	Single Rotor	Axial Rotor	Coaxial rotor	Tandem rotors	Quad Rotor	Blimp	Bird-like	Insect-like
Power cost	2	2	2	2	1	4	3	3
Control cost	1	1	4	2	3	3	2	1
Payload	2	2	4	3	3	1	2	1
Maneuverability	4	2	2	3	3	1	3	3
Mechanics simplicity	1	3	3	1	4	4	1	1
Aerodynamics complexity	1	1	1	1	4	3	1	1
Low speed flight	4	3	4	3	4	4	2	2
High speed	2	4	1	2	3	1	3	3
Miniaturization	2	3	4	2	3	1	2	4
Survivability	1	3	3	1	1	3	2	3
Stationary flight	4	4	2	4	4	3	1	2
Total	24	28	30	24	33	28	22	24

where, the values of 1, 2, 3 and 4 implies that poor, good, very good and excellent respectively.

A). **Single Rotor**: This type of UAV consists one main motor on the center and on small motor on the tail for the stability. It is identical to the helicopter configuration as shown in Figure 2.3.

Figure 2. 3: Single rotor

B). **Coaxial rotor**

Coaxial rotor consists two motors which are rotating in opposite directions and mounted to the same shaft as a shown in figure 2.4.

Figure 2. 4: Coaxial rotor

(C) **Quadrotor**

These consists four equal motors which are joined in a cross or plus configuration as a shown below in figure 2.5.

Figure 2. 5: Quadrotor

D). **Multi-rotors**

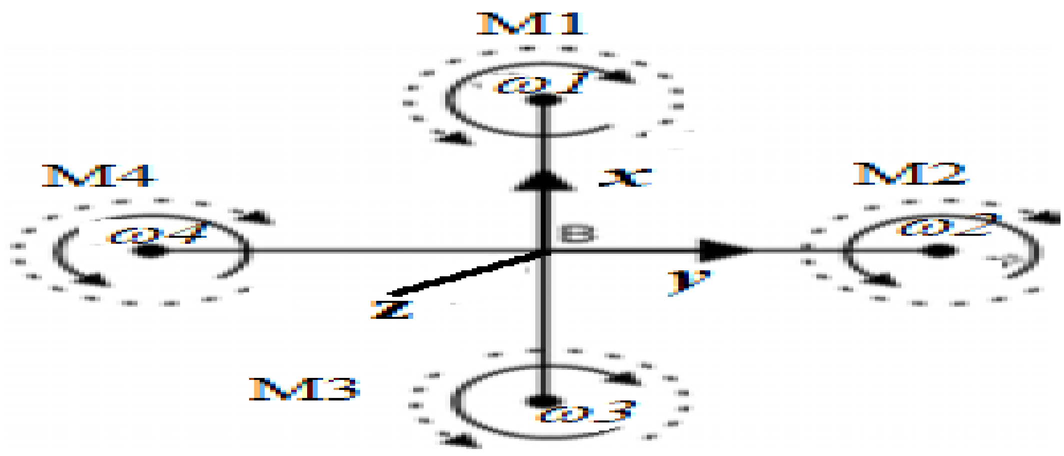
Are unmanned aerial vehicles consisting of six or more. They are agile type and fly even when a motor fails due to many numbers of rotors as a shown below in figure 2.6.

Figure 2. 6: Multi-rotor

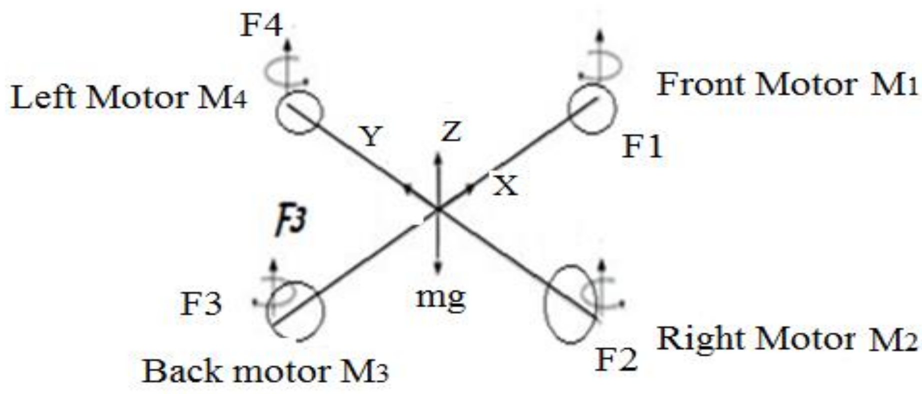
2.3 Mechanism of Flying Quadrotor

Quadrotors are an unmanned aerial vehicle which are designed with four motors in the plus (+) or cross (x) configuration. They able to move quickly and easily and controlled by angular speeds of each rotor. The position of motor arrangements relative to the body coordinate system cause two different types of quadrotor configurations as shown in figure 2.7 (a) and (b).

In this thesis we have focused the cross configuration because, it is quite thin and light. This shows robustness and stability comparison to the plus configuration.



a) Plus, configuration of UAV



b) Cross configuration of UAV

Figure 2. 7: Configuration of Quadrotor

Quadrotor have four poles, where each of the two are set symmetrically and perpendicularly to each other. To generate center of the quadrotor on the end of each pole actuators are arranged symmetrically. Each actuator consists rotor and propeller which is fixed to the shaft of the motor.

The force f_i (i = 1, 2, 3, 4) is generated by the i^{th} rotor. The rotors are assigned starting on the body frame of the x-axis with clockwise movement. Generally, the front motor (M_1) and back

(M_3) rotates in the clockwise direction while the left motor (M_4) and right motor (M_2) rotates in counterclockwise direction. The rotational motion of each motor is used to move the quadrotor. These motors are producing the force and torque about its center of rotation where the drag force acts opposite to the flight direction of the quadrotor. As a shown in figure 2.7 (b), all propellers are represented by straight and curved. The straight and curved arrows of propellers are used to represent the propeller velocity and direction of rotation respectively.

A system having small number of inputs than outputs is called underactuated system. Quadrotor have four inputs and six outputs that is why quadrotor is underactuated system.

2.4 Quadrotor Movement

Controlling of the input variables is very important to make the quadrotor easier to control. These variables could be controlled using different movements, which allows the quadrotor to reach a desired setpoint of the altitude, position along x- and y- axis, roll, pitch and yaw angles. The four basic movements of quadrotors are throttle movement, roll movement, pitch movement and yaw movement.

2.4.1 Throttle Movement

This movement is achieving by increasing or decreasing equal value of the angular speed of all motors. The throttle movement generates a vertical force (u_1) along z-axis with respect to the body frame which used to rise and let down the quadrotor.

2.4.2 Roll Movement

This movement is attaining by increasing the speed of back motor and decreasing speed of front motor. The change in speed generates a torque (τ_ϕ) in x-axis with respect to body-frame that makes the quadrotor turn and leads to roll angle acceleration ($\dot{\phi}$).

2.4.3 Pitch Movement

The pitch movement is reaching by increasing speed of the left motor and decreasing the speed of right motor. This difference in speed generates a torque (τ_θ) in y-axis with respect to body frame that makes the quadrotor turn and leads to pitch angle acceleration ($\dot{\theta}$).

2.4.4 Yaw Movement

This movement is achieved by increasing the speed of both front and back rotors and decreasing the speed of both right and left rotors. This change generates a torque (τ_{ψ}) in z-axis with respect to body frame, which makes the quadrotor turn and leads to yaw angle acceleration ($\dot{\psi}$).

We note that the symbols (ϕ, θ, ψ) are the roll angle, pitch angle and yaw angle around the x, y and z-axes respectively. Regarding of the axis representation, roll is the rotation around x-axis, which allows front and backward flying along x-axis, pitch is the rotation around y-axis, which allows right and left flying along x-axis. Yaw is the rotation around z-axis, which turns the quadrotor right or left by changing the heading as well as the direction of flight.

Quadrotor is hovering up when the total force generated by each motor is higher than gravitational force while flying down is when the total force less than gravitational force. Here the and rate remains zero. The detail comparison of the two types of quadrotor configuration is summarized in table 2.3.

Table 2. 3: Plus (+) and cross (x) configuration

Type of flight	Plus (+) Configuration	Cross (X) Configuration
Forward flight (Roll down)	M1 thrust is decreased M3 thrust is increased	M1 and M4 thrusts decreased M2 and M3 thrust increased
Backward flight (Roll up)	M1 thrust is increased M3 thrust is decreased	M1 and M4 thrusts increased M2 and M3 thrust decreased
Right flight (pitch Right)	M2 thrust decreased M4 thrust increased	M1 and M2 thrusts decreased M3and M4 thrust increased
Left flight (pitch Left)	M2 thrust increased M4 thrust decreased	M1 and M2 thrusts increased M3and M4 thrust decreased
Yaw Right (Turn Right)	M2 and M4 thrusts are decreased M1 and M3 thrusts are increased	M1 and M3 thrusts are decreased M2 and M4 thrusts are increased
Yaw left (Turn Left)	M2 and M4 thrusts are increased M1 and M3 thrusts are decreased	M1 and M3 thrusts are increased M2 and M4 thrusts are decreased

2.5 Actuators

In this thesis, we use four similar brushless DC motors, which are used to take an action based on the commands of the MRAC by converting electrical signal to mechanical signal if correction is needed. These are made up of two cooperative electromagnetic circuits namely, rotor which is freely rotate around the body of the quadrotor and stator is the fixed part.

2.6 Adaptive control system

Adaptive control system is the method of control that is used by the controller which adapts the behavior to the controlled system with variable parameters or initially uncertain. This method consists two loops. The first is the inner loop which is normal feedback with plant and the controller. The second loop is outer loop that is the parameter adjustment loop [7].

There are four types of adaptive schemes, namely gain scheduling, model reference adaptive control, self-tuning regulators and dual control [6].

2.6.1 Gain Scheduling

The concept of gain scheduling invented in linking with the growth of flight control systems. This scheme measures the gain and change, which is schedule to compensate the changes in the process gain by the controller [6]. The block diagram system with gain scheduling is shown in Figure 2.8, consists the inner loop composed of the process and the controller and an outer loop that adjusts the controller parameters on the basis of the operating points.

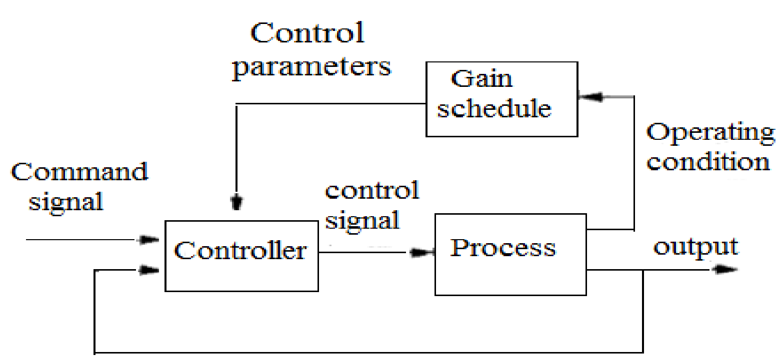


Figure 2. 8: Block digram of gain scheduling

The main problem of gain scheduling is difficulty of finding appropriate scheduled variables that characterize the operating conditions. The second problem of this scheme is it works only with predictable variable parameters.

2.6.2 Self-Tuning Regulators (STR)

STR is another type of adaptive control scheme which consists both the inner and outer loops. The inner loop includes process and feedback, and an outer loop consists controller, recursive parameter estimator and the design controller. The process parameters are estimated on-line, and the Estimation is used to estimate of the plant parameters [6].

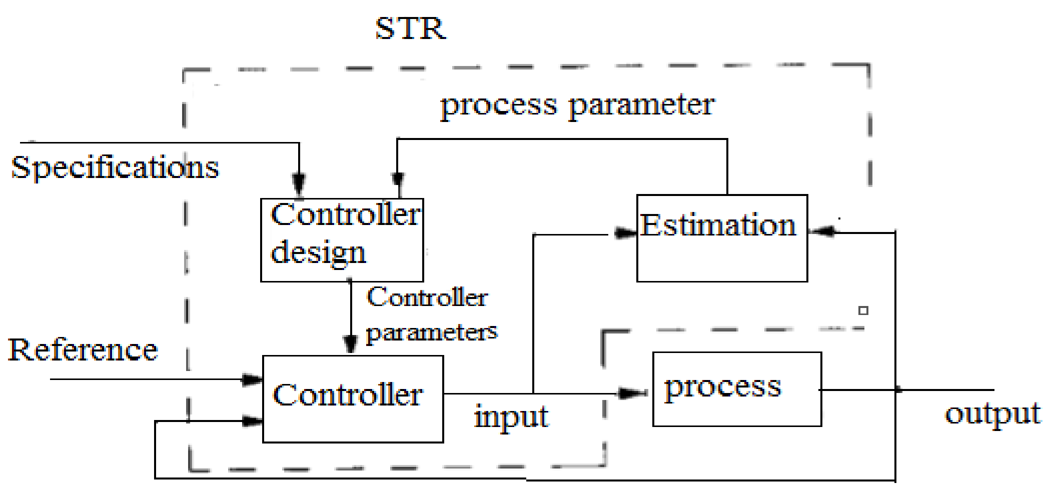


Figure 2. 9: Block Diagram of Self-tuning Regulator

Controller design is the component of the STR that characterizes on-line solution to a design problem for a system with known parameters. The controller parameters are updated indirectly via the design calculations in the self-tuner. The main problem of STR is it is only design for constant known parameters.

2.6.3 Dual Controller

The branch of control theory that deals with whose control system are initially unknown is called dual control theory. This scheme of adaptive system is difficult to design because it depends on dual control theory.

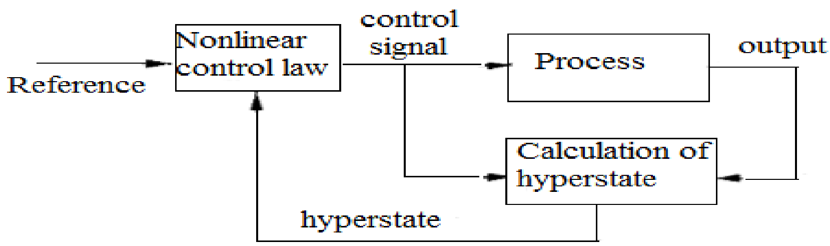


Figure 2. 10: Block diagram of Dual controller

2.6.4 Model Reference Adaptive Control

Model reference adaptive control (MRAC) was initially planned to answer a problem in which the performance specifications given in terms of model reference. This model tells how the output of the process ideally follow to the model reference output [6]. MRAC is one types of direct adaptive control architecture which is used to tune the controller in order to denote the closed loop dynamic model by an ideal model.

In this thesis, MRAC is design to control roll, pitch, yaw and altitude of the quadrotor UAV at hovering point. The purpose of the adaptation mechanism is to alter the controller parameters for the state quadrotor to follow reference model with undefined dynamic models of the physical parameters based on gradient method.

The overall structure of single input single output (SISO) system with direct type of adaptive control method (MRAC) will be summarizing as a shown in figure.2.11.

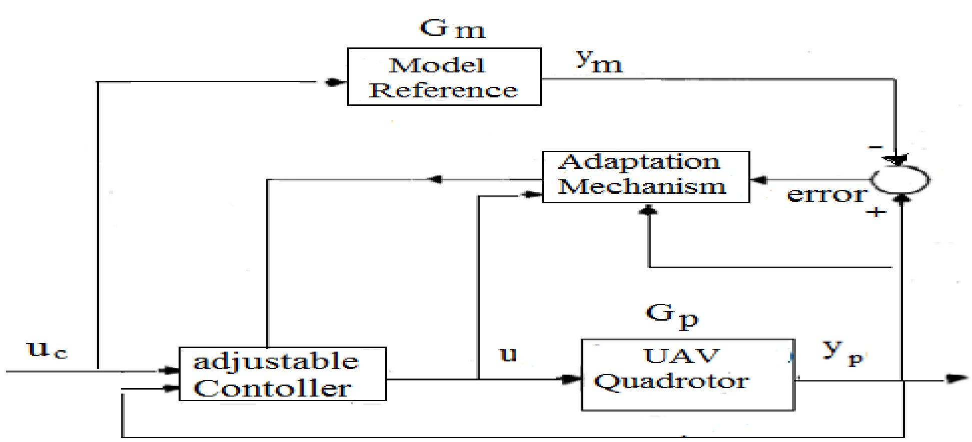


Figure 2. 11: Block diagram of model reference adaptive system

Model reference adaptive control system consists the following components.

a) **Model Reference**

It is used to give the ideal response of the adaptive control system to the reference input. This is chosen by the designer based on the desired performance. The choice of reference model is one part of the adaptive control system design and it must be filling the two necessities. The first condition is reflecting the transient performance specification of control tasks like overshoot (MP), settling time (ts), rising time (tr) steady state error (ess). Second condition, using adaptive control system the reference model should be achievable [6].

b) **Controller**

This component is defined by a set of adjustable parameters which are used to define the control law. The values of these parameter are depending on the adaptation gain (γ). In terms of the adjustable parameter the control law is linear because of design of adaptive controller requires linear parametrization to get adaptation mechanism with guaranteed stability and tracking convergence.

c) Adjustment Mechanism

This component is used to modify the parameters in the control law; so that the adaptation law pursuits the parameters such that the response of the plant could track the response of model reference.

In this thesis we have chosen to design model reference adaptive controller because it can be designed for the unknown and unpredictable variable parameters and uncertainty , so that MRAC is better than the other adaptive control schemes.

CHAPTER THREE

3 SYSTEM MODELING

For the designing of the proper feedback controller, it is obligatory to develop the model of the 6 DOF quadrotor UAV. In this thesis modeling of the system consist three main blocks as a shown in figure 3.1. These blocks are:

- Body dynamics block quadrotor UAV
- Rotor velocity to control input conversion block
- Motor dynamics block

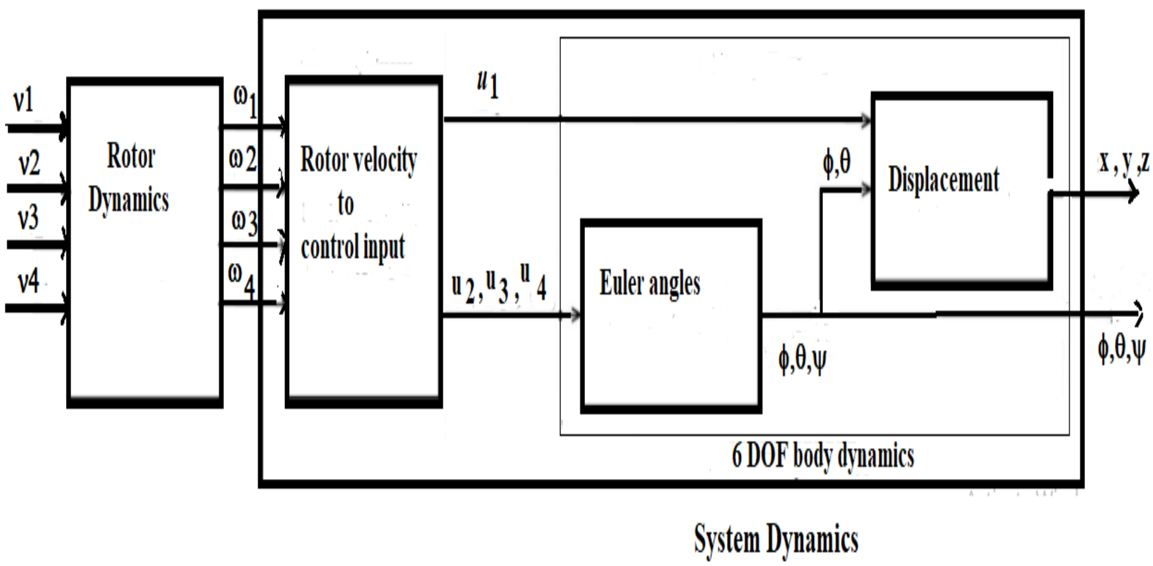


Figure 3. 1: Block diagram of open loop input output quadrotor system

To design a model reference adaptive controller (MRAC) for controlling of a quadrotor UAV system, the dynamic model of quadrotor must be adequately understood. Using the Newton-Euler law, the nonlinear differential equations that describes the motion of the quadrotor are derived.

For driving of the mathematical model of the quadrotor using Newton-Euler, some important assumptions are considered in order to accommodate design of MRAC. The assumptions are as follow:

- The quad rotor structure is supposed to be symmetrical and rigid body.
- The propellers are rigid i.e. propeller flapping does not happen.
- The center of mass does not coincide with the origin of the body fixed frame.
- Aerodynamic forces and moments are proportional to the rotor's speed.
- the Cross product of inertia matrix since x-z and y-z plane are symmetrical is neglected
- All motors and propellers are identical.
- Flat earth approximation and non-earth rotating to valid the operating work space is small and short duration of flight.
- Small angle approximation is used since the quadrotor is maneuvering near the hovering position.
- the effects of wind including ground and wall effects due to reflected wind from the spinning of the propeller are neglected.

3.1 Kinematics Model

Kinematics modeling the vehicle begins with defining of the reference frames and obtaining of the transformation matrices using cosine direction matrices (CDM).

a. Reference Frames

Here, we applied two types of reference frames that are used to describe the position and orientation of a 6-DOF the quadrotor. Reference frames are also used to explain the rotation matrix that is used to map any vector between these frames. The two reference frames are called earth and body reference frames as shown in figure 3.2. The Earth reference frame (E-frame) is fixed to the earth while body reference frame is the movable reference frame system which is attached to the quadrotor at its center of gravity (COG). As a shown in figure 3.2, the earth reference frame is denoted by (O_E, X_E, Y_E and Z_E). Where (O_E) is origin of the axis.

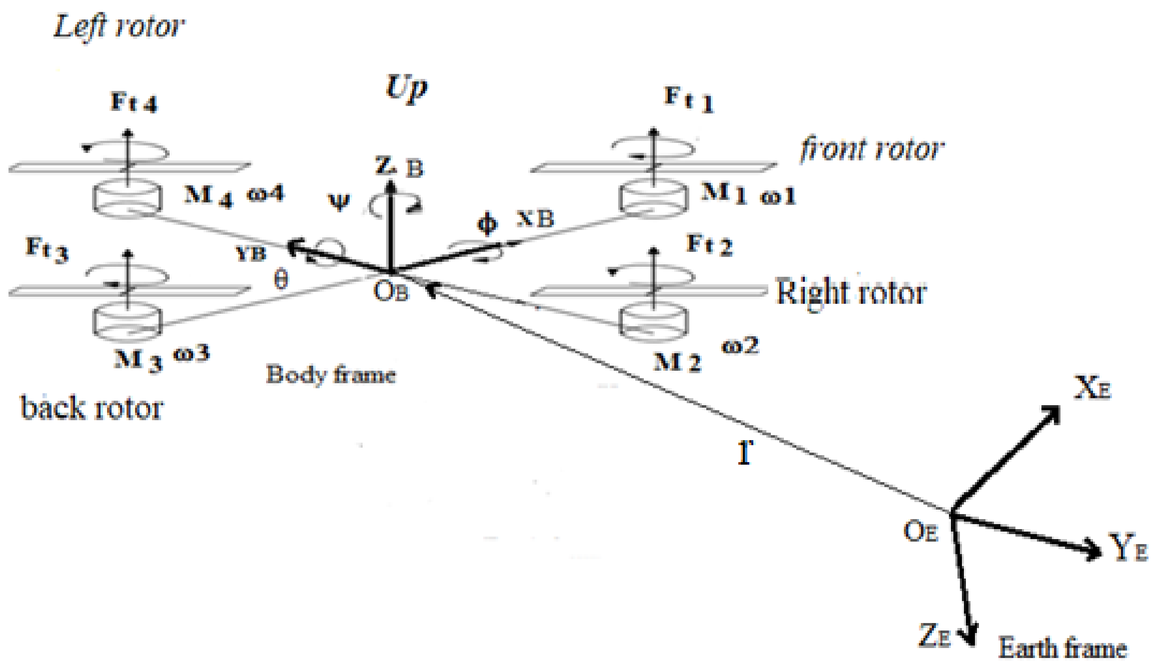


Figure 3. 2: Earth reference frame and body fixed reference frame

Using the E-frame, the translational position (α^E) in [m] and Euler angles (η^E) in [rad]) of the quadrotor are defined as:

$$\alpha^E = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \qquad \eta^E = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} \tag{3.1}$$

where x, y and z are the position of center of gravity E-frame and ϕ, θ and ψ the x-y-z Euler angles represent the roll, pitch and yaw in E-frame respectively.

The body-frame is right hand reference frame denoted by (O_B, X_B, Y_B and Z_B), that is attached to the body of the quadrotor. Where (O_B) is origin which has been chosen to coincide with center of the quad rotors cross structure, (X_B) points towards of the front, (Y_B) points towards right of the quadrotor and (Z_B) points upward of the quadrotor.

Using B-frame, the translational velocity (V_t^B [ms^{-1}]) and angular velocity (ω^B [rad s^{-1}]), the torque (τ^B [Nm]) and the force ($\mathcal{F}^B$ [N]) are defined as;

$$V_t^B = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

(3.2)

$$w^B = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

Where the symbols of u, v and w are the linear velocity of the center of gravity and *p*, *q* and *r* are the angular velocity of the quadrotor around x, y and z axis in B- frame respectively.

The Euler angles (η^E) are the attitude of the quadrotor that are defined by the orientation of B – frame with respect to E- frame. Here, some the physical properties of the quadrotor system are measured in terms E-frame while other properties are measured in B- frame. Therefore, Transformation of one frame to another frame is necessary.

b. **Rotation of Matrices (R)**

As defined before, two reference frames (E- frame and B- frame) will be used in the derivation of kinematics model. To find the components of a vector in both the frames, the rotation of matrices has to be first formulated. By making of a successive rotation around the corresponding axis's, reference frames can be move from one form to another form. Let R is the rotation of matrix that is obtained by using cosine direction matrices (CDM) from the product of the three planar matrices written in [3.3].

$$R_X(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -sin\phi \\ 0 & sin\phi & cos\phi \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ -sin\theta & 0 & cos\theta \end{bmatrix}$$

(3.3)

$$R_z(\psi) = \begin{bmatrix} cos\psi & -sin\psi & 0 \\ sin\psi & cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The symbols of $R_x(\phi)$, $R_y(\theta)$ and $R_z(\psi)$ are the planar rotation matrices.

In this thesis, we are used the Z-Y-X convention to move the body frame to earth frame as;

$$R = R_{zyx}(\phi, \theta, \psi) = R(\psi).R(\theta).R(\phi) \tag{3.4}$$

$$R = \begin{bmatrix} cos\psi & -sin\psi & 0 \\ sin\psi & cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ -sin\theta & 0 & cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\phi & -sin\phi \\ 0 & sin\phi & cos\phi \end{bmatrix}$$

$$= \begin{bmatrix} cos\psi cos\theta & sin\phi sin\theta cos\psi - sin\psi cos\theta & cos\phi sin\theta cos\psi + sin\psi sin\phi \\ sin\psi cos\theta & sin\phi sin\theta sin\psi + cos\psi cos\theta & cos\phi sin\theta sin\psi - sin\phi cos\psi \\ -sin\theta & sin\phi cos\theta & cos\phi cos\theta \end{bmatrix}$$

3.1.1 Translational Kinematics Model of Quadrotor System

The first point of studying the translational kinematics model of the quadrotor system derives location to get translational velocity in E-frame by applying rotation matrix (R). The translational kinematics modeling of the quadrotor system to get the velocity is written as:

$$\dot{\alpha}^E = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = Rv_t^B = R \begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{3.5}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} u(c\psi c\theta) + v(s\phi s\theta c\psi - s\psi c\theta) + w(c\phi s\theta c\psi + s\psi s\phi) \\ u(s\psi c\theta) + v(s\phi s\theta s\psi + c\psi c\theta) + w(c\phi s\theta s\psi - s\phi c\psi) \\ -u(s\theta) + v(s\phi c\theta) + w(c\phi c\theta) \end{bmatrix} \tag{3.6}$$

Equation (3.6) is called navigation (translational kinematics) equation of quadrotor system. where c and s, are the cos and sin respectively.

3.1.2 Rotational Kinematics Modeling of Quadrotor System

In this part studying, rotational kinematics modeling of quadrotor system the developing of the angular velocity is the primary phase in order to get the rate changes of the orientation angles. Using the transformation matrix (T), the angular velocity in the body frame is transformed in to the Euler rates as follow [2].

$$w^B = R_\phi R_\theta \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} + R_\phi \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} \tag{3.7a}$$

From (3.7a), we get the transformation matrix and its inverse as (3.7b) and (3,7c) respectively.

$$T = \begin{bmatrix} 1 & 0 & -s\phi \\ 0 & c\phi & c\theta s\psi \\ 0 & -s\phi & c\theta c\psi \end{bmatrix} \quad (3.7b)$$

$$T^{-1} = \begin{bmatrix} 1 & s\phi\, t\theta & c\phi t\theta \\ 0 & c\phi & -s\phi \\ 0 & \frac{s\phi}{c\theta} & \frac{c\phi}{c\theta} \end{bmatrix} \quad (3.7c)$$

$$\dot{\eta}^E = T^{-1} * w^B \quad (3.7d)$$

Using (3.7d), we get rotational kinematics equation of the quadrotor system as (3.8)

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} p + t\theta(qs\phi + rc\phi) \\ qc\phi - rs\phi \\ \frac{q*s\phi + rc\phi}{c\theta} \end{bmatrix} \quad (3.8)$$

By solving equation (3.5) and (3.7d) the kinematic (translational and rotational) model of quadrotor system is represented by equation (3.9).

$$\begin{aligned} \dot{x} &= u * c\psi * c\theta + v(s\phi * s\theta * c\psi - s\psi * c\theta) + w(c\phi * s\theta * c\psi + s\psi * s\phi) \\ \dot{y} &= u * s\psi * c\theta + v(s\phi * s\theta * s\psi + c\psi * c\theta) + w(c\phi * s\theta * s\psi - s\phi * c\psi) \\ \dot{z} &= -u * s\theta + v * s\phi * c\theta + w * c\phi * c\theta \\ \dot{\phi} &= p + t\theta * (q * s\phi + r * c\phi) \\ \dot{\theta} &= q * c\phi - r * s\phi \\ \dot{\psi} &= \frac{q * s\phi + r * c\phi}{c\theta} \end{aligned} \quad (3.9)$$

3.2 Dynamic Model of Quadrotor System

As a shown in figure 3.3, the rigid body motion of a quadrotor can be divided in to two subsystems namely; translational subsystem and rotational subsystem.

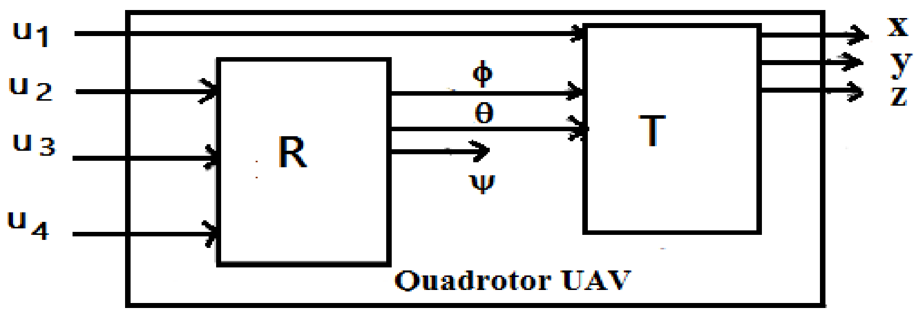


Figure 3. 3: Connection of rotational and translational subsystems of the quadrotor system

To develop the dynamic mathematical model the quadrotor, from the Euler's first and second axioms, the Newton's second law of motion must employ. The main aim this model is to develop the equations of the rate change of the angular velocity of rigid body quadrotor system.

3.2.1 Translational Dynamic model of quadrotor system

Using the Euler's first axioms of the Newton's second law

$$F^{B}_{net} = m\frac{d}{dt}{v_t}^{B} + w^{B}\ X\ (m{v_t}^{B}) \tag{3.10}$$

where X = cross product, F^{B}_{net} is the total force acting on the body of the quadrotor

Type of Force

In this thesis we are focused on the three forces which are acting on the body of the quadrotor. These forces are: thrust force, gravitational force and aerodynamic drag forces.

a) Thrust force

This force is the resultant force generated by the all motors with respect B-frame on z-axis. The force of each motor is proportional to angular speed square.

$$F_i=\frac{1}{2}\rho A C_T r^2 \omega_i^2 \approx k\omega_i^2\ ,\ k = \frac{1}{2}\rho A C_T r^2 \tag{3.11}$$

Where ρ is the air density, r and A are the radius and cross-sectional area of the propeller, C_T is aerodynamic thrust coefficient and k is the trust factor and ω_i is the i_{th} rotor angular speed. Then the thrust force (F_t) is B-frame can be Witten as;

$$F_t^B = \sum F_{i=1,2,3,4} = \begin{bmatrix} 0 \\ 0 \\ F_1+F_2+F_3+F_4 \end{bmatrix} = k \begin{bmatrix} 0 \\ 0 \\ \omega_1^2+\omega_2^2+\omega_3^2+\omega_4^2 \end{bmatrix} \qquad (3.12)$$

The thrust force in the E-frame can be obtained using the rotation matrix as follow:

$$F_t^E = u_1^E = Ru_1^B = \begin{bmatrix} c\psi c\theta & s\phi s\theta c\psi - s\psi c\theta & c\phi s\theta c\psi + s\psi s\phi \\ s\psi c\theta & s\phi s\theta s\psi + c\psi c\theta & c\phi s\theta s\psi - s\phi c\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ u_1 \end{bmatrix} \qquad (3.13)$$

After we solving (3.13), we get (3.14)

$$F_t^E = u_1^E = \begin{bmatrix} (c\phi s\theta c\psi + s\psi s\phi)u_1 \\ (c\phi s\theta s\psi - s\phi c\psi)\ u_1 \\ (c\phi c\theta)u_1 \end{bmatrix} \qquad (3.14)$$

Where u_1 is the control input of the system which is the thrust force i.e. $F_t = u_1$

b) **Gravitational force**

This force is generated by the gravitational acceleration acting along the z axis with respect to E-frame.

$$F_{g\ \text{E-frame}} = m \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix}, \qquad (3.15)$$

The gravitation force (F_g) in the B-frame can be Witten as;

$$F_{g\ \text{B-frame}} = R^T \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} = R^{-1} \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} = \begin{bmatrix} -mgs\theta \\ mgc\theta s\phi \\ mgc\theta c\phi \end{bmatrix} \qquad (3.16)$$

c) **Aerodynamics drag force**

In this thesis, the effect of aerodynamic drag is only considered for a translational dynamic. The aerodynamics drag force (F_{aed}) is proportional to the linear velocity as shown in (3.17).

$$F_{aed}{}^{E} = k_{dg}\, \dot{\alpha}^{E} = k_{dg} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} \qquad (3.17)$$

where the k_{dg} is the drag force constant diagonal matrix,

$$k_{dg} = \text{diag}\,[k_x\ k_y\ k_z]$$

The force F_{aed} can be also express in terms of the B-frame as;

$$F_{aed}{}^{B} = k_{dg}(v_t^B) = k_{dg} \begin{bmatrix} u \\ v \\ w \end{bmatrix} \qquad (3.18)$$

The forces that we considered are the gravitational force F_g and the thruster force F_t and aerodynamic force F_{aed} in the B- frame are written as;

$$F^{B}{}_{net} = F_t{}^{B} - F_g^B + F^{B}{}_{aed}$$

Then, from the Euler's first axiom Newton's law in (3.9), the rate change linear velocity of the quadrotor system is obtain as:

$$\dot{v}_t{}^{B} = \frac{F_{net}}{m} - (w^B \text{ X } v_t^B) \qquad (3.19)$$

$$\dot{v}_t{}^{B} = \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \frac{F_t{}^{B} - F_g{}^{B} + F_{aed}{}^{B}}{m} - (w^B \text{ X} v_t^B)$$

$$= \begin{bmatrix} 0 \\ 0 \\ \frac{u_1}{m} \end{bmatrix} + \begin{bmatrix} gs\theta \\ -gc\theta s\phi \\ -gc\theta c\phi \end{bmatrix} + \frac{1}{m} \begin{bmatrix} k_x u \\ k_y v \\ k_z w \end{bmatrix} + \begin{bmatrix} rv - qw \\ pw - ru \\ qu - pv \end{bmatrix}$$

After we solving (3.19), we get (3.19a)

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} gs\theta + \frac{k_x}{m}\ u + rv - qw \\ -gc\theta s\phi + \frac{k_y}{m} v + pw - ru \\ \frac{u_1}{m}(-gc\theta c\phi) + \frac{k_z}{m} w + qu - pv \end{bmatrix} \tag{3.19a}$$

The forces that we considered are the gravitational force F_g and the thruster force F_t and aerodynamic force F_{aed} in the E- frame are written as (3.19b).

$$F^{E}{}_{net} = F_t{}^{E} - F_g{}^{E} + F^{E}{}_{aed} \tag{319b}$$

Then (3.10) could be rewritten as:

$$\dot{v}_t{}^{E} = \frac{F^{E}{}_{net}}{m} - (w^B \times (v_t{}^B))$$

$$\dot{v}_t{}^{E} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \frac{F_t{}^E - F_g{}^E + F_{aed}{}^E}{m} - (w^B \times (v_t{}^B))$$

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} (c\phi s\theta c\psi + s\psi s\phi)\frac{u_1}{m} \\ (c\phi s\theta s\psi - s\phi c\psi)\frac{u_1}{m} \\ (c\phi c\theta)\frac{u_1}{m} \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} + \begin{bmatrix} \frac{k_x \dot{x}}{m} \\ \frac{k_y \dot{y}}{m} \\ \frac{k_z \dot{z}}{m} \end{bmatrix} + \begin{bmatrix} rv - qw \\ pw - ru \\ qu - pv \end{bmatrix} \tag{3.19c}$$

3.2.2 Rotational Dynamic Model of quadrotor system

By applying the Euler's second axioms of the Newton's law, the rotational equation of motion can be derived as follow:

$$M_{net} = I\frac{d}{dt}w^B + w^B \text{ X } (Iw^B) \tag{3.20}$$

Where I, is inertia matrix of the quadrotor, $w^B \in R^{3x1}$ are the body-fixed frame rotational velocities [p q r].

The inertia matrix of the quadrotor unmanned aerial vehicle is diagonal matrix and the off-diagonal elements are zero because the symmetry of the system.

$$I = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} = \text{diag}\,(I_{xx}, I_{yy}, I_{zz})$$

Torques

In this thesis we are considered two main sources of torques. The two sources of torques in the quadrotor system are torque of propellers and gyroscopic torques.

a) Torque of propellers

These torques are generated by the main movement control inputs which are proportional to the square of angular speed with respect of B- frame. Those types of torques can be mathematically expressed as;

$$\tau_{propeller} = \begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} = \begin{bmatrix} l(F_3 - F_1) \\ l(F - F_2) \\ d(F_1 - F_2 + F_3 - f_4) \end{bmatrix} = \begin{bmatrix} -l & 0 & l & 0 \\ 0 & -l & 0 & l \\ d & -d & d & -d \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} \quad (3.21)$$

where, l is the distance between center of the propellers and center of the quadrotor. Then, Equation (3.21) can be written in terms of angular speed of the rotors as:

$$T_{propler} = \begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} = \begin{bmatrix} lk(\omega_3^2 - \omega_1^2) \\ lk(\omega_4^2 - \omega_2^2) \\ d(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2) \end{bmatrix}$$

$$= \begin{bmatrix} -lk & 0 & lk & 0 \\ 0 & -lk & 0 & lk \\ d & -d & d & -d \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} \quad (3.22)$$

where $k>0$ and $d>0$ are two parameters of the force and torque constants respectively they depend on the air density, the geometry and lift and drag coefficients of the propeller.

b) Gyroscopic torque ($\tau_{gyroscopic}$)

The two propellers (M_1) and (M_3) are rotating clockwise while the remain two propellers ((M_2) and (M_4) are rotating counterclockwise. When the algebraic sum of speeds of the rotor is not equal to zero, an overall imbalance speed will be occurred. This imbalance speed will cause a gyroscopic effect proportional to the roll and pitch rates. The equation shown in (3.23) defines the overall propeller speed (ω_t) in [rad/s].

$$\omega_t = (\omega_1 + \omega_3)\text{-}(\omega_2 + \omega_4) \tag{3.23}$$

Therefor the gyroscopic torque τ_{gysc} can be mathematically model as:

$$\tau_{gysc} = \begin{bmatrix} \tau_{gysc\_x} \\ \tau_{gysc\_y} \\ \tau_{gysc\_z} \end{bmatrix} = \sum_{i=1}^{4} -J_p \left(w^B \text{ X } \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right) \omega_t = -J_r \left(w^B \text{ X } \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right) \omega_t \tag{3.24}$$

$$\begin{bmatrix} \tau_{gysc\_x} \\ \tau_{gysc\_y} \\ \tau_{gysc\_z} \end{bmatrix} = \begin{bmatrix} -J_r q \\ J_r p \\ 0 \end{bmatrix} \omega_t = \begin{bmatrix} -J_r q \;\; \omega_t \\ J_r p \;\; \omega_t \\ 0 \end{bmatrix}$$

$$M_{net} = M_{propler\ movment} + M_{gyroscopic\ movment}$$

$$\tau_{net} = \tau_{propler} + \tau_{gyroscopic} = \begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} + \begin{bmatrix} \tau_{gysc\_x} \\ \tau_{gysc\_y} \\ \tau_{gysc\_z} \end{bmatrix}$$

$$\tau_{net} = \begin{bmatrix} l(f_3 - f_1) \\ l(f_4 - f_2) \\ d\,(f_1 - f_2 + f_3 - f_4) \end{bmatrix} + \begin{bmatrix} -J_r q \;\; \omega_t \\ J_r p \;\; \omega_t \\ 0 \end{bmatrix}$$

Where J_p is the inertia of the i_{th} propellers and $\sum_{i=1}^{4} -J_p = J_r$ is the total rotational moment of inertia around the propeller's axis in [$N.m.s^2$].

From the rotational equation of motion Euler's second axiom and Newton's law, we get (3.25).

$$\dot{w}^B = I^{-1}(M_{net} - w^B \text{ X } (I w^B)) \tag{3.25}$$

$$\begin{bmatrix}\dot{p}\\\dot{q}\\\dot{r}\end{bmatrix}=\begin{bmatrix}I_{xx}^{-1} & 0 & 0\\0 & I_{yy}^{-1} & 0\\0 & 0 & I_{zz}^{-1}\end{bmatrix}\left(\begin{bmatrix}lu_2\\lu_3\\lu_4\end{bmatrix}+\begin{bmatrix}-J_r q\ \omega_t\\J_r p\ \omega_t\\0\end{bmatrix}-\begin{bmatrix}p\\q\\r\end{bmatrix}X\begin{bmatrix}I_{xx} & 0 & 0\\0 & I_{yy} & 0\\0 & 0 & I_{zz}\end{bmatrix}\begin{bmatrix}p\\q\\r\end{bmatrix}\right)$$

By solving (3.25), we get (3.26).

$$\begin{bmatrix}\dot{p}\\\dot{q}\\\dot{r}\end{bmatrix}=\begin{bmatrix}\frac{(I_{yy}-I_{zz})}{I_{xx}}rq+\frac{J_r q}{I_{xx}}\omega_t+\frac{lu_2}{I_{xx}}\\\frac{(I_{zz}-I_{xx}}{I_{yy}}pr+\frac{J_r p}{I_{yy}}\omega_t+\frac{lu_3}{I_{yy}}\\\frac{(I_{xx}-I_{yy})}{I_{zz}}pq+\frac{lu_4}{I_{zz}}\end{bmatrix} \quad (3.26)$$

Equation (3.26) is called the moment equation. Because of symmetry of the system i.e. $I_{xx}=I_{yy}$, we get (3.26a).

$$\dot{r}=\frac{lu_4}{I_{zz}} \quad (3.26a)$$

The general dynamic model (translational and rotational equation) of the quadrotor system is written in a state space form as follow;

$$\begin{aligned}\dot{u}&= gs\theta+\frac{k_x}{m}u+rv-qw\\\dot{v}&=-gc\theta s\phi+\frac{k_y}{m}v+pw-ru\\\dot{w}&=\frac{u_1}{m}(-gc\theta c\phi)+\frac{k_z}{m}w+qu-pv\\\dot{p}&=\frac{(I_{yy}-I_{zz})}{I_{xx}}rq+\frac{J_r q}{I_{xx}}\omega_t+\frac{u_2}{I_{xx}}\\\dot{q}&=\frac{(I_{zz}-I_{xx}}{I_{yy}}pr+\frac{J_r p}{I_{yy}}\omega_t+\frac{u_3}{I_{yy}}\\\dot{r}&=\frac{u_4}{I_{zz}}\end{aligned} \quad (3.27)$$

The kinematics and dynamic model of the quadrotor UAV system is represented by equation (3.19a) or (3.19c) and (3.27).

In this thesis, we follow the Z-Y-X convention to transform from B- frame to E-frame. The B-frame angular velocities (p q r) and rate change of the Euler angles ($\dot{\phi}$ $\dot{\theta}$ and $\dot{\psi}$) are related by considering unity transformation matrix (T) for small angle movement as;

$$[p\ q\ r] = [\dot{\phi}\ \ \dot{\theta}\ \ \dot{\psi}]$$

$$[\dot{p}\ \ \dot{q}\ \ \dot{r}] = [\ddot{\phi}\ \ \ddot{\theta}\ \ \ddot{\psi}] \tag{3.28}$$

Now the dynamic model (rotational) of the quadrotor UAV written in (3.26) can be rewrite in the E-frame as;

$$\begin{bmatrix}\ddot{\phi}\\ \ddot{\theta}\\ \ddot{\psi}\end{bmatrix} = \begin{bmatrix}\frac{(I_{yy}-I_{zz})}{I_{xx}}\dot{\theta}\dot{\psi} + \frac{-J_r\,\dot{\theta}\,\omega_t}{I_{xx}} + \frac{U_2}{I_{xx}}\\ \frac{(I_{zz}-I_{xx})}{I_{yy}}\dot{\phi}\dot{\psi} + \frac{J_r\,\dot{\phi}\,\omega_t}{I_{yy}} + \frac{U_3}{I_{yy}}\\ \frac{lU_4}{I_{zz}}\end{bmatrix} \tag{3.29}$$

Then using (3.28) the dynamic model (translational and rotational) of the quadrotor system in the E –frame is

$$\begin{aligned}
\ddot{\phi} &= \frac{(I_{yy}-I_{zz})}{I_{xx}}\dot{\theta}\dot{\psi} + \frac{-J_r\,\dot{\theta}\,\omega_t}{I_{xx}} + \frac{lU_2}{I_{xx}}\\
\ddot{\theta} &= \frac{(I_{zz}-I_{xx})}{I_{yy}}\dot{\phi}\dot{\psi} + \frac{J_r\,\dot{\phi}\,\omega_t}{I_{yy}} + \frac{lU_3}{I_{yy}}\\
\ddot{\psi} &= \frac{lU_4}{I_{zz}}\\
\ddot{x} &= \frac{u_1}{m}(c\phi s\theta c\psi + s\psi s\phi) + \frac{k_x\,\dot{x}}{m} + rv - qw\\
\ddot{y} &= \frac{u_1}{m}c\phi s\theta s\psi - s\phi c\psi) + \frac{k_y\,\dot{y}}{m} + pw - ru\\
\ddot{z} &= \frac{u_1}{m}(c\phi c\theta\,) + \frac{k_z\,\dot{z}}{m} + qu - pv - g
\end{aligned} \tag{3.30}$$

The dynamic model of the quadrotor describing in the Euler angles (roll, pitch and yaw) rotation consists three different terms. Those are:-

A). **Gyroscopic Effects Resulting from the Rigid Body Rotations**

1). **Rolling Moment**: The gyroscopic effect resulting from the rigid body rotation in the rolling moment is the term of $\frac{(I_{yy}-I_{zz})}{I_{xx}}\dot{\theta}\dot{\psi}$.

2). **Pitching moment:** The gyroscopic effect resulting from the rigid body rotation in the pitching moment is the term of $\frac{(I_{zz}-I_{xx})}{I_{yy}}\dot{\phi}\dot{\psi}$.

3). **Yawing moment:** There is no gyroscopic effect resulting from the rigid body rotation in the yawing moment because the x-z and y-z planes are symmetrical i.e.

$$I_{xx}=I_{yy}, \quad \frac{(I_{xx}-I_{yy})}{I_{zz}}\dot{\phi}\dot{\theta}=0$$

B). **Gyroscopic Effects Resulting of Rotation of Propeller Coupled with Body Rotations**

1). Rolling Moment

The gyroscopic effect resulting from the rotation of the propeller coupled with body rotation in this moment is represented by $\frac{J_r\dot{\theta}\omega_t}{I_{xx}}$.

2). **Pitching moment**

The gyroscopic effect resulting from the rotation of the propeller coupled with body rotation in pitching moment is represented by $\frac{J_r\dot{\phi}\omega_t}{I_{yy}}$.

3). Yawing moment

There is no gyroscopic effect resulting from the rotation of the propeller coupled with body rotation in yawing moment.

C). Actuators action:

1). **Rolling Moment:** The actuator action during the rolling moment is represented by

$$l(F_3-F_1)=\tau_\phi$$

2). **pitching moment:** The actuator action during the pitching moment is represented by

$$l(F_4-F_2)=\tau_\theta$$

3). **Yawing moment:** The actuator action during the yawing moment is represented by

$$d(F_3+F_1)-(F_2+F_4))=\tau_\psi$$

3.3 Control Inputs

The control inputs are the output of the actuators and the inputs to quadrotor UAV which are the forces and moments generated by the rotors. The control input vector (u_i) can be represented as follow;

$$\mathrm{u_i} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} F_t \\ \tau_\phi/l \\ \tau_\theta/l \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} F_1 + F_2 + F_3 + F_4 \\ l(F_3 - F_1) \\ l(F_4 - F_2) \\ d(F_1 + F_3) - (F_2 + F_4) \end{bmatrix}$$

$$= \begin{bmatrix} k & k & k & k \\ k & 0 & k & 0 \\ 0 & -k & 0 & k \\ d & -d & d & -d \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} \qquad (3.31)$$

Finally, in this thesis, the nonlinear dynamic model of the studied quadrotor system obtained in (3.30) would be written as:

$$\begin{aligned} \ddot{\phi} &= (a_1 x_6 - a_2 \omega_t) x_4 + b_1 u_2 \\ \ddot{\theta} &= (a_3 x_6 + a_4 \omega_t) x_2 + b_2 u_3 \\ \ddot{\psi} &= b_3 u_4 \\ \ddot{z} &= \frac{u_1}{m} (c x_1 c x_3) + (\frac{k_z}{m} + x_4) x_8 - x_2 x_{12} - g \\ \ddot{x} &= \frac{u_1}{m} (c x_1 s x_3 c\, x_5 + s x_1 s x_5) + \frac{k_x}{m} x_{10} + x_6 x_{12} - x_4 x_8 \\ \ddot{y} &= \frac{u_1}{m} (c x_1 s x_3 s x_5 - s x_1 c x_5) + (\frac{k_y}{m} - x_6) x_{12} + x_2 x_8 \end{aligned} \qquad (3.32)$$

where $\phi = x_1$, $\dot{\phi} = x_2$, $\theta = x_3$, $\dot{\theta} = x_4$, $\psi = x_5$, $\dot{\psi} = x_6$, $z = x_7$, $\dot{z} = x_8$, $x = x_9$, $\dot{x} = x_{10}$, $y = x_{11}$, $\dot{y} = x_{12}$, $a_1 = \frac{I_{yy} - I_{zz}}{I_{xx}}$, $a_2 = \frac{J_r}{I_{xx}}$, $b_1 = \frac{l}{I_{xx}}$, $a_3 = \frac{I_{xx} - I_{zz}}{I_{yy}}$, $a_4 = \frac{J_r}{I_{yy}}$, $b_2 = \frac{l}{I_{yy}}$,

$b_3 = \frac{1}{I_{zz}}$, $J_{xx} = J_{yy} = \frac{2 * M * r^2}{5} + 2 * l^{2} * m_M$, $J_{zz} = \frac{2 * M * r^2}{5} + 4 * l^{2} * m_M$

Mass of the quadrotor $(m) = 4 * m_M + M$

The equilibrium points of the nonlinear model are zero except, $u_{1e} = mg$

Where m_M total mass of the motor and propeller, M is mass of the rest part of the quadrotor, r and *l* are the radius of propellers and length of the arms respectively.

The nonlinear dynamic model of the rotational subsystem of the quadrotor written in (3.32) is represented in MATLAB Simulink as shown in figure 3.4.

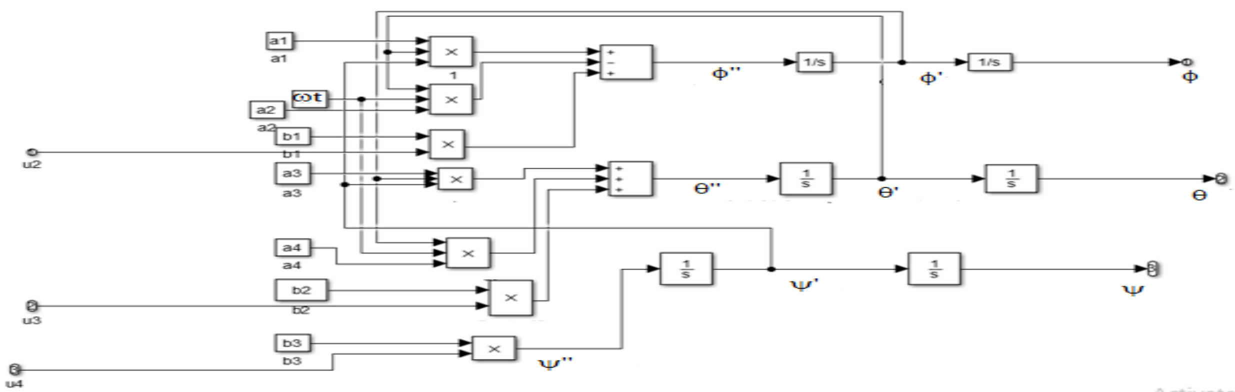


Figure 3. 4: Nonlinear dynamic rotational subsystem of the quadrotor in Simulink

The dynamic nonlinear modeling of the translational subsystem of the quadrotor in a Simulink is as a shown in figure 3.5.

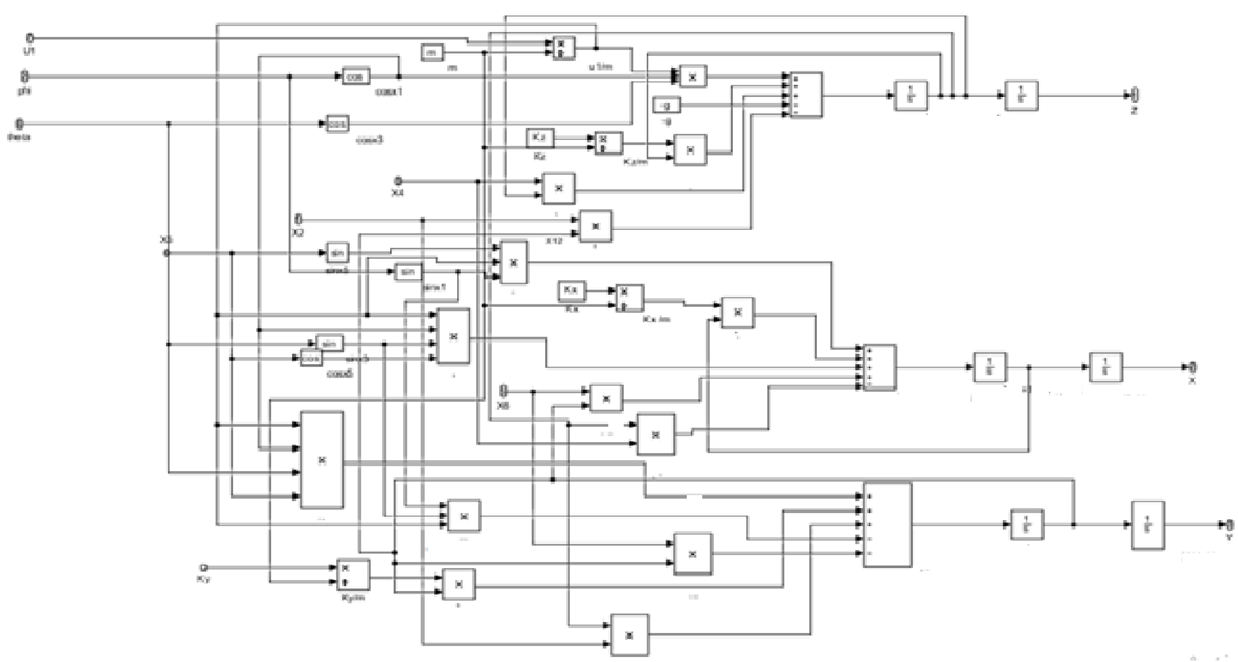


Figure 3. 5: Nonlinear dynamic translational sub system of the quadrotor Simulink

The connection of the two subsystems (rotational and translational) of the quadrotor using the MATLAB Simulink is represented as shown in figure 3.6.

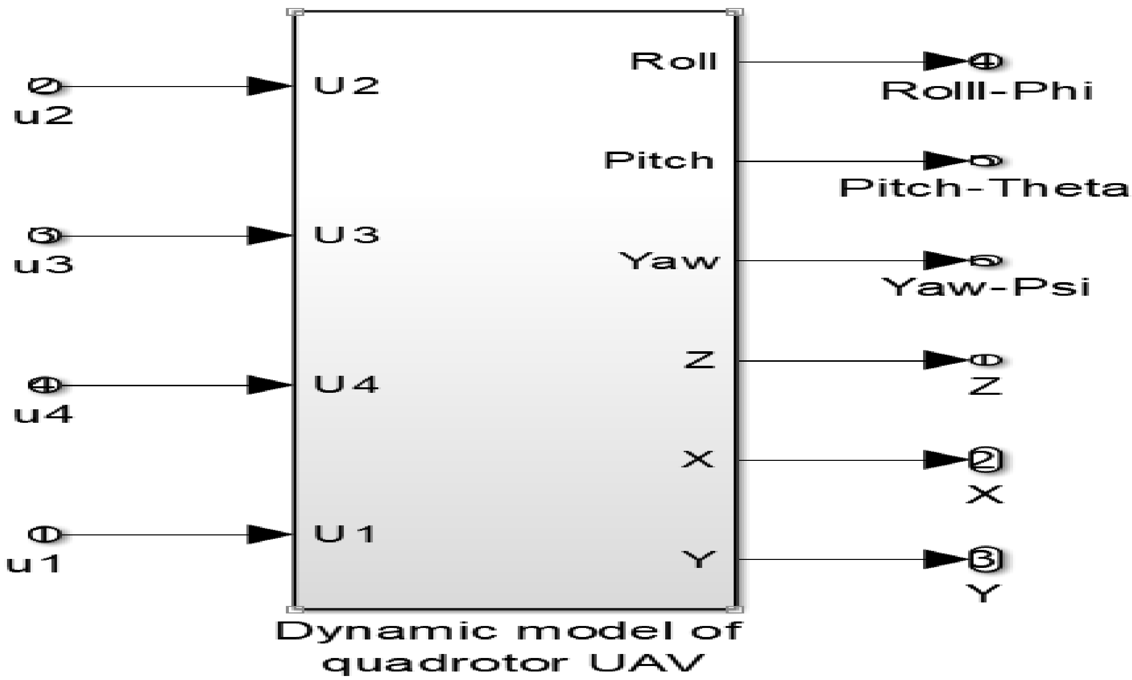


Figure 3. 6: nonlinear dynamic (rotational and translational) model of the quadrotor

3.4 Existence of limit cycle for nonlinear system

In a phase plane analysis of the nonlinear system, limit cycle is the isolated closed curve. The closed paths are specified the periodic landscape of the motion and isolation. Based on the motion shapes of the paths in the neighborhood of the limit cycle, one can separate in to three kinds of limit cycles [32]. These are; stable, unstable and semi-stable limit cycles.

3.4.1 Stable Limit Cycles

If all paths in the neighborhood of the limit cycle converges to it as $t \to \infty$ it is called stable limit cycle.

3.4.2 Unstable Limit Cycles

If all paths in the neighborhood of the limit cycle diverges from as $t \to \infty$ it is called unstable limit cycle.

3.4.3 Semi-stable Limit Cycles

The system is called semi-stable limit cycle if some of the lines in the neighborhood converge to it, while the others diverge from it as $t \to \infty$.

Generally, limit cycle is the most characteristics of nonlinear dynamic system that stand up in the quadrotor system with undefined delay of flight, friction and on-off actuator action. For altitude and attitude control of the quadrotor subsystems have the on-off actuator which converges to the stable limit cycle in steady state.

3.5 Linearization

In this thesis linearization of the nonlinear dynamic models of the quadrotor system is required for designing proper MRAC. The nonlinear dynamic equation of the quadrotor system defined in (3.32) will be linearize using the Jacobian method at the operating point. The linear dynamic equation of the quadrotor system is represented as:

$$\begin{aligned} \dot{X} &= Ax + Bu \\ y &= cx \end{aligned} \tag{3.33}$$

The Jacobian method of linearization works by performing implicit partial derivative of (3.32) with respect to the state vectors and input vectors evaluated at the equilibrium points as follow;

$$(A_{Xie}) = \frac{\partial F(x,u)}{\partial x} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \tag{3.33a}$$

$$(\mathrm{B_{ie}}, \mathrm{U_{ie}}) = \frac{\partial \mathrm{F(x,u)}}{\partial \mathrm{u}} = \begin{bmatrix} \frac{\partial \mathrm{f_1}}{\partial \mathrm{u_1}} & \cdots & \frac{\partial \mathrm{f_1}}{\partial \mathrm{u_n}} \\ \vdots & \ddots & \vdots \\ \frac{\partial \mathrm{f_n}}{\partial \mathrm{u_1}} & \cdots & \frac{\partial \mathrm{f_n}}{\partial \mathrm{u_m}} \end{bmatrix} \tag{3.33b}$$

where n is the number of state variables, matrix A is nxn, m the number of inputs to the quadrotor and matrix, B is nxm.

For the linearization the equilibrium state is chosen as;

$$\dot{x}_{ie} = 0 \tag{3.34}$$

In this thesis, we are going to control the attitude and altitude of the quadrotor system at a nominal hovering point and also, we are used small angle approximation technique to simplification of trigonometric function as;

$$cosx_1 = cosx_3 = 1\,,\ sinx_1 = x_1,\ sinx_3 = x_3,\ sinx_5 = x_5 \text{ and } cosx_5 = cosx_5 \tag{3.35}$$

The nonlinear dynamic equation of (3.32) the quadrotor system is rewritten as (3.36).

$$\begin{aligned}
\ddot{\phi} &= b_1 u_2 \\
\ddot{\theta} &= b_2 u_3 \\
\ddot{\psi} &= b_3 u_4 \\
\ddot{z} &= \frac{u_1}{m} - g \\
\ddot{x} &= \frac{u_1}{m}(x_3 * c\,x_5 + x_1 * sx_5) \\
\ddot{y} &= \frac{u_1}{m}(x_3 * sx_5 - x_{1*}cx_5)
\end{aligned} \tag{3.36}$$

From equation (3.36), x_5 can be obtain as:

$$x_3 * cx_5 + x_1 * sx_5 = 0 \tag{3.36 a}$$

$$x_3 * sx_5 - x_1 * cx_5 = 0 \tag{3.36b}$$

Then, from equation (3.36b), we obtain that,

$$cx_5 = \frac{x_3}{x_1} * sx_5 \tag{3.36c}$$

After we substitute (3.36c) into (3.36a), we get

$$x_3^2 * sx_5 + x_1^2 * sx_5 = 0 \tag{3.36d}$$

Therefore, from (3.36d), we get, $x_5 = 0$, then, at hovering point ($\omega_t = 0$), there is no imbalance of speed in the rotors. This implies that there are no gyroscopic effects on body quadrotor system; so that second derivative of x and y are neglected because the vehicle hovers at the hovering point.

Finally, equation (3.36) is rewritten as:

$$\begin{aligned} \ddot{\phi} &= b_1 u_2 \\ \ddot{\theta} &= b_2 u_3 \\ \ddot{\psi} &= b_3 u_4 \\ \ddot{z} &= \frac{u_1}{m} - g \end{aligned} \tag{3.37}$$

Now the dynamic model of the quadrotor system is linear and decoupled in to four SISO as a shown in (3.37). The equilibrium points of the quadrotor subsystems at hovering points are (0, 0, 0, mg).

From (3.37), the dynamic model of the vehicle consists 4 subsystems namely rolling subsystem, pitching subsystem, yaw subsystem and altitude subsystem.

3.5.1 Rolling subsystem

The open loop transfer function of this subsystem is a second order unstable system because of missing order one and zero on the denominator part. The transfer function of this subsystem is expressed as:

$$\frac{\phi(s)}{u_2(s)} = \frac{b_1}{s^2} \tag{3.38a}$$

The output of the rolling subsystem is

$$\phi(s) = \frac{b_1}{s^2} u_2(s) \tag{3.38b}$$

The MATLAB Simulink model of this subsystem is a shown in figure.3.7.

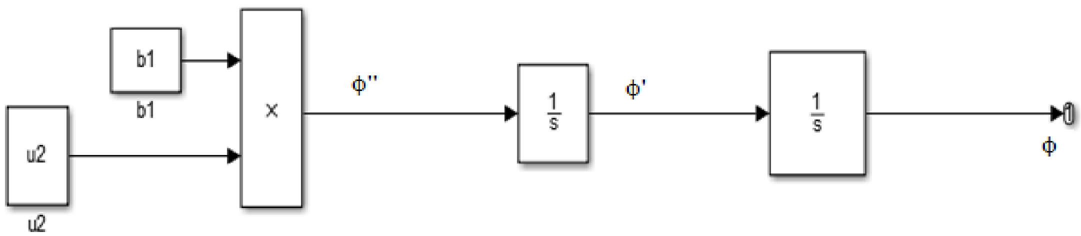


Figure 3. 7: Rolling Subsystem Simulink model

3.5.2 Pitching subsystem

The open loop transfer function of this subsystem is also a second order unstable similar to the rolling system. Then the transfer function of the pitching subsystem is;

$$\frac{\theta(s)}{u_3(s)} = \frac{b_2}{s^2} \tag{3.38c}$$

The open loop response of this system is

$$\theta(s) = \frac{b_2}{s^2} u_3(s) \tag{3.38d}$$

The MATLAB Simulink model of the pitch subsystem is a shown in figure 3.8.

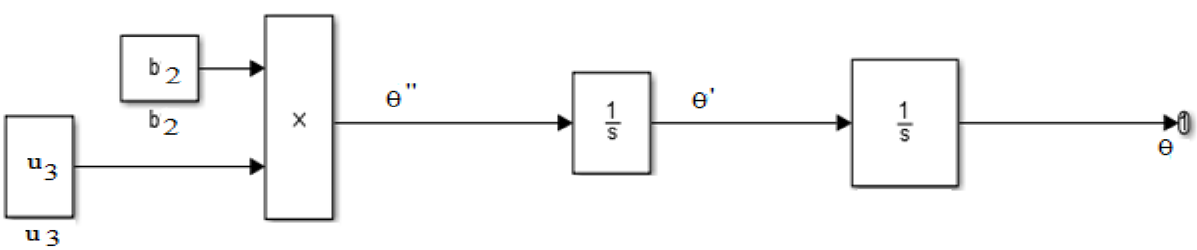


Figure 3. 8: Pitching Subsystem Simulink model

3.5.3 Yawing subsystem

The open loop transfer function of the yaw subsystem is;

$$\frac{\psi(s)}{u_4(s)} = \frac{b_3}{s^2} \tag{3.38e}$$

The output of the yaw subsystem is;

$$\psi(s) = \frac{b_3}{s^2} u_4(s) \tag{3.38f}$$

The MATLAB Simulink model of this subsystem is a shown in figure 3.9.

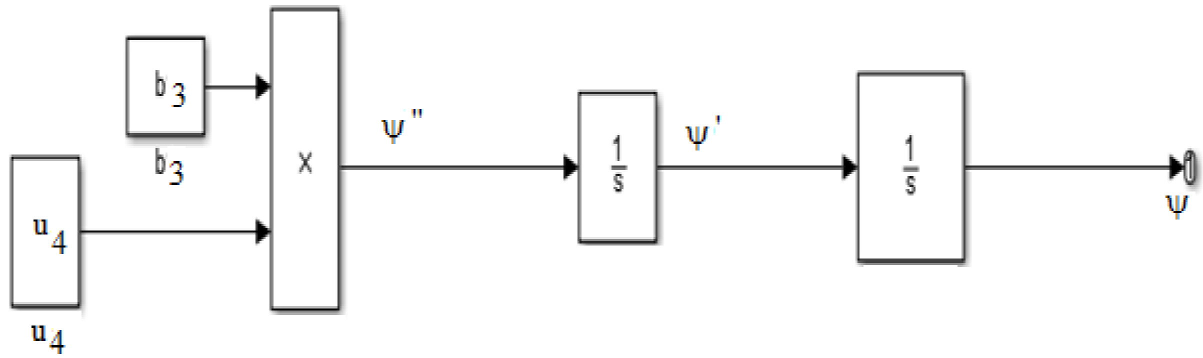


Figure 3. 9: Yawing Subsystem Simulink model

3.5.4 Altitude subsystem

The transfer function of the altitude subsystem is also second order and the MATLAB Simulink model of this subsystem is a shown in figure 3.10.

$$\frac{z(s)}{u_1(s)} = \frac{1}{ms^2} \tag{3.38g}$$

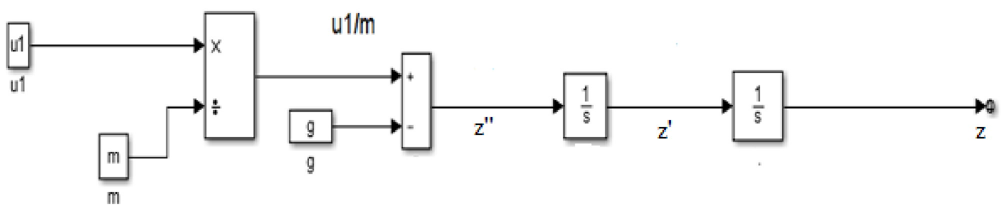


Figure 3. 10: Altitude Subsystem Simulink model

The stability of the linearized quadrotor system is checked from the eigen values of matrix A. The state space representation of roll, pitch, yaw and altitude subsystem are as:

$$\left.\begin{aligned}\dot{x}_1 &= x_{2r}\\ \dot{x}_2 &= b_1u_2\end{aligned}\right\}\text{rolling subsystem,} \qquad A = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix},$$

$$\left.\begin{aligned}\dot{x}_3 &= x_4\\ \dot{x}_4 &= b_2u_3\end{aligned}\right\}\text{pitching subsystem,} \qquad A=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix} \tag{3.39}$$

$$\left.\begin{aligned}\dot{x}_5 &= x_6\\ \dot{x}_6 &= b_3u_4\end{aligned}\right\}\text{yaw subsystem,} \qquad A= \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$$

$$\left.\begin{aligned}\dot{x}_7 &= x_6\\ \dot{x}_8 &= u_1 - g\end{aligned}\right\}\text{altitude subsystem,} \qquad A=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$$

Now matrix A and input matrix B, is equal for each subsystem; therefore, eigenvalues of these system also the same.

Eigen value of quadrotor system is obtained as;

$det(\lambda I - A) = 0$, then, $\lambda^2 + 1 = 0, \lambda = j1,$ (3.40)

This implies that eigen values of each quadrotor subsystem are pure imaginary, so that it is a center type or circle type stability, which means, the systems are oscillate around the equilibrium points, so that these subsystems are not asymptotic stable.

3.5 Rotor velocity to control input

This is obtained by taking the inverse of the control input of the quadrotor system which are written in equation (3.31) and we get;

$$\begin{aligned}\omega_1 &= \sqrt{\frac{u_1}{4kl} + \frac{u_3}{2kl} + \frac{u_4}{4d}}\\ \omega_2 &= \sqrt{\frac{u_1}{4kl} - \frac{u_3}{2kl} + \frac{u_4}{4d}}\\ \omega_3 &= \sqrt{\frac{u_1}{4kl} + \frac{u_2}{2kl} - \frac{u_4}{4d}}\end{aligned} \tag{3.41}$$

$$\omega_4 = \sqrt{\frac{u_1}{4kl} - \frac{u_2}{2kl} - \frac{u_4}{4d}}$$

The rotor velocity to the control inputs of the quadrotor systems using the MATLAB Simulink in the user defined function is expressed as a figure 3.13.

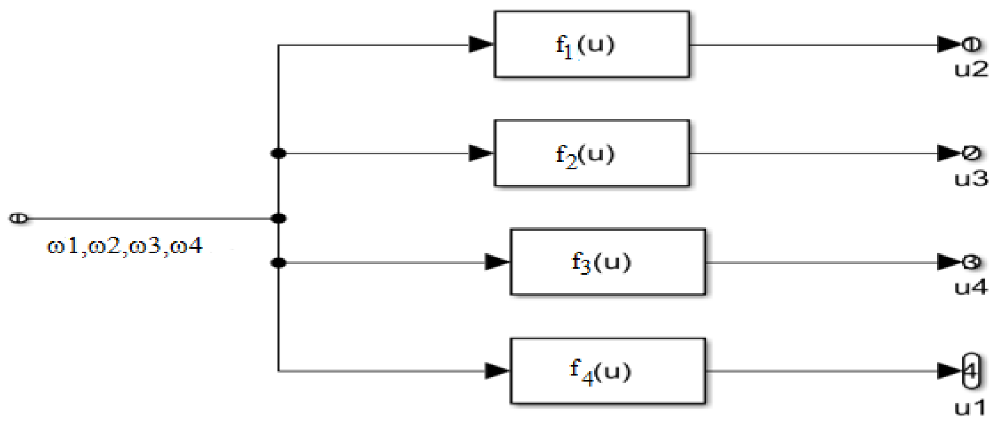


Figure 3. 11: Rotor Velocity to Control input block

3.6 Motor Dynamic Model

In this thesis, we use four identical BLDC motors for the quadrotor system that are provide a small friction and large torques. At the steady state, the dynamics model of brushless direct current motor is equal to the conventional DC motor as a shown in figure 3.14.

At steady state, the schematic circuit of brushless DC motor is represented by the DC motor circuit equivalent as a shown in figure 3.14.

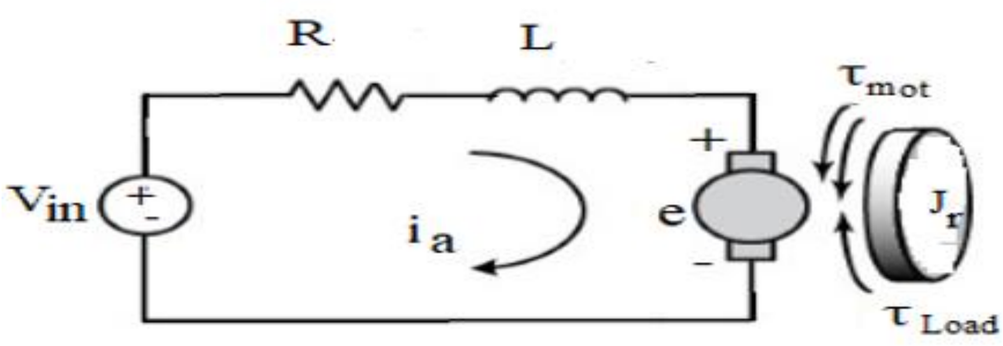


Figure 3. 12: Conventional DC motor

The mathematical modeling of the DC motor is divided in to two parts namely electrical system and mechanical system. The mathematical model of electrical system is done using the mesh analysis technique as;

$$v_i = R * i_a + L\frac{di_a}{dt} + e \tag{3.41}$$

$$v_i = i_a + L\frac{di_a}{dt} + k_m * \omega \tag{3.42}$$

where R is i_{th} motors resistance, L is motor inductance, i_a is armature current, v_i is the i_{th} input voltage, e is generated emf and the term of $k_m * \omega$ represents the e, with k_m is the constant torque of the motor, ω is the angular velocity of the motor.

The current is DC, the voltage drop across the inductance at steady state is zero. The inductance of quadrotor system is very small ; so that the inductance is neglected in the dynamic state.

$$v_i = R i_a + k_m \omega_i \tag{3.43}$$

The mathematical modeling of the mechanical part of the motor is done as;

$$J_r \dot{\omega} = \tau_{mot} - \tau_{load} \tag{3.44}$$

$$\tau_{mot} = K_e * i_a = k_m * i_a \tag{3.45}$$

where the τ_{mot} is torque produced by the motor, K_e is the motor's electric constant but for a small motor it approximately equal to the k_m and the τ_{load} is torque the load which is neglected, then the mechanical derivation part is rewrite as;

$$J_r \dot{\omega} = k_m * i_a \tag{3.46}$$

After we take the Laplace transform of equation (3.46), we get;

$$i_a(s) = \frac{J_r}{k_m} s\omega(s) \tag{3.47}$$

When we substitute (3.47) into (3.43) and we get,

$$v_i(s) = (k_1 s + k_m)\omega_i(s) \tag{3.48}$$

where $k_1 = \frac{J_r * R}{k_m}$, which is a constant gain.

Then, the dynamic modeling of the motor is first order lag transfer function as shown in (3.49).

$$\frac{\omega_i(s)}{v_i(s)} = \frac{1}{k_1 s + k_m} \tag{3.49}$$

CHAPTER FOUR

4 CONTROLLER DESIGN

4.1 Introduction

In this chapter, we have designing model reference adaptive control for each subsystems of the quadrotor. This controller was initially planned to overcome the problem with the performance specifications which are given in the form of model reference. This model tells process how the process output ideally should respond to th model reference [6].

The strategy of MRAC is used to design the adaptive controller that works on the principle of adjusting controller parameters; so that the output of each quadrotor subsystem follows output of model reference. The output of the system is compared to a desired response from reference model, then the control parameters are updated based on this error. The goal is to make the parameters to converge to ideal values that cause the plant response to match the response of the reference model.

To develop the adaptation mechanism, there are some mathematical techniques like gradient method, Lyapunov stability theory, hyper stability and passivity theory, augmented error and a model following MRAC. The first two methods are the most common methods of adaptation mechanism techniques.

In this thesis we use only the gradient method for the developing of adaptation mechanisms to control the quadrotor system.

4.1 Design of MRAC using Gradient method

Gradient method (MIT rule) was developed by the Instrumentation laboratory at Massachusetts Institute of Technology (MIT) which used to apply the MRAC method for any application. The name is derived from the laboratory’s name that is why we call it MIT rule [6]. During designing of MRAC using this mathematical technique, the designer choses the tuning gains for adjustment mechanism, structure of the controller and reference models.

In gradient based MRAC starts by defining the tracking error (e) as;

$$e = y_p - y_m = 0 \tag{4.1}$$

where y_p is output of the plant and y_m is model reference output.

In the gradient method, the cost function is defined as;

$$J(\theta_1, \theta_2) = \frac{1}{2}e^2 \tag{4.2}$$

In this rule, to minimize the cost function it is reasonable to adjust the parameters in the direction of negative gradient of J, that is

$$\frac{d\theta_i}{dt} = -\gamma \frac{\partial J}{\partial \theta_i} = -\gamma\, e \frac{\partial}{\partial \theta_i} e \tag{4.3}$$

Equation (4.3) is called gradient method or MIT rule.

The partial derivative $\frac{\partial}{\partial \theta_i}$ is the sensitivity derivative of the system which tells how error is influenced by the adjustable parameters θ_i.

The first step during designing of MRAC based on gradient method is choosing structure of the controller. The second step finding adaptation mechanism of the controller parameters i.e. how the controller parameters must adapt or update their values with system parameter changing, operating condition changing and when disturbance happened to bring back the system to its reference input.

In this thesis, we are selected a combination of feedback and feedforward controller type as follow;

$$u = \theta_1 u_c - \theta_2 y_p \tag{4.4}$$

where $\theta_1$and θ_2 are the parameters of the controller to be updated using MRAC, u_c is the reference input and u is the controller signal.

In MRAC, the controller parameters are updated as function of the error between the actual plant and model reference. So, the controller parameters are updated in terms of model reference (y_m), plant output (y_p), error ($y_p - y_m$).

In this thesis, the second order quadrotor subsystems are defined as;

$$\frac{d^2 y_p}{dt^2} = -a_1 \frac{d}{dt} y_p(t) - a_2 y_p(t) + bu \tag{4.5}$$

The reference model also defined as;

$$\frac{d^2 y_m}{dt^2} = -a_{1m} \frac{dy}{dt} y_m(t) - a_{2m} y_m(t) + b_m u_c \tag{4.6}$$

After collecting similar terms and taking the Laplace transformation with setting zero all initial conditions, the transfer function of the plant and the reference model are written in (4.7) and (4.8) respectively.

$$G_p = \frac{b}{s^2 + a_1 s + a_2} \tag{4.7}$$

$$G_m = \frac{b_m}{s^2 + a_{1m} s + a_{2m}} \tag{4.8}$$

where G_p and G_m are transfer function of the plant and reference model respectively.

Substituting equation (4.4) into (4.5), then the response of the plant is

$$y_p = \frac{b\theta_1}{s^2 + a_1 s + a_2 + b\theta_2} u_c \tag{4.9}$$

Similarly, the response of the reference model is

$$y_m = \frac{b_m}{s^2 + a_{1m} s + a_{2m}} u_c \tag{4.10}$$

Then the optimal values of the controller parameters are;

$$\theta_1 = \frac{b_m}{b}\ ,\ \theta_2 = \frac{a_{2m} - a_2}{b} \tag{4.11}$$

Now to apply gradient rule, the sensitivity derivatives are obtained by taking partial derivatives of error with respect the controller parameters to be updated θ_1 and θ_2. Introduce the error signal equation that is

$$e = y_p - y_m$$

$$e = \left(\frac{b\theta_1}{s^2+a_1 s+a_2+b\theta_2} - \frac{b_m}{s^2+a_{1m}s+a_{2m}}\right)u_c \tag{4.12}$$

Take the partial derivative of equation (4.12) with respect of θ_1, θ_2 and we get,

$$\frac{\partial}{\partial\theta_1} e = \frac{b}{s^2+a_1 s+a_2+b\theta_2} u_c \tag{4.13}$$

$$\frac{\partial}{\partial\theta_2} e = \frac{-b^2\,\theta_1}{(s^2+a_1 s+a_2+b\theta_2\,)^2} u_c \tag{4.14}$$

Equation (4.14) can be written as;

$$\frac{\partial}{\partial\theta_2} e = \frac{-b}{(s^2+a_1 s+a_2+b\theta_2)} * \frac{b\,\theta_1}{(s^2+a_1 s+a_2+b\theta_2)} u_c$$

$$= \frac{-b}{(s^2+a_1 s+a_2+b\theta_2)} y_p \tag{4.15}$$

The partial derivative of error signal with respect of θ is used to obtain how the parameter θ is updated.

Now the updated parameters are obtaining as;

$$\frac{d\theta_1}{dt} = -\gamma e \frac{\partial}{\partial\theta_1} e \tag{4.16}$$

$$= -\gamma * \left(\frac{b\theta_1}{s^2+a_1s+a_2+b\theta_2} - \frac{b_m}{s^2+a_{1m}s+a_{2m}}\right) * \frac{b}{s^2+a_1s+a_2+b\theta_2}u_c$$

$$\frac{d\theta_2}{dt} = -\gamma e\frac{\partial}{\partial\theta_2}\ e \tag{4.17}$$

$$= -\gamma * \left(\frac{b\theta_1}{s^2+a_1s+a_2+b\theta_2} - \frac{b_m}{s^2+a_{1m}s+a_{2m}}\right)u_c * \frac{-b}{(s^2+a_1s+a_2+b\theta_2}y_p \tag{4.18}$$

Then the updated parameters are;

$$\frac{d\theta_1}{dt} = -\gamma e\frac{b}{s^2+a_1s+a_2+b\theta_2}u_c \tag{4.19}$$

$$\frac{d\theta_2}{dt} = -\gamma e\frac{-b}{(s^2+a_1s+a_2+b\theta_2}y_p \tag{4.20}$$

Gradient method is the relationship of change of theta and cost function that is a central to adaptive nature of controller. In this thesis, we see how the general structure of MRAC based on gradient method can be used as adaptive controller, considering of each quadrotor subsystem including motor dynamic model with an adaptive feed forward gain is as shown in figure 4.1.

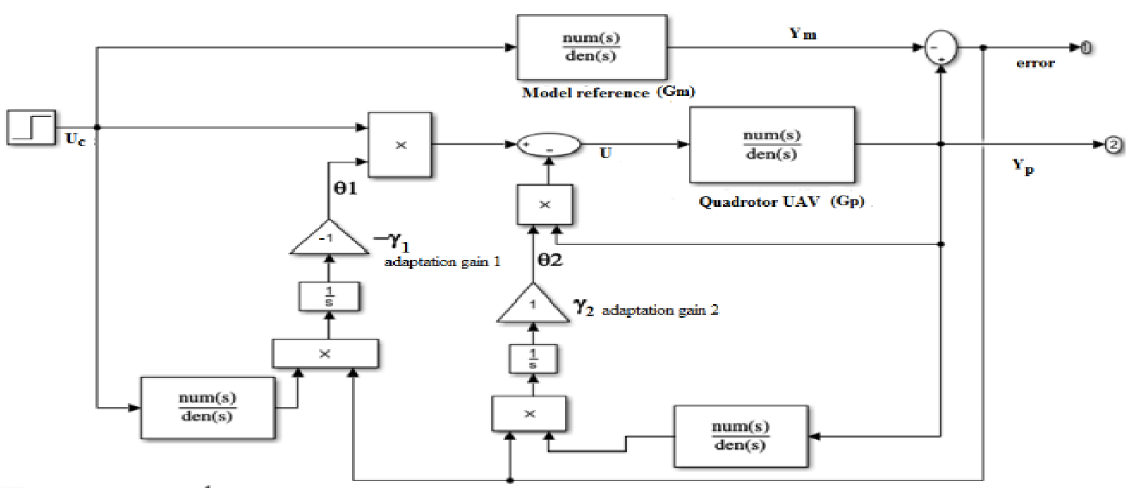


Figure 4. 1: General structure of MRAC based on the gradient method

CHAPTER FIVE

5 SIMULATION RESULTS AND DISCUSSION

In chapter three, mathematical modeling and linearization of the quadrotor UAV are discussed in detail. Similarly, in chapter four, we have designed four MRAC based on gradient method for the four SISO quadrotor subsystems. In this chapter simulation results of each subsystem will be discussed in brief without any controller and with MRAC are discussed.

A simulation result for each quadrotor subsystem is carried out whose design parameters are specified in Table 5.1, which are taken from the literature review of the quadrotor system parameters and constants [30].

Table 5. 1: quadrotor parameter's and constants

Parameters of quadrotor	Description	Value	Unit
Ixx	Inertia matrix of the B –frame around x –axis	$7.5e^{-3}$	$Kg.m^2$
Iyy	Inertia matrix of the B –frame around y –axis	$7.\ 5e^{-3}$	$Kg.m^2$
Izz	Inertia matrix of the B –frame around z –axis	$1.3e^{-5}$	$Kg.m^2$
L	Length of the arm	1	*M*
Jr	Rotor inertia	$3.13e^{-3}$	
M	Total mass of the quadrotor	0.58	Kg
K	Thrust factor	2	$N.s^2$
Kdg=Diag (Kx, Ky Kz)^T	Aerodynamic translational Diagonal matrix drags force constant	Diag (0.1,0.1, 0.15)^T	Ns/m
M	Gravitational force	9.8	m/s^2
R	Motor circuit resistance	0.6	**Ω**
Km	Motor torque constant	5.2	mNm/A

The specifications of transient performance characteristics of each subsystem are summarized in table 5.2.

Table 5.2: Specification for transient performances characteristics

Dynamic performance characteristics	Values
Maximum overshoot(%MP)	<0.20%
Settling time (ts)	<0.15 sec
Steady state error (ess)	0

5.1 Open Loop Response of Quadrotor subsystems

The approximated open loop transfer function of the roll, pitch, yaw, altitude and position along x and y axes including motor dynamics models of the quadrotor subsystems with the values of the design parameters are represented from equation (5.1) up to (5.4) respectively.

$$G_{\phi}(s) = \frac{365.42}{s^2+1368.4s} \quad (5.1)$$

$$G_{\theta}(s) = \frac{365.42}{s^2+1368.4s} \quad (5.2)$$

$$G_{\psi}(s) = \frac{18.97}{s^2+1368.45s} \quad (5.3)$$

$$G_{z}(s) = \frac{129.31}{s^2+1368.4s} \quad (5.4)$$

The open loop responses of four independent subsystems are shown in figure (5.1) up to (5.4).

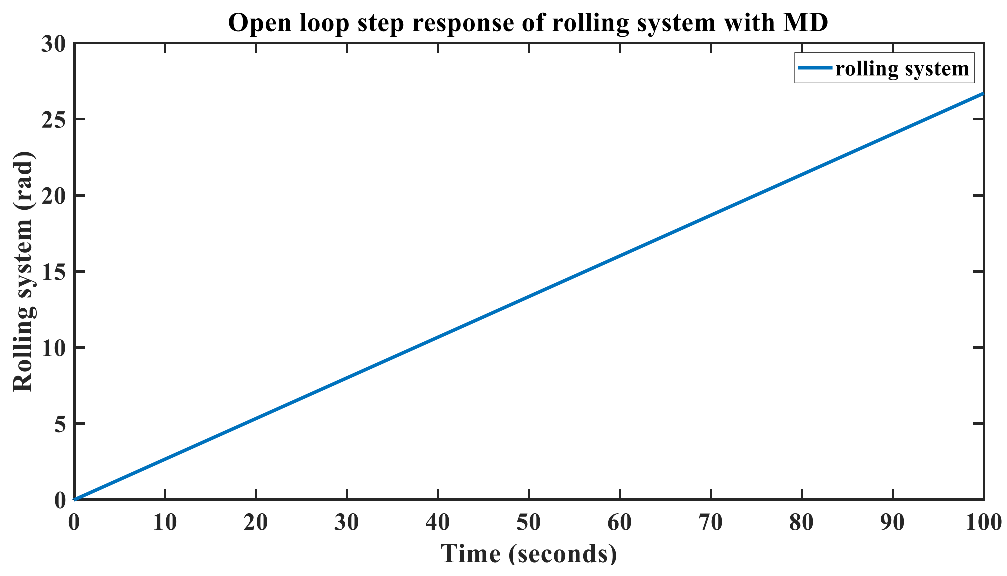


Figure 5. 1: Open loop unit step response of Rolling subsystem

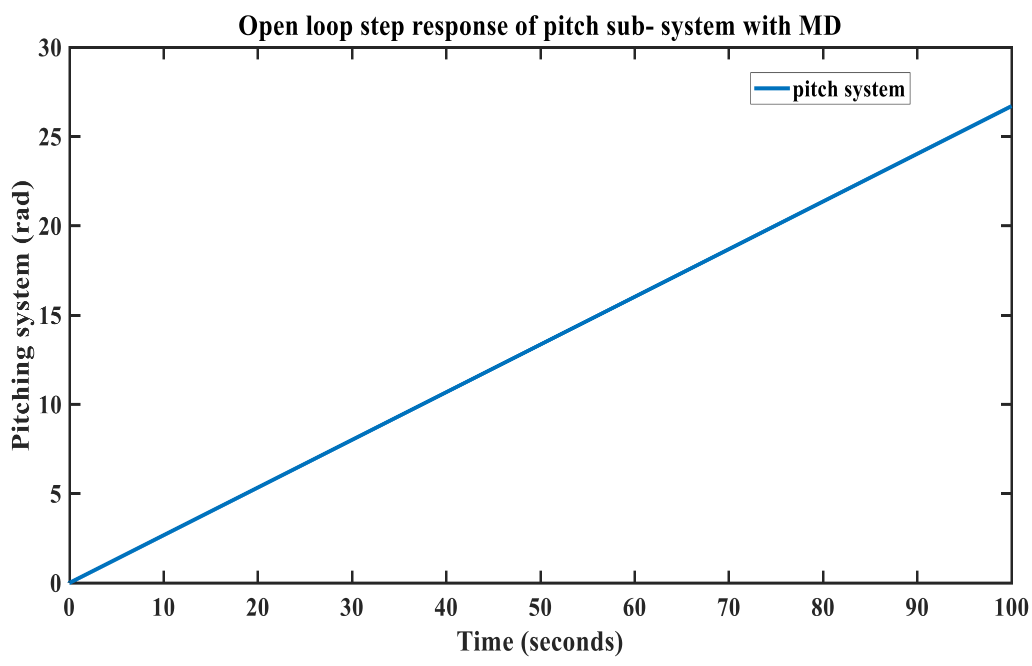


Figure 5. 2: Open loop unit step response of Pitching subsystem

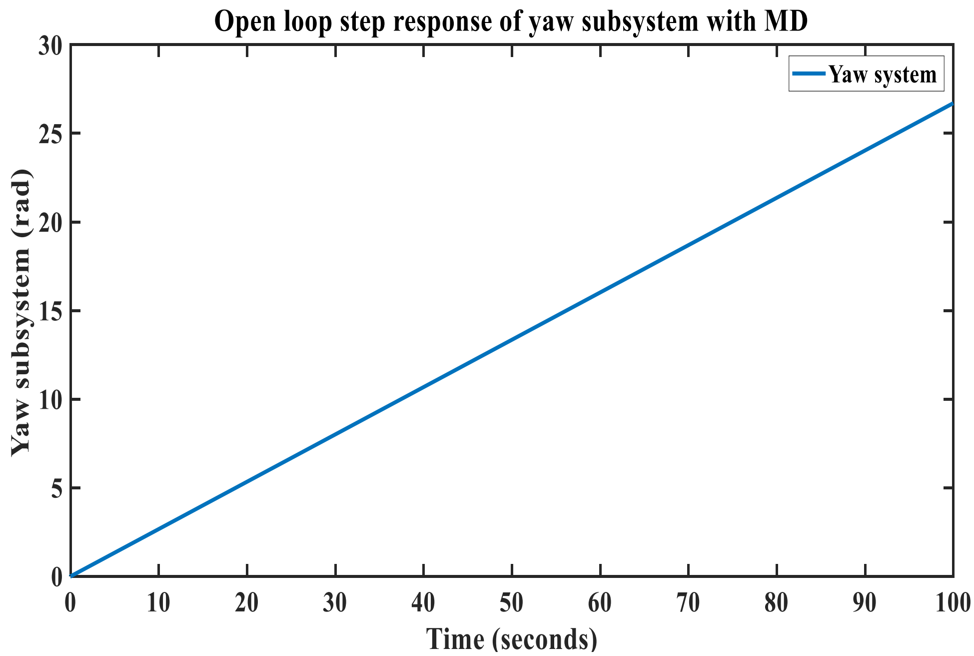


Figure 5. 3 :Open loop unit step response of Yaw subsystem

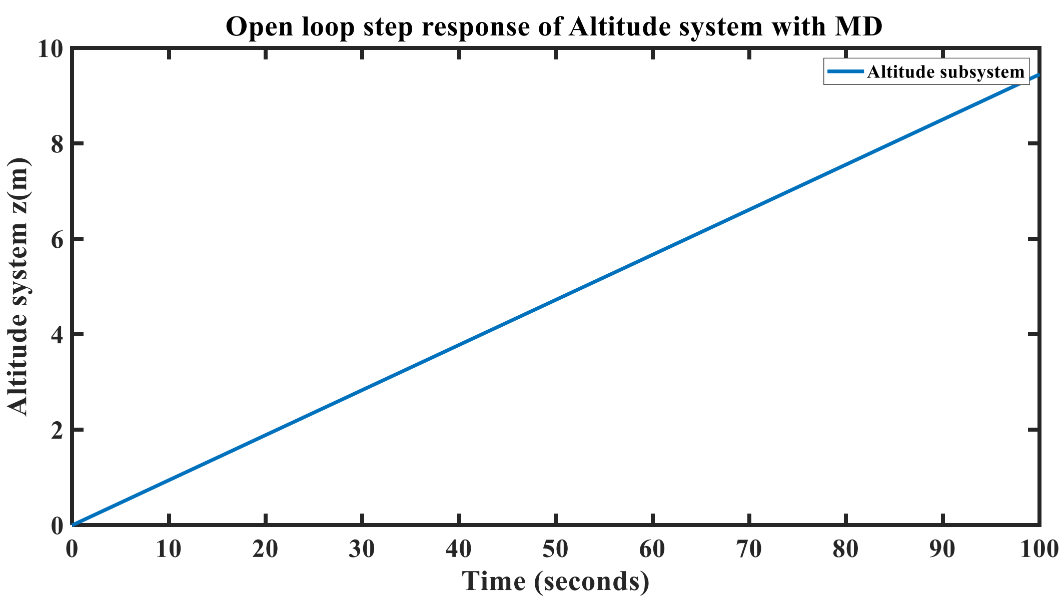


Figure 5. 4: Open loop unit step response of Altitude subsystem

The open loop unit step responses of four quadrotor subsystem are continuously increased. So that the open loop responses of these subsystem are not at the setpoint, unsettled and unstable. Therefore, it needs a closed loop system with controller to control the values at the desired value, settle with short time and to minimize the maximum overshoot.

5.2 Step Responses of Quadrotor System with MRAC

To overcome the simulation result problems of the open loop response of Quadrotor UAV subsystems, selection of model reference is important step in design of MRAC in which the plant is required to follow it. So, in order to have the plant good transient performance characteristic, reference model must have good characteristics. Considering this implication, the system given in (5.7) is taken as a model reference to stabilize each quadrotor subsystem by MRAC based on gradient method.

$$G_m = \frac{\omega_n^2}{s^2+2\sigma\omega_n s+\omega_n^2} = \frac{900}{s^2+54s+900} \quad (5.7)$$

Settling time is obtained as

$$t_s = \frac{4}{\sigma\omega_n} \quad \text{for } 2\% \text{ tolerance criteria}$$

$$t_s = \frac{3}{\sigma\omega_n} \quad \text{for } 5\% \text{ tolerance criteria}$$

In this thesis, we have considered the 2% tolerance criteria to calculate the settling time.

5.2.1 Step Response of SISO Rolling Subsystem with MRAC

The step response of rolling subsystem of the quadrotor UAV with the gradient based MRAC is shown in figure 5.7.

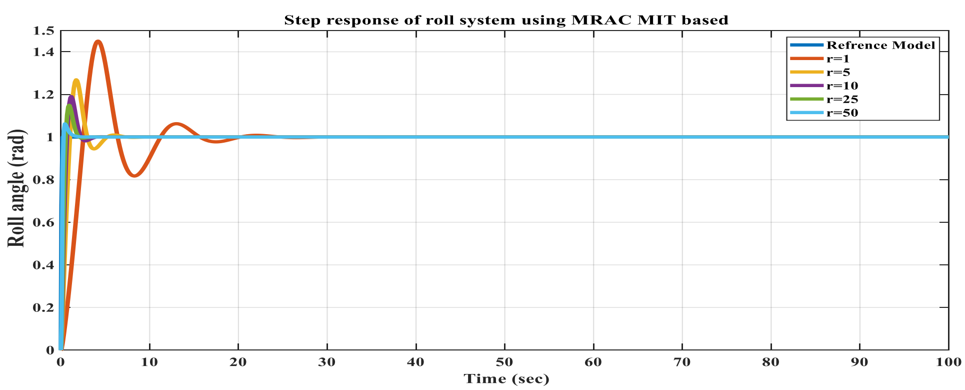


Figure 5. 5: Step response of Rolling system (ϕ(t)) using MRAC based on gradient method

Figure 5.5, Shows that the closed loop response of rolling subsystem and reference model under gradient based MRAC with different values of adaptation gain (γ). From this figure, the performance characteristics of the rolling system are different for different adaptation gain**.**

In this thesis, the range of adaptation gain is chosen from 1 up to 50 for the rolling subsystem to get satisfactory performance characteristics. From the simulation result when the γ is less than 10, the performance characteristics are not satisfactory, but with adaptation higher than 10 it gives acceptable performance with fast response. So, the response of rolling system is fast, settled and stable using the MRAC based on gradient method with range of $10 \leq \gamma \leq 50$, beyond of this range it is unacceptable because higher value of adaptation gain causes instability.

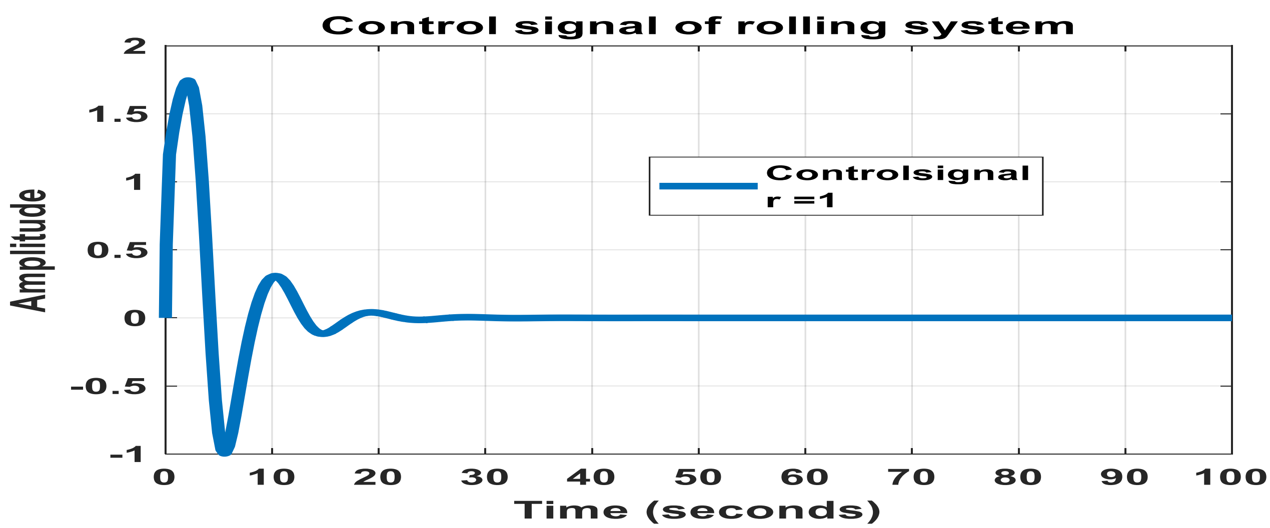


Figure 5. 6: Control signal for step change of Roll when γ = 1

The control signal for step change of rolling system under MRAC shown in figure 5.6, becomes zero after 30 seconds with unity adaptation gain. This implies that, the plant out is not exactly follows the reference model output that taking of control action is needed. From figure 5.7, the control signal for step change of rolling system with adaptation gain =10 becomes zero after one second. So, the plant response exactly follows the refence output. Therefore, there is no need of taking of control action.

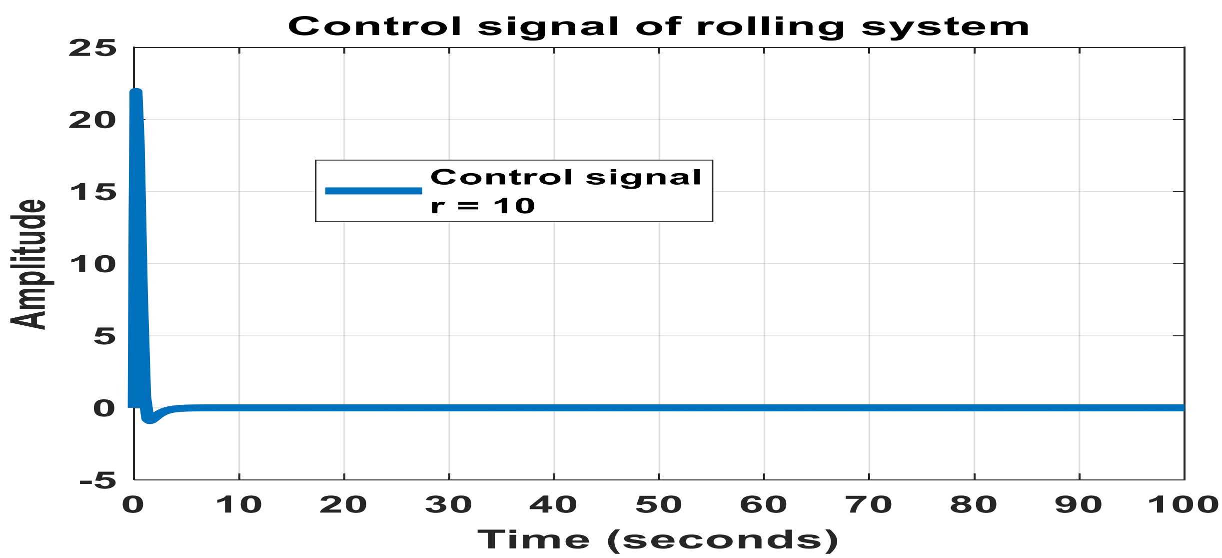


Figure 5. 7: Control signal for step change of Roll, when γ = 10

5.2.2 Step Response of SISO Pitching Subsystem with MRAC

The closed loop step response of pitch subsystem with is shown in figure 5.8.

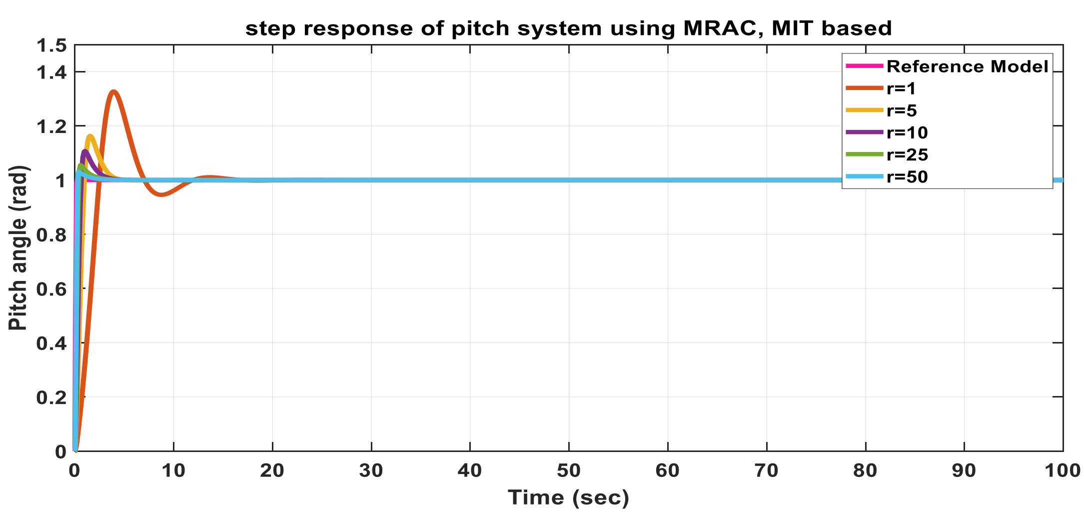


Figure 5. 8: Step response of Pitching system ($\theta(\mathbf{t})$) using MRAC based on gradient method

To control pitch subsystem, MRAC is designed based on gradient method with adaptation gains 1up to 50. The response of this system is different at different values of adaptation gain, so that selection of proper adaptation gain is necessary to get good performance characteristics. From figure 5.8, the performance characteristics of the system with adaptation gain less than 5 is unsatisfactory and satisfactory higher than 5. Therefore with $5 \leq \gamma \leq 50$, the plant response follows to the reference model response.

The control signal for step change of pitch system under gradient based MRAC with $\gamma = 1$, is shown in figure 5.9.

The control signal of pitch system of the quadrotor using MRAC based on gradient method shown in figure 5.9 becomes zero after 20 seconds. This implies that the response of pitch is not ideally follow the reference model, it is still unstable and unsettled; so that taking of a control action is required.

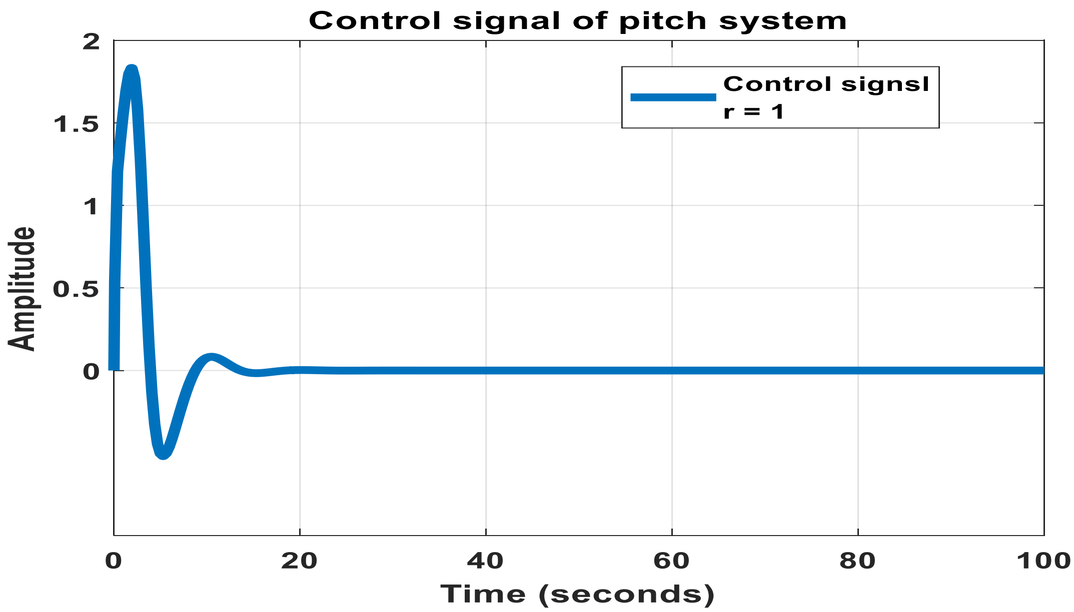


Figure 5. 9: Control signal for step change of Pitch system with $\gamma = 1$

The control signal for step change of pitch system under gradient based MRAC with $\gamma = 25$, is shown in figure 5.10. The control signal for step change of the pitch system under the developed controller becomes zero with in one second. This implies that the plant output follows exactly to the reference output which means the plant is stable, settled and fast. Therefore, taking of any controller action is not needed.

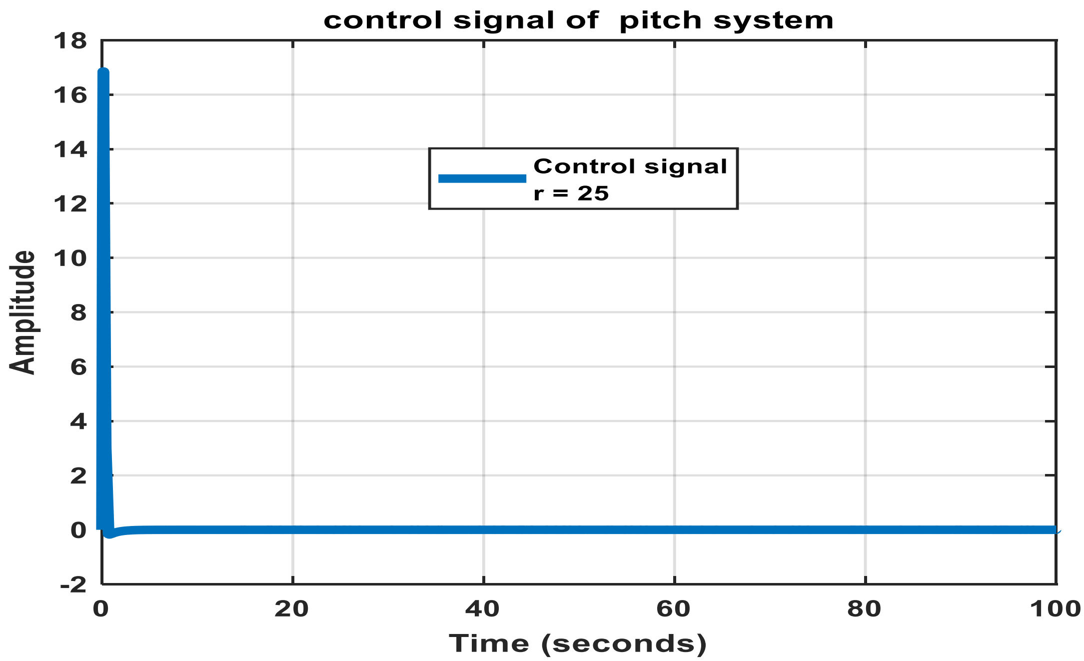


Figure 5. 10: Control signal for step change of Pitch system with $\gamma = 25$

5.2.3 Step Response of SISO Yaw Subsystem with MRAC

The closed loop step response of the yaw system with MRAC based on gradient method is shown in figure 5.11. This Shows that the response of yaw system for different values of adaptation gain. Here, the controller is applied to control yaw system with adaptation gain from 1 up to 200. From figure 5.11, the performance characteristics of yaw subsystem with adaptation gain less than 100 is not good. So, the plant output does not follow the reference output, but MRAC with $\gamma \geq 100$ gives satisfactory performance. Therefore $100 \leq \gamma \leq 200$ are the range of adaptation gain to get good performance.

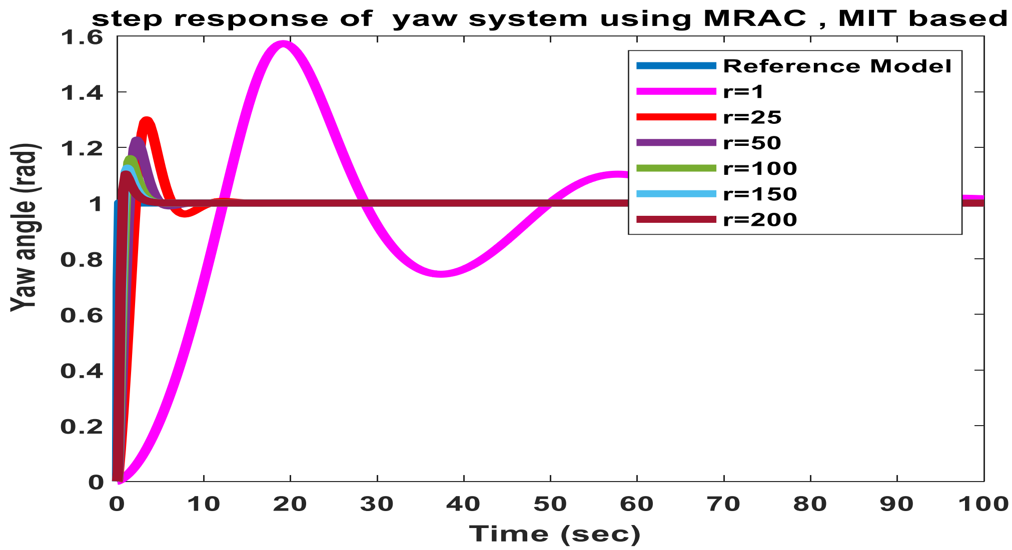


Figure 5. 11: Step response of Yaw system using MRAC based of gradient method.

The control signal for step change of yaw subsystem under the developed controller is shown in figure (5.12) and (5.13).

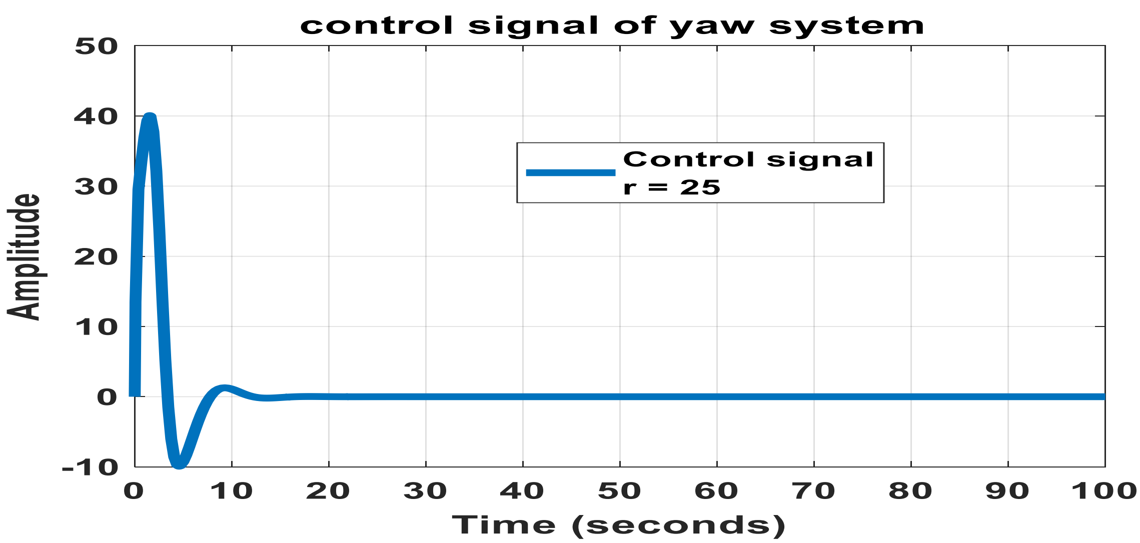


Figure 5. 12: Control signal for step change of yaw system with γ=25

The control signal for step change of yaw system under gradient based MRAC with $\gamma =$ 25 as shown in figure 5.12 becomes zero after 13 seconds. This implies that taking of a control action is needed in order to stabilize, settle and to make fast response. So, the system is unstable, unsettled, having unacceptable overshoot and slow response so that taking control action is needed.

The control signal of the yaw system under the developed controller becomes zero in less than one second with $\gamma = 200$ as shown in figure 5.13. This implies that output of plant exactly follows the reference output. Here, the plant is stable, settle and fast so, taking of any control action is not needed.

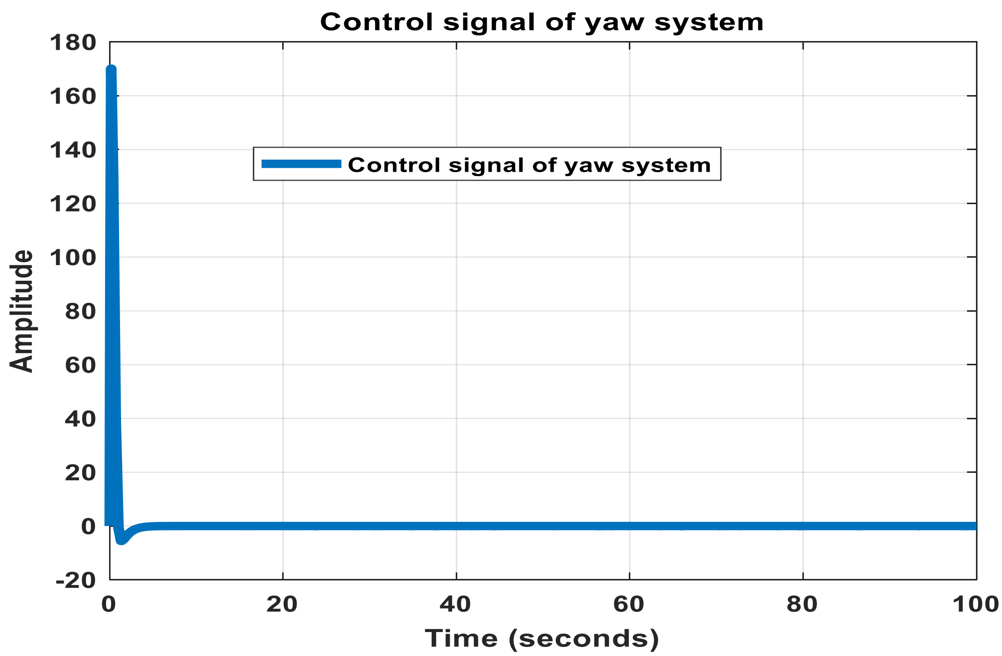


Figure 5. 13: Control signal for step change of Yaw system with $\gamma = 200$

5.2.4 Step Response of SISO Altitude Subsystem with MRAC

The instability of the altitude subsystem of the quadrotor is stabilize using the model reference adaptive control system based on gradient method of adaptation mechanism. The closed loop step response of the altitude subsystem using MRAC based on gradient Method is shown in figure 5.14. Here, MRAC is applied with adaptation gains 1 up to 50. The response of this system is not good performance characteristic under adaptation gain less than 15, but it gives a satisfactory performance with gain higher than 15.

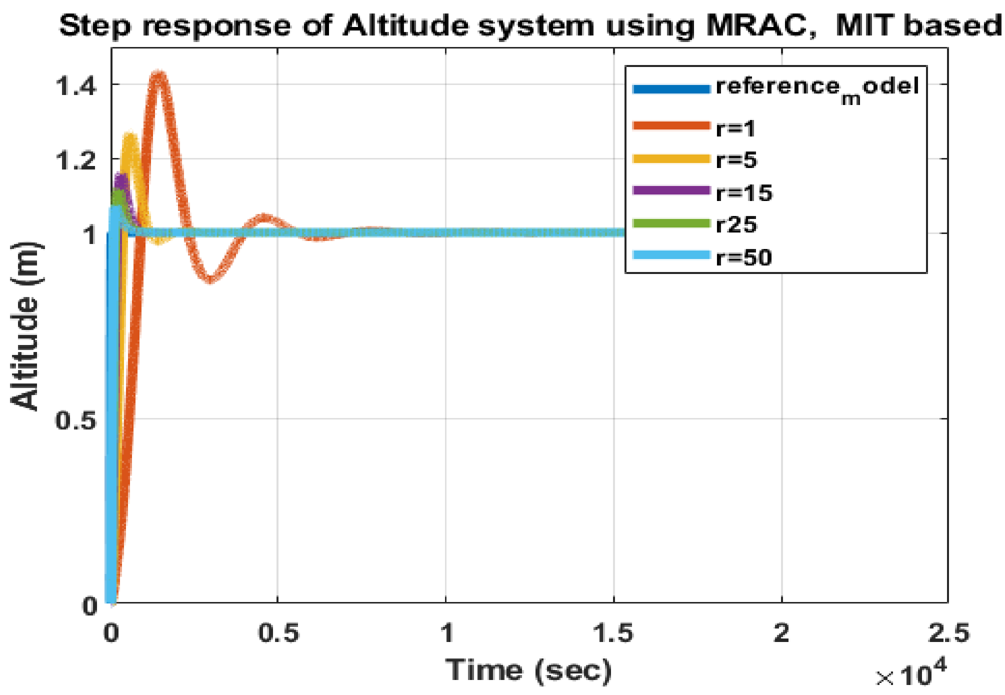


Figure 5. 14: Step response of Altitude system using MRAC based on Gradient

The control signal for step change of altitude system is shown in figure 5.15 and 5.16 under $\gamma = 1$ and $\gamma = 25$.

From figure 5.15, the control signal for step change of altitude system becomes zero after 30 seconds. So, taking of control action is required in order to stabilize and settle the altitude subsystem.

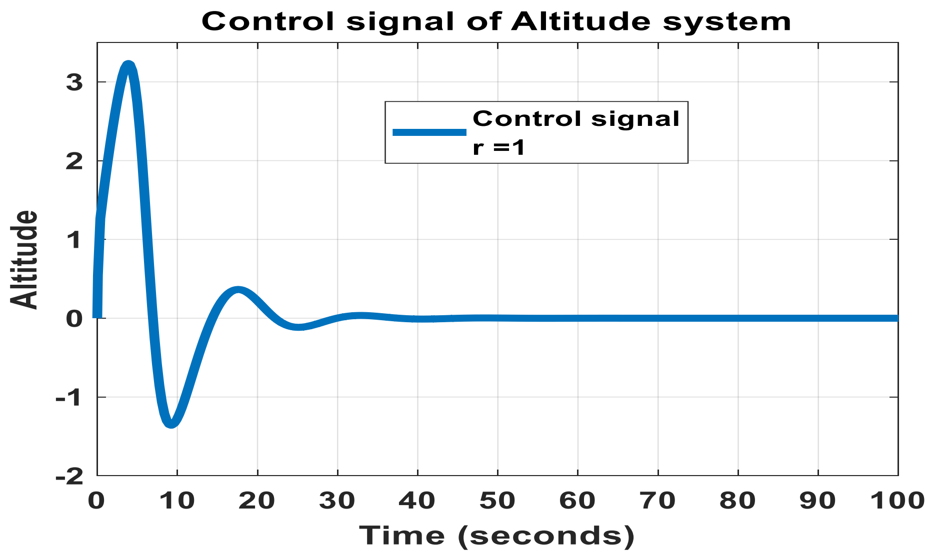


Figure 5. 15: Control signal for step change of Altitude system with $\gamma = 1$

From figure 5.16, the control signal for step change of altitude signal becomes zero almost after one second. This implies that the output of plant follows exactly to the reference output and the system is stable, settled and fast response. So that there is no any need of taking a control action

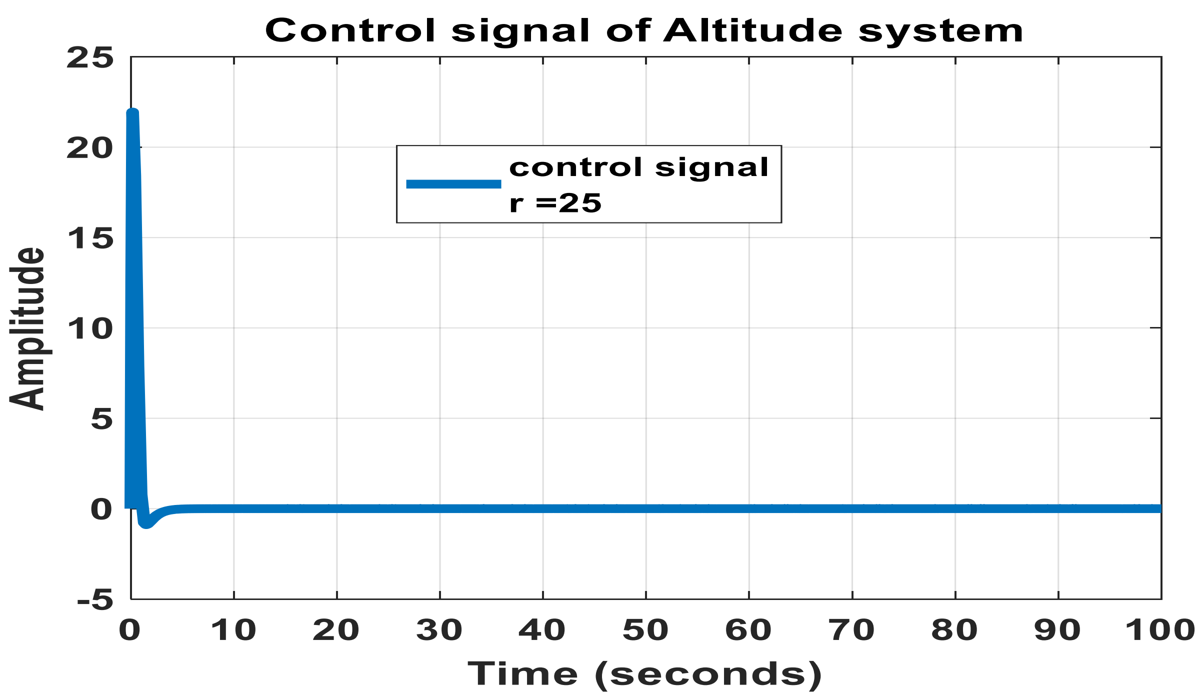


Figure 5. 16: Control signal for step change of Altitude system with $\gamma = 25$

CHAPTER SIX

6 CONCLUSION AND RECOMMENDATION

6.1 Conclusion

In this thesis, we have designed four MRAC based on gradient method to control the roll, pitch, yaw and altitude subsystems of the quadrotor. The design procedure consists the following steps. First small angle approximation technique is applied to the model to simplify the trigonometric function as much as possible. Second, linearization of the dynamic model is done using the jacobian method at the selected operating points. Finally, MRAC based on gradient method of adaptation mechanism is developed for the linearized model.

Generally, from the simulation results the dynamic performance characteristics of the systems are greatly improved under gradient based MRAC. The improvements are as follows:

- The rolling and altitude subsystem are improved by overshoot < 20%, settling time < 0.15 second, rising time < 0 .5 second with the range of adaptation gain is $10 \leq \gamma \leq 50$
- The pitch subsystem is improved by overshoot < 20%, settling time < 0.15 second, rising time < 0 .5second with the range of adaptation gain is $5 \leq \gamma \leq 50$
- The yaw subsystem is improved by overshoot < 20%, settling time < 0.15 second, rising time < 0.5second with the range of adaptation gain is $100 \leq \gamma \leq 200$

Therefore, the obtained transient performance characteristics using MRAC based on gradient method is good. The system is also stable and fast with zero steady state error. In general, the expected result from MRAC is attained effectively.

6.2 Recommendation

In this thesis, the MRAC based of gradient method is designed for the linearized dynamic model of the quadrotor. As a future work, since dynamic model is available, without linearization technique, system performance can be further improved by using more advanced nonlinear controller like back-stepping controller and Considering the disturbance to the dynamic model of the system is great for further improvement.

Finally, if all components are available this thesis can be implemented practically.

References

[1] http://krossblade.com/history-of-quadcopters-and-multirotors/.

[2] KimonP.Valavanis, GeorgeJ.Vachtsevanos, ‘’Handbook of Unmanned Aerial Vehicles ‘’second edition, vol.4 pp 234-356, June 2014.

[3] T. Yucelen and A. J. Calise, Derivative-free model reference adaptive control, Journal of Guidance, Control and Dynamics, 2011.

[4] Anderson, S. B., “Historical Overview of V/STOL Aircraft Technology,” NASA Technical Memorandum 81280, Ames Research Center, Moffett Field, CA, March 1981.

[5] M. A. Ma’sum, M. K. Arrofi, G. Jati, F. Arifin, M. N. Kurniawan, P. Mursanto, and W. Jatmiko, ”Simulation of intelligent unmanned aerial vehicle (UAV) for military surveillance”, International Conference on Advanced Computer Science and Information Systems, pp. 161-166, 2013.K.

[6] J. Astrom, B. Wittenmark, “Adaptive Control”, Lund Institute of Technology New York, 2nd ed, 2008.

[7] Y. B. He, H. B. Zhou, H. W. Yue, G. T. Qiu, and Z. Z. Lin, Proceedings Chinese Guidance, Navigation and Control Conference, Acceleration-based Multiple-loop Control of Unmanned Quadrotors with Disturbances, Nanjing, China, IEEE August 12-14, 2016

[8] Yuebang He, Hongtao Wang, Hong Man, Tianlei Wang, Min Yang, Quadrotor Control Design with Attitude Constraints, Proceedings of the 37th Chinese Control Conference Wuhan, china, July 25-27, 2018.

[9] Hossein Bolandi1, Mohammad Rezaei1, Reza Mohsenipour2, Hossein Nemati1, Seed Majid Smailzadeh1, Attitude Control of a Quadrotor with Optimized PID Controller, scientific research Intelligent Control and Automation, December 9, 2012.

[10] Qingsong Jiao1,2, Jia Liu1,2, Yunxi Zhang1,2, Wenyu Lian1,2,’’ Analysis and Design the Controller for Quadrotors Based on PID Control Method’’, the 33rd Youth Academic Annual Conference of Chinese Association of Automation (YAC) Nanjing, china; May 18-20, 2018.

[11] Farhad Parivash and Ali Ghasemi,’’Trajectory Tracking control For A Quadrotor Using Fuzzy PID control Scheme’’, IEEE 4th International conference on knowledge based - Engineering and Innovation (KBEI)’’, Iran University of Science and Technology, Tehran, December, 22nd, 2017.

[12] Wenyan Yu, Jun Li and Kunlin Yang,'' Research on Fuzzy Adaptive Stabilization PID Control System'', 2018 IEEE 3rd Advanced Information Technology, Electronic and Automation Control Conference (IAEAC 2018).

[13] Khoi Nguyen Dangl[1], Gigun Lee[2] and Taesam Kang[3], Linear Quadrotor Modelling and Attitude Controller Design Based on Experimental Data, 2015 15th International Conference on Control, Automation and Systems (ICCAS 2015) Oct. 13-16, 2015 in BEXCO, Busan, Korea.

[14] Bin Fan, Jia Sun and Yao Yu, ''LQR controller for a Quadrotor Design and Experiment'',31st youth Academic Annual Conference of Chinese Association of Automation Wuhan, china; November vol 4, pp.11-13, 2016.

[15] B. Panomrattanarug, K. Higuchi and F. Mora-Camino," Attitude control of a quad rotor aircraft using LQR state feedback controller with full order state observer", Proceedings of the SICE Annual Conference 2013, Nagoya, Japan, September 2013.

[16] Parthibi Dey, Shailaja R. Kurod and Raghu Ramachandran, Robust Attitude Control of Quadrotor using Sliding Mode, International Conference on Automatic Control and Dynamic Optimization Techniques (ICACDOT) International Institute of Information Technology (I²IT), Pune., December, 2016

[17] L. Luque-Vega, B. Castillo-Toledo and Alexander G. Loukianov,'' Block Linearization Control of a Quadrotor Via Sliding Mode'', American Control Conference Fairmont Queen Elizabeth, Montréal, Canada, June 27- 29, 2012.

[18] Hamid Saeed Khan and Muhammad Bilal Kadri, ''Attitude and Altitude Control of Quadrotor by Discrete PID control and Non-linear Model Predictive Control''.

[19] Zhou Fang, Zhang Zhi, Liang Jun and Wang Jian,'' Feedback Linearization and Continuous Sliding Mode Control for a Quadrotor UAV'', Proceedings of the 27th Chinese Control Conference, Kunming, Yunnan, China; July 16-18, 2008.

[20] M. Huang, B. Xian, C. Diao, K. Yang and Yu Feng, Adaptive tracking control of underactuated quadrotor unmanned aerial vehicles via backstepping, 2010 American Control Conference, Baltimore, MD, USA, June 30-July 02, pp. 2076-2081.

[21] Zhou Fang, Zhang Zhi, Liang Jun and Wang Jian,'' Feedback Linearization and Continuous Sliding Mode Control for a Quadrotor UAV'', Proceedings of the 27th Chinese Control Conference, Kunming, Yunnan, China; July 16-18, 2008.

[22] R. Xu and U. Ozguner," Sliding mode control of a quad-rotor helicopter", Proceedings of the IEEE 45th Conference on Decision & Control, San Diego, CA, USA, December 2006.

[23] Matthias Schreier, Modeling and Adaptive Control of a Quadrotor, Proceedings of 2012 IEEE International Conference on Mechatronics and Automation, August 5 - 8, Chengdu, China.

[24] T. Yucelen and A. J. Calise, Derivative-free model reference adaptive control, Journal of Guidance, Control and Dynamics, Chengdu, china, September, 2011.

[25] K. J. Astrom, B. Wittenmark, "Adaptive Control", Lund Institute of Technology New York, 2nd ed, 2008.

[26] C.-C. Hang and P. Parks, Comparative studies of model reference adaptive control systems, Automatic Control, IEEE Transactions, vol. 96 no. 1 pp. 2956-2641, 2011.

[27] F. Solc, "Modelling and control of a quadcopter", Advanced in Military Technology, vol. 5, pp. 29-38, 2010.

[28] Qingsong Jiao1,2, Jia Liu1,2, Yunxi Zhang1,2, Wenyu Lian1,2,'' Analysis and Design the Controller for Quadrotors Based on adaptive Control Method'', the 33rd Youth Academic Annual Conference of Chinese Association of Automation (YAC) Nanjing, china; May 18-20, 2018.

[29] Ivana Palunko, Rafael Fierro," Adaptive Control of a Quadrotor with Dynamic Changes in the Center of Gravity", The International Federation of Automatic Control, vol. 44 no. 1 pp. 2626-2631, fedhyia, Russia, august, 2011.

[30] S. Bouabdallah, A. Noth and R. Siegwan, "PID vs LQ Control Techniques Applied to an Indoor Micro Quadrotor," in Proceedings of 2004 1EEElRS.J Internationel Conference on Intelligent Robots and Systems, Sendal, Japan, September 28. October 2,2004.

[31] Online: https://www.e-education.psu.edu/geog892/node/5.

[32] EAN-JACQUES E. SLOTINE Weiping Li'' Applied nonlinear control'' Massachusetts Institute of Technology, second edition, vol.4 pp.30-45.